Asian Research Journal of Mathematics

8(1): 1-14, 2018; Article no.ARJOM.37767
ISSN: 2456-477X

Modeling of Blood Flow through Stenosed Artery with Heat in the Presence of Magnetic Field

K. W. Bunonyo[1*], C. Israel-Cookey[2] and E. Amos[3]

[1]*Department of Mathematics and Statistics, Federal University Otuoke, Yenagoa, Nigeria.*
[2]*Department of Physics, Rivers State University, Port Harcourt, Nigeria.*
[3]*Department of Mathematics, Rivers State University, Port Harcourt, Nigeria.*

Authors' contributions

This work was carried out in collaboration between all authors. Authors CIC and KWB designed the study, performed the mathematical formulations. Authors EA and KWB proffered the solutions, presented the plots and interpreted the results. Author KWB managed literature searches and wrote the first draft of the manuscript. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ARJOM/2018/37767
Editor(s):
(1) Cenap Ozel, Mathematics, King Abdulaziz University (KAU), Saudi Arabia and Dokuz Eylul University, Turkey.
Reviewers:
(1) Farhad Ali, City University of Science & Information Technology, Pakistan.
(2) Grienggrai Rajchakit, Maejo University, Thailand.
(3) Jagdish Prakash, University of Botswana, Botswana.
(4) John Abraham, University of St. Thomas, USA.
(5) Don Liu, Louisiana Tech University, USA.
Complete Peer review History: http://www.sciencedomain.org/review-history/22654

Original Research Article

Received: 28th October 2017
Accepted: 16th December 2017
Published: 9th January 2018

Abstract

An investigation of an oscillatory blood flow in an indented artery with heat source in the presence of magnetic field was carried out. The formulated governing models are solved using Frobenius method where the solutions are transformed into Bessel functions $I_0(\beta r)$ and $K_0(\beta r)$ of order zero of the first and second kind. The computational results are presented graphically for the velocity profile $w(r,t)$, the temperature profile $\theta(r)$. The study reveals that the blood flow is appreciably influenced by the presence of a magnetic field and also by the value of the Grashof Gr number. It is observed that the presence of the magnetic field M retards the velocity profile as well as the flow rate; the Grasof number Gr causes an increment in the velocity profile which is consistent with the existing laws of physics. Furthermore, the radiation parameter Rd does affect the velocity profile which means, it

*Corresponding author: E-mail: wilcoxonnetb@yahoo.com, wilcoxbk@fuotuoke.edu.ng;

is a welcome development in thinning of the blood for effective treatment of cardiovascular disease considering the standard Prandtl number of blood ($Pr = 21$). In addition, the temperature of the fluid is caused to rise due to an increase in radiation parameter irrespective of the height of the stenosis.

Keywords: Modeling; blood; Stenosis; heat; Darcy number.

Nomenclatures

(u', v', w') :	*Dimensional velocity components*
z' :	*Dimensional axisymetric direction of the flow*
r' :	*Dimensional radius of the artery*
t' :	*Dimensional time*
T_w :	*Dimensional wall temperature*
T' :	*Dimensional Fluid Temperature*
T_∞ :	*Fluid temperature at the stent region*
τ_w :	*Shear Stress at the wall of the artery*
M :	*Hartmann number*
B_0 :	*Magnetic field parameter*
k :	*Porosity*
Rd :	*Radiative heat source parameter*
Pr :	*Prandtl number*
Gr :	*Grashoff number*
C_p :	*Specific heat capacity of the fluid at constant pressure*
q_r' :	*Radiative heat flux*
Da :	*Darcy number or porosity parameter*
$R(z)$:	*Radius of the indented region of the artery*
R_0 :	*Radius of the normal artery*
L_0 :	*Length of the artery under investigation*
a :	$a = \frac{R}{R_0}$
d :	*Distance of the onset of stenosis*
k_T :	*Thermal conductivity*
$w_0(r)$:	*Dimensionless velocity*
$w(r,t)$:	*Velocity profile of the fluid*

Greek Symbols

$\theta_w = \frac{T_w}{T_\infty}$:	*Dimensionless wall temperature distribution*
ν :	*Kinematic viscosity*

μ :	*Dynamic viscosity of the fluid*
g :	*Acceleration due to gravity*
ρ :	*Density of the fluid*
δ :	*Maximum height of stenosis*
σ_c :	*Electrical conductivity*
ω:	*Oscillatory frequency*
β_T :	*Coefficient of volume expansion due to temperature*
β, λ :	*Modified radiation parameters*
P_0 :	*Oscillatory pressure*
θ_0 :	*Dimensionless temperature*
$\theta(r)$:	*Temperature profile*

1 Introduction

Mathematical investigation of blood flow through a permeable channel/vessel is of great concern for clinical researchers and has drawn genuine consideration of specialists. The investigation of a permeable medium is imperative both from in theoretical and practical point of view; in light of the majority of the characteristic stream, issues are joined with the permeable medium.

