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On Fuzzy Soft G-Metric Spaces

A. F. Sayed[1*], A. Alahmari[2] and Saleh Omran[3#]

[1] *Department of Mathematics, Al-Lith University College, Umm Al-Qura University, P.O. Box 112, Al-Lith 21961, Makkah Al Mukarramah, Kingdom of Saudi Arabia.*

[2] *Department of Mathematics, College of Applied Sciences, Umm Al-Qura University, Kingdom of Saudi Arabia.*

[3] *Department of Mathematics, Faculty of Science, South Valley University, Faculty of Science, Qena, Egypt.*

Authors' contributions

This work was carried out in collaboration between all authors. Author AFS initiated and designed the study, provided the technigues behind the proofs of the theorems and propositions and wrote the first draft of the manuscript and also, He reviewed and corrected the manuscript. Authors AA and SO provided some important background of the study and managed the literature searches. All authors read and approved the final manuscript.

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Abstract

In this paper, we introduce the notion of fuzzy soft G-metric space via fuzzy soft elements. Then, fuzzy soft convergence and fuzzy soft continuity are studied in fuzzy soft G-metric space. Finally, some fixed points theorems are proved.

Keywords: Soft set; fuzzy set; fuzzy soft set; fuzzy soft G-metric space; fixed point.

**Corresponding author: E-mail: dr.afsayed@hotmail.com, afssayed@uqu.edu.sa;*

 # Current address: Department of Mathematics, Faculty of Science, Taif University, Taif, Kingdom of Saudi Arabia.

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1 Introduction

In daily life, the problems in many fields such as engineering, physics, computer sciences, economics, social sciences, medical sciences and many other diverse fields deal with uncertain data and that may not be successfully modeled by the classical mathematics. There are two types of mathematical tools to deal with uncertainties namely fuzzy set theory introduced by Zadeh [1] and the theory of soft sets initiated by Molodstov [2] which helps to solve problems in all areas. Fuzzy soft set which is a combination of fuzzy set and soft set was introduced by Maji et al. [3]. Roy and Maji [4] gave some results on an application of fuzzy soft sets in decision making problem. Fuzzy soft sets and their applications have been investigated intensively (see, e.g., [5, 6, 7, 8, 9, 10, 11, 12, 13]). In [14], Beaulaa and Gunaseeli introduced a definition of the fuzzy soft metric space, also in [15], Sayed and Alahmari introduced the notions of some mappings and proved some fixed point theorems in fuzzy soft metric spaces.

The concept of G-metric space was introduced by Mustafa and Sims [16] in order to extend and generalize the notion of metric space, a number of authors have studied and characterized many well-known results in the setting of G-metric space (see, e.g., [17, 18, 19, 20, 21, 22, 23, 24, 25]).

Güler et al. [26] introduced the concept of soft G-metric space according to a soft element and obtained some of its properties. Then, they defined soft G-convergence and soft G-continuity. Moreover, they proved existence and uniqueness of fixed points in soft G-metric spaces. In [27] Güler and Yildirim introduced the notion of soft G-Cauchy sequences and soft G-complete spaces and investigated some properties of such spaces. In this paper, we introduce the notion of fuzzy soft G-metric space via fuzzy soft elements. Then, fuzzy soft convergence and fuzzy soft continuity are studied in fuzzy soft G-metric space, Finally, some fixed points theorems are proved.

2 Preliminaries

In this section we present some basic definitions of fuzzy soft set, fuzzy soft metric space and soft G-metric space.

Throughout our discussion, X refers to an initial universe, E the set of all parameters for X, $P(X)$ denotes the power set of X and $I = [0, 1]$.

Definition 2.1. [1] Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function $\mu_A : X \to I$ whose value $\mu_A(x)$ represents the "degree of membership" of x in A, for every x in X. Let I^X denotes the family of all fuzzy sets on X.

A member A in I^X is contained in a member B of I^X, denoted by $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ for every $x \in X$ (see [1]).

Let $A, B \in I^X$, we have the following properties on fuzzy sets (see [1]).

(1) Equality: $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ for all $x \in X$,

(2) Intersection: $C = A \wedge B \in I^X$ by $\mu_C(x) = min\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$,

(3) Union: $D = A \vee B \in I^X$ by $\mu_C(x) = max\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$,

(4) Complement: $E = A^c \in I^X$ by $\mu_E(x) = 1 - \mu_A(x)$ for all $x \in X$.