Ahmadi and Manvi [1] inferred a general condition of movement for the stream of thick liquid through a permeable medium. Numerous organic tissues, for example, bones and vascular tissues, the renal framework and in addition the veins containing greasy stores are thought to be permeable by nature. Mekheimer and Al-Arabi [2] investigated "the mathematical model to decide the attributes of peristaltic transport of magnetohydrodynamic flow through a permeable medium. A pulsatile stream of blood through a stenosed permeable medium has been studied by El-Shahed [3]. A numerical model to think about the impact of permeable parameter and stature stenosis on the divider shear push has been examined by Misra and Verma [4]. Bhargava et al. [5] numerically examined the pulsatile stream and mass exchange of an electrically conducting Newtonian biofluid through a channel with a permeable medium. Rita and Jyoti Das [6] have studied the effect of heat transfer on MHD oscillatory viscoelastic fluid flow in a channel through a porous medium. The body acceleration assumes an essential part of the blood stream in the artery, such body increasing acceleration is typically caused unexpectedly, e.g. move by road vehicles, aircraft or space shuttle. Sud and Sekhon [7] examined the pulsatile stream of blood through an inflexible circular tube subject to body quick movement, regarding blood as a Newtonian fluid. Mishra and Sahu [8] analyzed the stream of blood through extensive artery under the activity of intermittent body increasing acceleration. Chaturani and Palanisamy [9] investigated the pulsatile flow affected by occasional body acceleration up by assuming blood as a Power-law liquid. Kyoung et al. [10] examined the impact of an occasional speeding up and turning impacts in a stenosed vein. The blood stream and divider shear push are changed under body development or quickening variety. Biswas and Chakraborty [11] investigated pulsatile stream of blood in a tightened vein with body acceleration and watched that the acceleration and stream rate increments yet successful thickness diminish, because of a slip divider.

A mathematical model with a specific aim was considered the qualities of the non-Newtonian blood flow through an adaptable decreased artery in the presence of stenosis subject to the pulsatile pressure gradient has been examined by Mandal [12]. Makinde and Mhone [13] have considered heat transfer to MHD oscillatory flow in a channel filled with porous medium. Unsteady heat transfer to MHD oscillatory flow through a porous medium under slip condition was investigated by Hamza et al. [14]. Many researchers Rao et al. [15] feel that the hydrodynamic factors may be helpful in the diagnosis, treatment and fundamental understanding of many disorders. Mathur & Jain [16] developed a mathematical model for studying the non-Newtonian flow of blood through a stenosed arterial segment. The problem was solved using analytical and results displayed graphically for different flow characteristics like shear stress and velocity profile. Misra & Shit Misra, [17] investigated blood flow through arterial segment assuming blood as Hershal-Bulkley fluid. They obtained that the skin-friction and the resistance to flow is maximum at the throat of the stenosis and minimum at the end. Ali et al. [18] analyzed the effect of an axially symmetric time-dependent growth into the lumen of a tube for the constant cross-section through which a Newtonian fluid is steadily flowing. They investigated the structure of flow through the arterial model with one or two sinusoidal stenosis, assuming the arterial blood flow is quasi-steady.

Al Khatib and Wilson [19] have studied the Poiseuille flow of a yield stress fluid in a channel. The flow of a visco-plastic fluid in a channel of slowly varying width was studied by Frigaard and Ryan [20]. Aamir Ali and Saleem Asghar [21] have analyzed by oscillatory channel flow for non- Newtonian fluid. Mekheimer and El Kot [22] investigated an axisymmetric blood flow through an axially nonsymmetric but radially symmetric mild stenosis tapered artery.

Sun et al. [23] investigated the interaction between components in the cardiovascular system; it observed that changes to vascular pressure induce a compliant response in the vessel during systolic and diastolic stages. They concluded that interaction could be influenced by the disease state of the artery and plaque reduces arterial compliance. Vallez et al. [24] carried out numerical analysis of the influence of plaque on the compliance of an arterial wall. In the investigation the systemic variation of transluminal pressure difference and plaque thickness using four different transluminal pressure variations. It is shown that healthy artery possessed the largest arterial compliance.

Plourde et al. [25] carried out a numerical study to quantify plaque removal on blood flow in the popliteal artery. They used atherectomy device to partially remove a calcified plaque layer. It is found that the removal of plaque layer by orbital atherectomy increases the blood flow through the artery due to the drop in systolic pressure by 2.5 times less than prior to treatment. Plourde et al. [26] carried out a simulation of pressure and velocity fields throughout an artery before and after removal of plaque using orbital atherectomy plus adjunctive balloon angioplasty or stenting. Calculations were carried out with unsteady computational fluid dynamic solver that allows the fluid to naturally transit to turbulence. The results showed that the atherectomy procedure leads to an increased flow rate through the stenotic region with a coincident decrease in pressure drop across the stenosis.

Sparrow et al. [27] examined the infusion catheters with a balloon; it is observed that the shear stress is increased in the fluid that passed through the lumen. The study further investigated the effect of a new injection catheter design which is indented to resist the deleterious effect of balloon compression on cell viability for various flow rates.

Liu et al. [28] investigated the sediment of particles in flow with shear oscillations. The developed a mathematical model to simulate sediment particles motion in shear and oscillatory flow by combining Eulerian and Lagrangian approaches, the model is capable of simulating the motion of each individual particle grain as well as a bulk of numerous grains in the fluid.

The aim of the article is to investigate an oscillatory blood flow in an indented artery with a heat source in the presence of magnetic field. The expression for velocity profile and temperature profile are obtained analytically and discussed graphically. In addition, the effect of the pertinent parameters on the velocity profile and temperature profile are also discussed.

2 Mathematical Formulation

We consider an artery as presented in as a cylindrical channel that is porous and allows absorption of heat to promote physiotherapeutic treatment as a means of controlling stenotic blood flow as shown schematically in Fig. 1. The blood is also considered to be a Newtonian fluid, incompressible, but flow is in the horizontal directions that are, in parallel direction, based on the fact that there are four basic valves in the cardiovascular system that prevents back flow in the presence of magnetic field with heat source are presented and to be solved analytical.

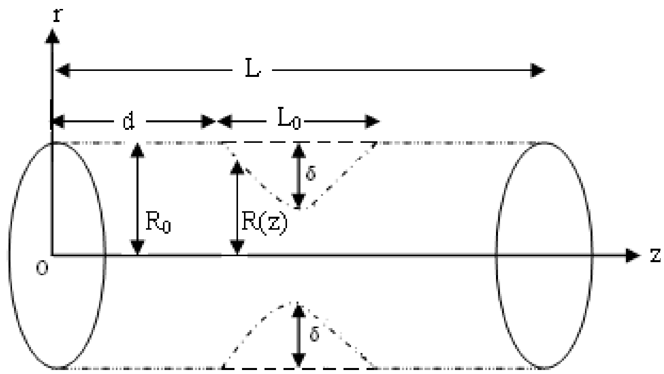

Fig. 1. Geometry of the problem

It is assumed that the fluid has small electrical conductivity and electromagnetic force produced is also very small. Take a cylindrical coordinate system (r,θ,z) where r lies along the centre of the channel, $R \approx R(\mathrm{z})$ is the artery radius in the obstructed region, R_0 is radius of normal artery, L_0 is the length of stenosis, d is the location of the stenosis onset, L is the length of the artery, δ is the maximum height of the stenosis, the dimensional axial velocity of the fluid (blood) is in the direction of $z \equiv z'$ is the axial coordinate at time.

Then, assuming a Boussinesq incompressible fluid model, the governing equations of the motion are given as:

$$\rho\frac{\partial w'}{\partial t'} = -\frac{\partial p'}{\partial z'} + \mu\left(\frac{\partial w'^2}{\partial r'^2} + \frac{1}{r'}\frac{\partial w'}{\partial r'}\right) - \sigma_c B_0'^2 w' + \rho\beta_T(T' - T_\infty)\vec{g} - \frac{\mu}{k}w' \tag{1.1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k_T}{\rho C_p}\left(\frac{\partial T'^2}{\partial r'^2} + \frac{1}{r'}\frac{\partial T'}{\partial r'}\right) - \frac{q_r'}{\rho C_p}(T' - T_\infty') \tag{1.2}$$

$$\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'u'\right) + \frac{1}{r'}\frac{\partial v'}{\partial \theta'} + \frac{\partial w'}{\partial z'} = 0 \tag{1.3}$$

$$\overrightarrow{J'} \times \overrightarrow{B'} = \sigma_c(\overrightarrow{V'} \times \overrightarrow{B'}) \times \overrightarrow{B'} = \sigma_c(\overrightarrow{V'} \times B_0') \times (0,0,B_0') = -\sigma_c B_0'^2 w' \tag{1.4}$$

The corresponding boundary conditions to equations (1.1) – (1.4) are

$$\begin{aligned} &w' = 0\ ,\ \ T' = T_w \ \text{ at } r' = R(z) \\ &w' = 0,\ \ T' = T_\infty \ \text{ at } r' = R_0 \end{aligned} \tag{1.5}$$

where w' is the dimensional axial velocity of the fluid (blood) in the direction of $z \equiv z'$ at time t' in the flow field, ρ is the density of blood, k_T is the thermal conductivity, T' is the temperature in the radial direction i.e. $r \equiv r'$, k is the porosity of the stenosed region, β_T is the volumetric expansion parameter, σ is the electrical conductivity of the blood, μ is the dynamic viscosity of the fluid under investigation, q_r' is the quantity of heat and C_p is the specific heat capacity of at constant pressure.