Definition 2.2. [1] The empty fuzzy set, denoted by $\tilde{0}$, is the function which maps each $x \in X$ to 0. That is, $\tilde{0}(x) = 0$ for all $x \in X$. A universal fuzzy set denoted by $\tilde{1}$ is a function which maps each $x \in X$ to 1. That is, $\tilde{1}(x) = 1$ for all $x \in X$.

Definition 2.3. [2] Let $A \subseteq E$. A pair (F, A) is called a soft set over X if F is a mapping $F : A \rightarrow P(X)$.

In other words, a soft set over X is a parameterized family of subsets of the universe X. For $\epsilon \in A$, $F(\epsilon)$ may be considered as the set of ϵ-approximate elements of the soft set (F, A), or as the set of ϵ-approximate elements of the soft set.

Definition 2.4. [28] A pair (f, A), denoted by f_A, is called a fuzzy soft set over X, where f is a mapping $f : A \rightarrow I^X$ defined by $f_A(e) = \mu^e_{f_A}$ where

$$\mu^e_{f_A} = \begin{cases} \tilde{0}, & \text{if } e \notin A; \\ otherwise, & \text{if } e \in A. \end{cases}$$

$\widetilde{(X, E)}$ denotes the class of all fuzzy soft sets over (X, E) and is called a fuzzy soft universe [3].

Definition 2.5. [29] A fuzzy soft set f_A over X is said to be:

(a) NULL fuzzy soft set, denoted by $\tilde{\phi}$, if for all $e \in A, f_A(e) = \tilde{0}$,

(b) absolute fuzzy soft set, denoted by $\tilde{E}$, if for all $e \in A, f_A(e) = \tilde{1}$.

Definition 2.6. [30] The complement of a fuzzy soft set f_A, denoted by f^c_A, where $f^c_A : E \rightarrow I^X$ is a mapping given by $\mu^e_{f^c_A} = \tilde{1} - \mu^e_{f_A}$ for all $e \in E$ and where $\tilde{1}(x) = 1$ for all $x \in X$. Clearly $(f^c_A)^c = f_A$.

Definition 2.7. [30] Let $f_A, g_B \in \widetilde{(X, E)}$. We say that f_A is fuzzy soft subset of g_B, denoted by $f_A \tilde{\subseteq} g_B$, if $A \subseteq B$ and $\mu^e_{f_A} \leq \mu^e_{g_B}$ for all $e \in A$, i.e. $\mu^e_{f_A}(x) \leq \mu^e_{g_B}(x)$ for all $x \in X$ and for all $e \in A$.

Definition 2.8. [30] Let $f_A, g_B \in \widetilde{(X, E)}$. The union of f_A and g_B is the fuzzy soft set h_C, where $C = A \cup B$ and for all $e \in C, h_C(e) = \mu^e_{h_c} = \mu^e_{f_A} \vee \mu^e_{g_B}$. Here we write $h_C = f_A \tilde{\cup} g_B$.

Definition 2.9. [30] Let $f_A, g_B \in \widetilde{(X, E)}$. The intersection of f_A and g_B is the fuzzy soft set d_C, where $C = A \cap B$ and for all $e \in C, d_C(e) = \mu^e_{d_c} = \mu^e_{f_A} \wedge \mu^e_{g_B}$. Here we write $d_C = f_A \tilde{\cap} g_B$.

Definition 2.10. [31] The fuzzy soft set $f_A \in \widetilde{(X, E)}$ is called a fuzzy soft point if there exist $x \in X$ and $e \in E$ such that $\mu^e_{f_A}(x) = \alpha (0 < \alpha \leq 1)$ and $\mu^e_{f_A}(y) = 0$ for each $y \in X - \{x\}$, and this fuzzy soft point is denoted by x^e_α or f_e.

Definition 2.11. [31] The fuzzy soft point x^e_α is said to be belonging to the fuzzy soft set g_A, denoted by $x^e_\alpha \tilde{\in} (g, A)$, if for the element $e \in A, \alpha \leq \mu^e_{g_A}(x)$.