The following dimensionless variables and parameters were introduced:

$$\left.\begin{aligned} & r=\frac{r'}{R_0}, z=\frac{z'}{R_0}, \theta=\frac{T'-T_\infty}{T_\infty}, w=\frac{w'R_0}{\nu}, t=\frac{t'\nu}{R_0^2}, P=\frac{p^* R_0^2}{\mu\nu}, \\ & Pr=\frac{\mu C_p}{k_T}, Gr=\frac{g\beta R_0^3}{\nu^2}T_\infty, Da=\frac{k}{R_0^2}, \\ & M=B_0 R_0\sqrt{\frac{\sigma_c}{\mu_e}}, Rd=\frac{q_r' R_0^2}{\mu C_p} \end{aligned}\right\} \tag{1.6}$$

where w is the stream velocity. The dimensionless governing equations together with the appropriate boundary conditions, (neglecting the bars for the sake of mathematical simplification) can be written as:

$$\frac{\partial w}{\partial t}=-\frac{\partial p*}{\partial z}+\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)-M^2 w+Gr\theta-\frac{w}{Da} \tag{1.7}$$

$$Pr\frac{\partial \theta}{\partial t}=\left(\frac{\partial^2 \theta}{\partial r^2}+\frac{1}{r}\frac{\partial \theta}{\partial r}\right)-RdPr\theta \tag{1.8}$$

The boundary conditions in dimensionless forms can be written as:

$$\begin{aligned} & w=0, \theta=\theta_w \quad at \quad r=a \\ & w=0, \theta=1 \quad at \quad r=1 \end{aligned} \tag{1.9}$$

where M, Gr, Da, Pr, Rd are Hartmann number, Grashof number, Darcy’s number, Prandtl's number and Radiation parameters respectively.

2.1 Method of solution

Since the flow of blood through an artery is largely dependent on the pumping action of the heart and it gives rise to an oscillatory pressure gradient, we can then represent the pressure gradient as

$$-\frac{\partial p*}{\partial z}=P_0 e^{i\omega t} \tag{1.10}$$

and define the velocity and temperature profiles as

$$w(r,t)=w_0(r)e^{i\omega t} \tag{1.11}$$

$$\theta(r,t) = \theta_0(r)e^{i\omega t} \tag{1.12}$$

where P_0 is constant pressure, ω is the angular frequency of the oscillation. Substituting the above expression in equations (1.10 – 1.12) into equations (1.7) – (1.9), we obtain:

$$\left(\frac{\partial^2 w_0}{\partial r^2} + \frac{1}{r}\frac{\partial w_0}{\partial r}\right) - \beta^2 w_0 + Gr\theta_0 = -P_0 \tag{1.13}$$

$$\left(\frac{\partial^2 \theta_0}{\partial r^2} + \frac{1}{r}\frac{\partial \theta_0}{\partial r}\right) - \lambda^2 \theta_0 = 0 \tag{1.14}$$

with the corresponding boundary:

$$\begin{aligned} w_0 &= 0, \theta_0 = \theta_w \mathrm{e}^{-i\omega t} \; at \;\; r = a \\ w_0 &= 0, \theta_0 = e^{-i\omega t} \quad at \;\; r = 1 \end{aligned} \tag{1.15}$$

where $\beta^2 = \left(M^2 + \frac{1}{Da} + i\omega\right)$ and $\lambda^2 = \left(Rd + i\omega\right)Pr$

Equations (1.13) to (1.14) are solved and the solutions for fluid velocity and temperature profiles are given as follows:

$$\theta(r) = \begin{bmatrix} \left(\dfrac{\theta_w K_0(\lambda) - K_0(\lambda a)}{I_0(\lambda a)K_0(\lambda) - K_0(\lambda a)I_0(\lambda)}\right) I_0(\lambda r) \\ + \left(\dfrac{I_0(\lambda a) - \theta_w I_0(\lambda)}{I_0(\lambda a)K_0(\lambda) - K_0(\lambda a)I_0(\lambda)}\right) K_0(\lambda r) \end{bmatrix} \tag{1.16}$$

$$w(r,t) = \begin{bmatrix} \left\{\left(\dfrac{B_1}{A_4}\right)K_0(\beta) - \left(\dfrac{B_2}{A_4}\right)K_0(\beta a)\right\} I_0(\beta r) + \left\{\left(\dfrac{B_2}{A_4}\right)I_0(\beta a) - \left(\dfrac{B_1}{A_4}\right)I_0(\beta)\right\} K_0(\beta r) \\ + \dfrac{P_0}{\beta^2} + \dfrac{Gr}{\beta^2}\left\{\left(\dfrac{A_1}{A_3}\right)I_0(\lambda r) + \left(\dfrac{A_2}{A_3}\right)K_0(\lambda r)\right\} \end{bmatrix} e^{i\omega t} \tag{1.17}$$

where the value for the constants $A_1, A_2, A_3, A_4, B_1, B_2$ obtained using the boundary conditions and is stated in appendix A.

3 Results

We would discuss the graphical results presented in the preceding section; from the formulated, analytically solved governing equations of the problem, modelling of an oscillatory blood flow with heat source in the presence of magnetic field and carry out the investigation using Mathematic 10.3.

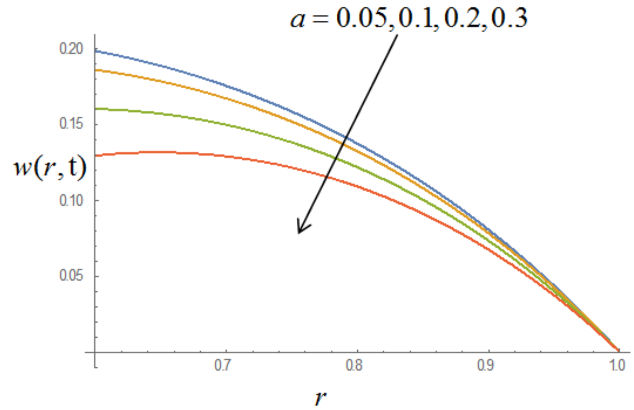

Fig. 2. Velocity profile $w(r,t)$ **against** r **with variation of** a **, leaving** $\theta_w = 0.2, Pr = 21, Gr = 10, Da = 0.5\ \omega = 0.01, Rd = 0.2,\ \text{M} = 2, t = 0.5$.

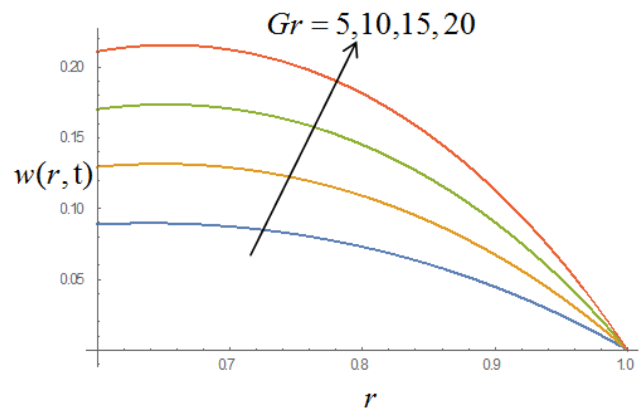

Fig. 3. Velocity profile $w(r,t)$ **against** r **with variation of** Gr**, leaving** $a = 0.3, \theta_w = 0.2, \text{Pr} = 21, Da = 0.05\ \omega = 0.2,\ \text{M} = 2, t = 0.5, Rd = 0.2$.

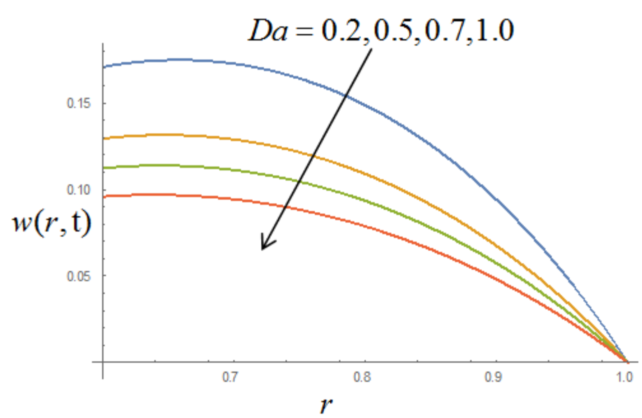

Fig. 4. Velocity profile $w(r,t)$ **against** r **with variation of** Da**, leaving** $a = 0.3, Rd = 0.2, \theta_w = 0.2\ \omega = 0.01\ \text{M} = 2, t = 0.5, Pr = 21, Gr = 10$ **constant**

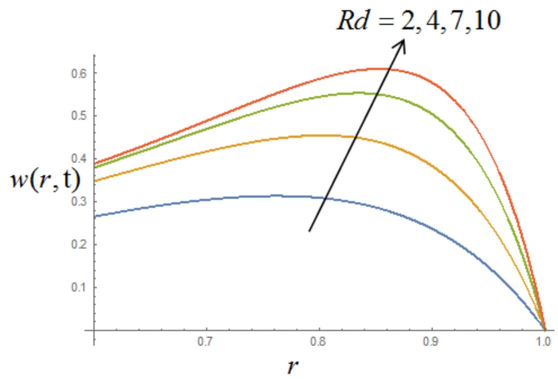

Fig. 5. Velocity profile $w(r,t)$ against r with variation of Rd, leaving
$\theta_w = 0.2, Gr = 10,\ Da = 0.2\ \omega = 0.01\ \mathrm{M} = 2, t = 0.5, Pr = 21, a = 0.3$

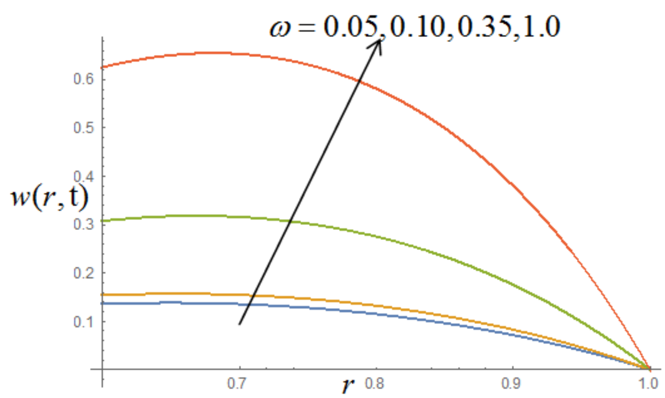

Fig. 6. Velocity profile $w(r,t)$ against r with variation of ω, $a = 0.3,\ Rd = 0.2$
$Da = 0.5\ \theta_w = 0.2, Gr = 10,\ t = 0.01, Pr = 21,\ \mathrm{M} = 2,$ **constant**