Definition 2.12. [14] Let f_A be fuzzy soft set over X. The two fuzzy soft points $F_{e_1}, F_{e_2} \in f_A$ are said to be equal if $\mu_{f_{e_1}}(x) = \mu_{f_{e_2}}(x)$ for all $x \in X$. Thus $f_{e_1} \neq f_{e_2}$ if and only $\mu_{f_{e_1}}(x) \neq \mu_{f_{e_2}}(x)$ for all $x \in X$.

Proposition 2.1. *[14] The union of any collection of fuzzy soft points can be considered as a fuzzy soft set and every fuzzy soft set can be expressed as the union of all fuzzy soft points.*

$$f_A = \{\tilde{\cup}_{f_e \tilde{\in} f_A} f_e : e \in E\}$$

Proposition 2.2. *[14] Let f_A, f_B be two fuzzy soft sets then $f_A \tilde{\subseteq} f_B$ if and only if $f_e \tilde{\in} f_A$ implies $f_e \tilde{\in} f_B$ and hence $f_A = f_B$ if and only if $f_e \tilde{\in} f_A$ and only if $f_e \tilde{\in} f_B$.*

Definition 2.13. [32] Let $\mathbb{R}$ be the set of real numbers and $B(\mathbb{R})$ be the collection of all nonempty bounded subsets of $\mathbb{R}$ and E be taken as a set of parameters, $A \subseteq E$. Then a mapping $f : A \to B(\mathbb{R})$ is called a soft real set. If a soft real set is a singleton soft set, it will be called a soft real number and denoted by $\tilde{r}, \tilde{s}, \tilde{t}$ etc. $\tilde{0}$ and $\tilde{1}$ are the soft real numbers where $\tilde{0}(e) = 0, \tilde{1}(e) = 1$ for all $e \in E$ respectively.

The set of all soft real numbers is denoted by $\mathbb{R}$ and the set of all nonnegative soft real numbers by $\mathbb{R}(A)^*$.

The soft real number $\bar{r}$ will denote a particular type of soft real number such that $\bar{r}(e) = r$ for all $e \in E$.

Definition 2.14. [33] A (nonnegative) fuzzy soft real number is a fuzzy set on the set of all (nonnegative) soft real numbers $\mathcal{R}(A)$, that is, a mapping $\tilde{\tilde{\lambda}} : \mathcal{R}(A) \to [0, 1]$, associating with each (nonnegative) soft real number $\tilde{t}$, its grade of membership $\tilde{\tilde{\lambda}}(\tilde{t})$ satisfying the following conditions:

(i) $\tilde{\tilde{\lambda}}$ is convex
that is, $\tilde{\tilde{\lambda}}(\tilde{t}) \geq \min\ (\tilde{\tilde{\lambda}}(\tilde{s}), \tilde{\tilde{\lambda}}(\tilde{r}))$ for $\tilde{s} \subseteq \tilde{t} \subseteq \tilde{r}$;

(ii) $\tilde{\tilde{\lambda}}$ is normal
that is, there exists $\tilde{t_0} \in \mathcal{R}(A)^*$ such that $\tilde{\tilde{\lambda}}(\tilde{t_0}) = 1$;

(iii) $\tilde{\tilde{\lambda}}$ is upper semi continuous provided for all $\tilde{t} \in \mathcal{R}(A)$ and $\alpha \in [0, 1]$
$\tilde{\tilde{\lambda}}(\tilde{t}) < \alpha$, there is a $\delta > 0$ such that $\parallel \tilde{s} - \tilde{t} \parallel \leq \delta$ implies that $\tilde{\tilde{\lambda}}(\tilde{s}) < \alpha$.

The fuzzy soft real numbers be denoted by $\tilde{\tilde{r}}, \tilde{\tilde{s}}, \tilde{\tilde{t}}$ etc, while $\bar{\bar{r}}, \bar{\bar{s}}, \bar{\bar{t}}$ will be denoted in particular type of fuzzy soft real numbers such that $\bar{\bar{r}}(e) = \mu^e \bar{r}$ that is a fuzzy number for all $e \in E$.

Let $A \subseteq E$. The set of all nonnegative fuzzy soft real numbers be denoted by $\mathcal{R}(A)^*$ and the collection of all fuzzy soft points of a fuzzy soft set f_A over X be denoted by $FSC(f_A)$.