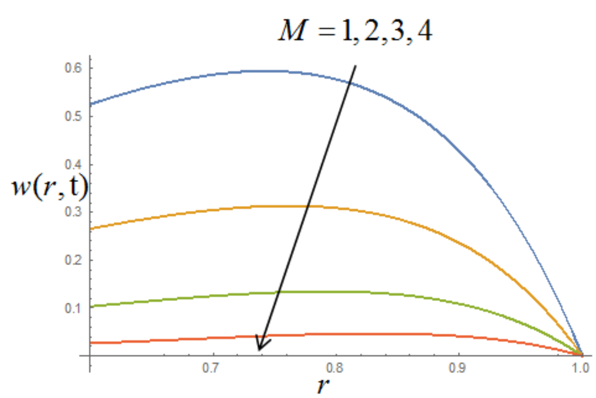

Fig. 7. Velocity profile $w(r,t)$ against r with variation of M, leaving $a = 0.3,$
$Da = 0.2,\ \omega = 0.01,\ Rd = 0.2,\ Gr = 10,\ \theta_w = 0.2,\ Pr = 21,\ t = 0.5$

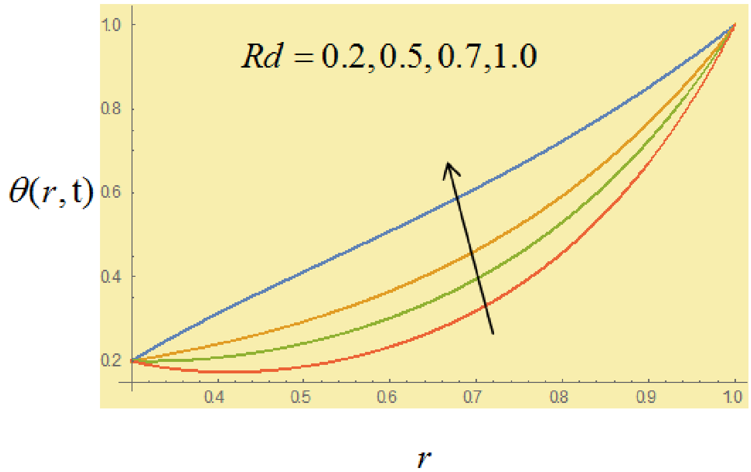

Fig. 8. Temperature profile $\theta(r,t)$ **against** r **with variation of** Rd **, leaving** $a = 0.3,\ Pr = 21$ $\omega = 0.01,\ \theta_w = 0.2$.

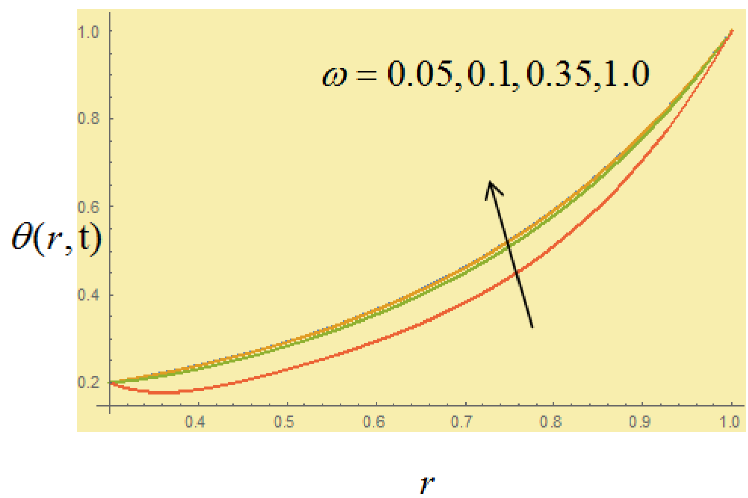

Fig. 9. Temperature profile $\theta(r,t)$ **against** r **, with variation of** ω **leaving** $a = 0.3,\ Pr = 21$ $Rd = 0.5,\ \theta_w = 0.2$

4 Discussion

Fig. 2 shows that an increase in the stenotic region while maintaining other physical parameters $\theta_w = 0.2, Pr = 21, M = 2, Gr = 10, Da = 0.2, Rd = 0.2$ causes retardation of blood flow and could lead to constriction/blockage of the blood vessel. The adverse implication is that it increases the heart rate; cause the rise in blood pressure and enlargement of tissues/ damage.

Fig. 3 uniquely indicates that Grasof number Gr causes the blood flow to accelerate since $Gr > 0$ and this is in good agreement with the observation of known theory. We conclude that the behavior of the velocity profile increases and is consistent with the known laws of physics and thus say that Grashof number $Gr > 0$ is an important physical parameter for controlling blood flow.

The Hartmann number M is associated with the ratio of the magnetic field body force (Lorentz force) to viscous force. Fig. 7 depicts that an increase in the values of the Hartmann number causes retardation of blood flow indicating the fact that the imposition of the azimuthal magnetic field decelerates the flow and consequently the thickness of the velocity boundary layer gets diminished. This observation is consistent with the physical fact that the Lorentz force that appears due to the interaction of the magnetic field and the blood velocity resists the corresponding blood flow, resulting in the velocity to decrease gradually.

Fig. 4, it is observed that as the stenotic height is increased the velocity profile decreases with the other valuable parameters $\theta_w = 0.2, Pr = 21,\ Gr = 10, M = 2,\ Da = 0.2,\ Rd = 0.2,\ \omega = 0.01, t = 0.5$ remained unchanged. This simply means the increased stenosis region adversely affects the flow of the blood in the human cardiovascular system; there is a need for all human beings to undergo some form of cardiovascular checks at a point in their lives.

The radiation parameter Rd shows the effect of radiation on the blood flow. Acceleration of blood flow under the effect of radiative heat is also observed in Fig. 5. The radiation results in an increase in the heat of the system resulting in the gain in kinetic energy of the blood molecules, that actually leads to thinning of the blood; as a consequence, the blood velocity is enhanced substantially. This is very important in the treatment cardiovascular disease in its little way. Fig. 6 suggests that blood flow can be improved by increasing the frequency parameter ω associated with blood. This presents us with an insight that blood flow can be enhanced by consistently improving the magnitude of the frequency parameter while the other physical parameters $\theta_w = 0.2,\ Pr = 21, Gr = 10,\ Da = 0.2, Rd = 0.2,\ a = 0.3,\ t = 0.5\ M = 2,$ are maintained.

Figs. 8 – 9 shows temperature profile because of radiative heat on human blood while maintaining other physical parameters $\theta_w = 0.2, a = 0.3, Pr = 21, \omega = 0.01$. It is observed that the radiative heat produces more oscillation in the blood. Physically, large heat radiation would lead to an improved surface heat flux. It may lead to the increase in the number of oscillation of the temperature of the blood; in consequence, it a good technique of supporting blood thinning for effective circulation of hemoglobin to all parts of the body.
The theoretical investigation of heat transfer blood flow through an indented artery with radiation effect and slip condition has been studied. The governing equations are solved analytically by employing the Frobenius series method of which our equations were later transformed to Bessel differential equations. We obtained our modified Bessel functions of order zero of the first and second kind. The effects of governing parameters such as heat parameter, Rd, the temperature parameter, θ_w, oscillatory frequency, ω, Darcy number, Da, Magnetic parameter, M, Prandtl number, Pr, for blood and radius of stenosis, a on the velocity flow, temperature characteristics are presented graphically and quantitatively.

We conclude from these results that:
We conclude from these results that:

- The velocity profile enhances for the increasing values of Gr, Rd, ω and M, Da, a caused the velocity to decrease.
- The temperature profile increases with increasing values of Rd, ω and a.

Thus, blood velocity can be controlled by suitably adjusting (increasing/decreasing) the magnetic field strength and other physical parameter. The results presented should be of sufficient interest to surgeons who usually want to keep the blood flow rate at a desired level during the entire surgical procedure.

Acknowledgement

The authors gratefully acknowledge the reviewers' comments that improved the quality of the article.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Ahmadi G, Manvi R. Equation of motion for viscous flow through a rigid porous medium. Indian Journal of Technology. 1971;9(12):441.

[2] Mekheimer KS, Al-Arabi TH. Nonlinear peristaltic transport of MHD flow through a porous medium. International Journal of Mathematics and Mathematical Sciences. 2003;2003(26):1663-82.

[3] El-Shahed M. Pulsatile flow of blood through a stenosed porous medium under periodic body acceleration. Applied Mathematics and Computation. 2003;138(2):479-88.

[4] Misra M, Mittal M, Singh RK, Verma AM, Rai R, Chandra G, Singh DP, Chauhan R, Chowdhary V, Mall A, Khan MJ. Prevalence of rheumatic heart disease in school-going children of Eastern Uttar Pradesh. Indian Heart Journal. 2007;59(1):42-3.

[5] Bhargava P, Bogy DB. Effect of shock pulse width on the shock response of small form factor disk drives. Microsystem Technologies. 2007;13(8-10):1107-15.

[6] Choudhury R, Das UJ. Heat transfer to MHD oscillatory viscoelastic flow in a channel filled with porous medium. Physics Research International; 2012.

[7] Sud VK, Sekhon GS. Arterial flow under periodic body acceleration. Bulletin of Mathematical Biology. 1985;47(1):35-52.

[8] Misra JC, Sahu BK. Flow through blood vessels under the action of a periodic acceleration field: A mathematical analysis. Computers & Mathematics with Applications. 1988;16(12):993-1016.

[9] Chaturani P, Palanisamy V. Casson fluid model for pulsatile flow of blood under periodic body acceleration. Biorheology. 1990;27(5):619-30.