Let X be a nonempty set. It is well known that a map $d : X^2 \to \mathbb{R}^+$ is called a metric if and only if it satisfies the following:

(a) $d(x, y) \geq 0$ and $d(x, y) = 0$ iff $x = y$, for all $x, y \in X$;

(b) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(c) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

If we replace the set X with the set of all fuzzy soft points of an absolute fuzzy soft set, and $\mathbb{R}^+$ with the set of all nonnegative fuzzy soft real numbers, then we get a refinement of metric space called fuzzy soft metric space [14] as follows:

Definition 2.15. [14] Let $A \subseteq E$ and $\tilde{E}$ be the absolute fuzzy soft set. A mapping $\tilde{d} : FSC(\tilde{E}) \times FSC(\tilde{E}) \to \mathcal{R}(A)^*$ is said to be a fuzzy soft metric on $\tilde{E}$ if $\tilde{d}$ satisfies the following conditions:
$(FSM_1) : \tilde{d}(f_{e_1}, f_{e_2}) \tilde{\geq} \tilde{0}$ for all $f_{e_1}, f_{e_2} \tilde{\in} \tilde{E}$;
$(FSM_2) : \tilde{d}(f_{e_1}, f_{e_2}) = \tilde{0}$ if and only if $f_{e_1} = f_{e_2}$ for al $f_{e_1}, f_{e_2} \tilde{\in} \tilde{E}$;
$(FSM_3) : \tilde{d}(f_{e_1}, f_{e_2}) = \tilde{d}(f_{e_2}, f_{e_1})$ for all $f_{e_1}, f_{e_2} \tilde{\in} \tilde{E}$;
$(FSM_4) : \tilde{d}(f_{e_1}, f_{e_3}) \tilde{\leq} \tilde{d}(f_{e_1}, f_{e_2}) + \tilde{d}(f_{e_2}, f_{e_3})$ for all $f_{e_1}, f_{e_2}, f_{e_3} \tilde{\in} \tilde{E}$.

The fuzzy soft set $\tilde{E}$ with the fuzzy soft metric $\tilde{d}$ is called the fuzzy soft metric space and is denoted by $(\tilde{E}, \tilde{d})$.

Definition 2.16. [26] Let X be a nonempty set and E be the nonempty set of parameters. A mapping $\tilde{G} : SE(\tilde{X}) \times SE(\tilde{X}) \times SE(\tilde{X}) \to \mathcal{R}(A)^*$ is said to be a soft generalized metric or soft G-metric on $\tilde{X}$ if $\tilde{G}$ satisfies the following conditions:

$(\tilde{G}_1) : \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) = \tilde{0}$ if $\tilde{x} = \tilde{y} = \tilde{z}$;
$(\tilde{G}_2) : \tilde{0} \tilde{<} \tilde{G}(\tilde{x}, \tilde{x}, \tilde{y})$ for all $\tilde{x}, \tilde{y} \tilde{\in} SE(\tilde{X})$ with $\tilde{x} \neq \tilde{y}$;
$(\tilde{G}_3) : \tilde{G}(\tilde{x}, \tilde{x}, \tilde{y}) \tilde{\leq} \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})$ for all $\tilde{x}, \tilde{y}, \tilde{z} \tilde{\in} SE(\tilde{X})$ with $\tilde{y} \neq \tilde{z}$;
$(\tilde{G}_4) : \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) = \tilde{G}(\tilde{x}, \tilde{z}, \tilde{y}) = \tilde{G}(\tilde{y}, \tilde{z}, \tilde{x}) = ...$;
$(\tilde{G}_5) : \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) \tilde{\leq} \tilde{G}(\tilde{x}, \tilde{a}, \tilde{a}) + \tilde{G}(\tilde{a}, \tilde{y}, \tilde{z})$ for all $\tilde{x}, \tilde{y}, \tilde{z}, \tilde{a} \tilde{\in} SE(\tilde{X})$.

The soft set $\tilde{X}$ with a soft G-metric $\tilde{G}$ on $\tilde{X}$is said to be a soft G-metric space and is denoted by $(\tilde{X}, \tilde{G}, \tilde{E})$.

3 Fuzzy Soft G-metric Spaces

Definition 3.1. Let E be a nonempty set of parameters, $A \subseteq E$ and $\tilde{E}$ be the absolute fuzzy soft. A mapping $\tilde{G} : FSC(\tilde{E}) \times FSC(\tilde{E}) \times FSC(\tilde{E}) \to \mathcal{R}(A)^*$ is said to be a fuzzy soft generalized metric or fuzzy soft G-metric on $\tilde{E}$ if $\tilde{G}$ satisfies the following conditions:

$(FS\tilde{G}_1) : \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) = \tilde{0}$ if $f_{e_1} = f_{e_2} = f_{e_3}$;
$(FS\tilde{G}_2) : \tilde{0} \tilde{<} \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})$ for all $f_{e_1}, f_{e_2} \tilde{\in} FSC(\tilde{E})$ with $f_{e_1} \neq f_{e_2}$;
$(FS\tilde{G}_3) : \tilde{G}(f_{e_1}, f_{e_1}, f_{e_2}) \tilde{\leq} \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})$ for all $f_{e_1}, f_{e_2}, f_{e_3} \tilde{\in} FSC(\tilde{E})$ with $f_{e_2} \neq f_{e_3}$;
$(FS\tilde{G}_4) : \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) = \tilde{G}(f_{e_1}, f_{e_3}, f_{e_2}) = \tilde{G}(f_{e_2}, f_{e_3}, f_{e_1}) = ...$;
$(FS\tilde{G}_5) : \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}(f_{e_1}, f_e, f_e) + \tilde{G}(f_e, f_{e_2}, f_{e_3})$ for all $f_{e_1}, f_{e_2}, f_{e_3}, f_e \tilde{\in} FSC(\tilde{E})$.

The fuzzy soft set $\tilde{E}$ with a fuzzy soft G-metric $\tilde{G}$ on $\tilde{E}$ is said to be a fuzzy soft G-metric space and is denoted by $(\tilde{E}, \tilde{G})$.

Example 3.1. *Let E be a nonempty set of parameters and $\tilde{E}$ be absolute fuzzy soft set. We define $\tilde{G} : FSC(\tilde{E}) \times FSC(\tilde{E}) \times FSC(\tilde{E}) \to \mathcal{R}(A)^*$ by*

$$\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) = \begin{cases} \tilde{0}, & \text{if } f_{e_1} = f_{e_2} = f_{e_3}; \\ \tilde{1}, & \text{if } f_{e_1} = f_{e_2} \neq f_{e_3} \text{ or } f_{e_1} \neq f_{e_2} = f_{e_3} \text{ or } f_{e_2} \neq f_{e_1} = f_{e_3}; \\ \tilde{\tilde{2}}, & \text{if } f_{e_1} \neq f_{e_2} \neq f_{e_3}. \end{cases}$$

for all $f_{e_1}, f_{e_2}, f_{e_3} \tilde{\in} FSC(\tilde{E})$. Then $\tilde{G}$ satisfies all the fuzzy soft G-metric axioms $(FS\tilde{G}_1)-(FS\tilde{G}_5)$.

Definition 3.2. A fuzzy soft G-metric space $(\tilde{E}, \tilde{G})$ is symmetric if it satisfies the following condition:

$(FS\tilde{G}_6) : \tilde{G}(f_{e_1}, f_{e_2}, f_{e_2}) = \tilde{G}(f_{e_1}, f_{e_1}, f_{e_2})$ for all $f_{e_1}, f_{e_2} \tilde{\in} FSC(\tilde{E})$.

Remark 3.1. It is obvious that the fuzzy soft G-metric space defined in Example 3.1 satisfies the condition $(FS\tilde{G}_6)$ and so it is symmetric.

Proposition 3.1. *Let $(\tilde{E}, \tilde{G})$ be a fuzzy soft G-metric space. Then the following hold for all $f_{e_1}, f_{e_2}, f_{e_3}, f_e \tilde{\in} FSC(\tilde{E})$:*
(a) If $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) = \tilde{0}$, then $f_{e_1} = f_{e_2} = f_{e_3}$.
(b) $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}(f_{e_1}, f_{e_1}, f_{e_2}) + \tilde{G}(f_{e_1}, f_{e_1}, f_{e_3})$.
(c) $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_2}) \tilde{\leq} \tilde{\tilde{2}} \tilde{G}(f_{e_2}, f_{e_1}, f_{e_1})$.
(d) $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}(f_{e_1}, f_e, f_{e_3}) + \tilde{G}(f_e, f_{e_2}, f_{e_3})$.
(e) $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{\frac{\tilde{2}}{3}} [\tilde{G}(f_{e_1}, f_{e_2}, f_e) + \tilde{G}(f_{e_1}, f_e, f_{e_3}) + \tilde{G}(f_e, f_{e_1}, f_{e_3})]$.
(f) $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}(f_{e_1}, f_e, f_e) + \tilde{G}(f_{e_2}, f_e, f_e) + \tilde{G}(f_{e_3}, f_e, f_e)]$.
(g) $|\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) - \tilde{G}(f_{e_1}, f_{e_2}, f_e)| \tilde{\leq}$ max$\{\tilde{G}(f_e, f_{e_3}, f_{e_3}), \tilde{G}(f_{e_3}, f_e, f_e)\}$.