[10] Kyoung M, Sheets ED. Vesicle diffusion close to a membrane: intermembrane interactions measured with fluorescence correlation spectroscopy. Biophysical Journal. 2008;95(12):5789-97.

[11] Biswas D, Chakraborty US. Pulsatile flow of blood in a constricted artery with body acceleration. Int. J. of Application and Applied Mathematics. 2009;4(2):329-42.

[12] Mandal PK. Dioxin: a review of its environmental effects and its aryl hydrocarbon receptor biology. Journal of Comparative Physiology B. 2005;175(4):221-30.

[13] Makinde OD, Mhone PY. Heat transfer to MHD oscillatory flow in a channel filled with porous medium. Romanian Journal of Physics. 2005;50(9/10):931.

[14] Hamza MM, Isah BY, Usman H. Unsteady Heat transfer to MHD oscillatory flow through a porous medium under slip condition. Int. J. of Computer Applications. 2011;33:12-7.

[15] Shukla JB, Parihar RS, Rao BR. Effects of stenosis on non-Newtonian flow of the blood in an artery. Bulletin of Mathematical Biology. 1980;42(3):283-94.

[16] Mathur P, Jain S. Mathematical modeling of non-Newtonian blood flow through artery in the presence of stenosis. Applied Mathematical Biosciences. 2013;4(1):1-2.

[17] Misra JC, Shit GC. Blood flow through arteries in a pathological state: A theoretical study. International Journal of Engineering Science. 2006;44(10):662-71.

[18] Ali R, Kaur R, Katiyar VK, Singh MP. Mathematical modeling of blood flow through vertebral artery with stenoses. Indian Journal of Biomechanics. 2009:151.

[19] Al Khatib MA, Wilson SD. The development of Poiseuille flow of a yield-stress fluid. Journal of Non-newtonian Fluid Mechanics. 2001;100(1):1-8.

[20] Frigaard IA, Ryan DP. Flow of a visco-plastic fluid in a channel of slowly varying width. Journal of Non-newtonian Fluid Mechanics. 2004;123(1):67-83.

[21] Ali A, Asghar S. Oscillatory channel flow for non-Newtonian fluid. International Journal of Physical Sciences. 2011;6(36):8036-43.

[22] Mekheimer KS, El Kot MA. The micropolar fluid model for blood flow through a tapered artery with a stenosis. Acta Mechanica Sinica. 2008;24(6):637-44.

[23] Sun B, Vallez LJ, Plourde BD, Abraham JP, Stark JR. Influence of supporting tissue on the deformation and compliance of healthy and diseased arteries. Journal of Biomedical Science and Engineering. 2015;8(08):490.

[24] Vallez LJ, Sun B, Plourde BD, Abraham JP, Staniloae CS. Numerical analysis of arterial plaque thickness and its impact on artery wall compliance. J Cardiovasc Med Cardiol. 2015;2(2):026-034. DOI: 10.17352/2455.

[25] Plourde BD, Vallez LJ, Sun B, Abraham JP, Staniloe CS. The effect of plaque removal on pressure drop and flow rate through an idealized stenotic lesion. Biology and Medicine. 2016;8(1):1.

[26] Plourde BD, Vallez LJ, Sun B, Nelson-Cheeseman BB, Abraham JP, Staniloae CS. Alterations of blood flow through arteries following atherectomy and the impact on pressure variation and velocity. Cardiovascular Engineering and Technology. 2016;7(3):280-9.

[27] Sparrow EM, Nelson-Cheeseman BB, Minkowycz WJ, Gorman JM, Abraham JP. Use of multi-lumen catheters to preserve injected stem cell viability and injectant dispersion. Cardiovascular Revascularization Medicine; 2017.

[28] Liu D, Zheng Y, Chen Q. Grain-resolved simulation of micro-particle dynamics in shear and oscillatory flows. Computers & Fluids. 2015;108:129-41.

Appendix A

$$A_1 = \theta_w K_0(\lambda) - K_0(\lambda a)\,;\; A_2 = I_0(\lambda a) - \theta_w I_0(\lambda)\,;\; A_3 = I_0(\lambda a)K_0(\lambda) - K_0(\lambda a)I_0(\lambda)$$

$$A_4 = I_0(\beta a)K_0(\beta) - K_0(\beta a)I_0(\beta),\; A_5 = \frac{\beta\left(B_2 K_0(\beta a) - B_1 K_0(\beta)\right)}{A_4},$$

$$A_6 = \frac{\beta\left(B_2 I_0(\beta a) - B_1 I_0(\beta)\right)}{A_4}$$

$$B_1 = -\frac{P_0}{\beta^2} - \frac{Gr}{\beta^2} H(\lambda a) \text{ and } B_2 = -\frac{P_0}{\beta^2} - \frac{Gr}{\beta^2} H(\lambda)$$

$$B_3 = \left(\frac{A_2}{A_3}\right) K_1(\lambda a) - \left(\frac{A_1}{A_3}\right) I_1(\lambda a),$$

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://sciencedomain.org/review-history/22654