(h) $|\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) - \tilde{G}(f_{e_1}, f_{e_2}, f_e)| \tilde{\leq} \tilde{G}(f_{e_1}, f_e, f_{e_3})$.
(i) $|\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) - \tilde{G}(f_{e_2}, f_{e_3}, f_{e_3})| \tilde{\leq}$ $max\{\tilde{G}(f_{e_1}, f_{e_3}, f_{e_3}), \tilde{G}(f_{e_3}, f_{e_1}, f_{e_1})\}$.
(j) $|\tilde{G}(f_{e_1}, f_{e_2}, f_{e_2}) - \tilde{G}(f_{e_2}, f_{e_1}, f_{e_1})| \tilde{\leq}$ $max\{\tilde{G}(f_{e_2}, f_{e_1}, f_{e_1}), \tilde{G}(f_{e_1}, f_{e_2}, f_{e_2})\}$.

Proof. (a) Case 1: Let all of the fuzzy soft elements be distinct. Then, we have $\tilde{0} \tilde{\leq} \tilde{G}(f_{e_1}, f_{e_1}, f_{e_2}) \tilde{\leq} \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})$ from $(FS\tilde{G}_2)$ and $(FS\tilde{G}_3)$, respectively.

Case 2: Let two of the fuzzy soft elements be equal and the remaining one be distinct. Thus, we have $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{>} \tilde{0}$ by $(FS\tilde{G}_4)$ and $(FS\tilde{G}_2)$. From the two cases, we obtain $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \neq \tilde{0}$.

(b) We have $\tilde{G}(f_{e_1}, f_{e_1}, f_{e_2}) + \tilde{G}(f_{e_1}, f_{e_1}, f_{e_3}) = \tilde{G}(f_{e_2}, f_{e_1}, f_{e_1}) + \tilde{G}(f_{e_1}, f_{e_1}, f_{e_3}) \tilde{\geq}$ $\tilde{G}(f_{e_2}, f_{e_1}, f_{e_3}) = \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})$ from $(FS\tilde{G}_4)$ and $(FS\tilde{G}_5)$.

(c) By using (b), we have $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_2}) \tilde{\leq} \tilde{G}(f_{e_1}, f_{e_1}, f_{e_2}) + \tilde{G}(f_{e_1}, f_{e_1}, f_{e_2}) =$ $\tilde{\tilde{2}}\tilde{G}(f_{e_2}, f_{e_1}, f_{e_1})$ from $(FS\tilde{G}_4)$.

(d) Case 1: Let $f_{e_1} \neq f_{e_3}$. Then, we have $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}(f_e, f_e, f_{e_1}) + \tilde{G}(f_e, f_{e_2}, f_{e_3}) \tilde{\leq}$ $\tilde{G}(f_e, f_{e_1}, f_{e_3}) + \tilde{G}(f_e, f_{e_2}, f_{e_3})$ from $(FS\tilde{G}_5), (FS\tilde{G}_4)$ and $(FS\tilde{G}_3)$, respectively. So, we obtain $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}(f_{e_1}, f_e, f_{e_3}) + \tilde{G}(f_e, f_{e_2}, f_{e_3})$ by $(FS\tilde{G}_4)$.

Case 2: Let $f_{e_1} = f_{e_3}$ and $f_{e_2} \neq f_e$.

Thus we get $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) = \tilde{G}(f_{e_1}, f_{e_1}, f_{e_3}) \tilde{\leq} \tilde{G}(f_{e_1}, f_{e_2}, f_e) \tilde{\leq} \tilde{G}(f_{e_1}, f_{e_2}, f_e) + \tilde{G}(f_{e_1}, f_e, f_{e_1})$ $= \tilde{G}(f_e, f_{e_2}, f_{e_1}) + \tilde{G}(f_{e_1}, f_e, f_{e_1})$ from $(FS\tilde{G}_4)$ and $(FS\tilde{G}_3)$.

Case 3: Let $f_{e_1} = f_{e_3}$ and $f_{e_2} = f_e$.

It is obvious.

(e) By (d) and $(FS\tilde{G}_4)$, we have
$\tilde{\tilde{3}}\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{\tilde{2}}\tilde{G}(f_{e_1}, f_{e_2}, f_e) + \tilde{\tilde{2}}\tilde{G}(f_{e_1}, f_e, f_{e_3}) + \tilde{\tilde{2}}\tilde{G}(f_e, f_{e_2}, f_{e_3})$. Thus,
$\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{\tilde{\frac{2}{3}}}[\tilde{G}(f_{e_1}, f_{e_2}, f_e) + \tilde{G}(f_{e_1}, f_e, f_{e_3}) + \tilde{G}(f_e, f_{e_2}, f_{e_3})]$.

(f) By $(FS\tilde{G}_5)$ and (b), we obtain
$\tilde{\tilde{3}}\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{\tilde{3}}\tilde{G}(f_{e_1}, f_e, f_e) + \tilde{\tilde{3}}\tilde{G}(f_{e_2}, f_e, f_e) + \tilde{\tilde{3}}\tilde{G}(f_{e_3}, f_e, f_e)$.
Hence, $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}(f_{e_1}, f_e, f_e) + \tilde{G}(f_{e_2}, f_e, f_e) + \tilde{G}(f_{e_3}, f_e, f_e)$.

(g) We have $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}(f_{e_3}, f_e, f_e) + \tilde{G}(f_e, f_{e_1}, f_{e_2})$ from $(FS\tilde{G}_5)$. Then, $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) - \tilde{G}(f_e, f_{e_1}, f_{e_2}) \tilde{\leq} \tilde{G}(f_{e_3}, f_e, f_e) \tilde{\leq} max\{\tilde{G}(f_{e_3}, f_e, f_e), \tilde{G}(f_e, f_{e_3}, f_{e_3})\}$.

In a similar way, we get

$$-max\{\tilde{G}(f_{e_3}, f_e, f_e), \tilde{G}(f_e, f_{e_3}, f_{e_3})\} \tilde{\leq} \tilde{G}(f_e, f_{e_3}, f_{e_3}) \tilde{\leq} -[\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) - \tilde{G}(f_e, f_{e_1}, f_{e_2})].$$

Thus, we obtain

$$|\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) - \tilde{G}(f_{e_1}, f_{e_2}, f_e)| \tilde{\leq} \max\{\tilde{G}(f_e, f_{e_3}, f_{e_3}), \tilde{G}(f_{e_3}, f_e, f_e)\}.$$

(h) The proof is clear, by (d) and $(FS\tilde{G}_4)$.

(i) It is obvious, by $(FS\tilde{G}_5)$ and $(FS\tilde{G}_4)$.

(j) It follows from (c) and $(FS\tilde{G}_4)$. □

Proposition 3.2. *Let $(\tilde{E}, \tilde{G})$ be a fuzzy soft G-metric space. Then the following are equivalent.*
(a) $(\tilde{E}, \tilde{G})$ is symmetric.
(b) $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_2}) \tilde{\leq} \tilde{G}(f_{e_1}, f_{e_2}, f_e)$, for all $f_{e_1}, f_{e_2}, f_e \tilde{\in} FSC(\tilde{E})$.
(c) $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}(f_{e_1}, f_{e_2}, f_e) + \tilde{G}(f_{e_3}, f_{e_2}, f_{\acute{e}})$, for all $f_{e_1}, f_{e_2}, f_{e_3}, f_e, f_{\acute{e}} \tilde{\in} FSC(\tilde{E})$.

Proof. $(a) \Rightarrow (b)$: Since $(\tilde{E}, \tilde{G})$ is symmetric, we have $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_2}) = \tilde{G}(f_{e_1}, f_{e_1}, f_{e_2})$.

Case 1: Let $f_{e_1} \neq f_e$. We have $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_2}) \tilde{\leq} \tilde{G}(f_{e_1}, f_{e_2}, f_e)$ by $(FS\tilde{G}_3)$.

Case 2: Let $f_{e_1} = f_e$. It is clear by $(FS\tilde{G}_3)$.

$(b) \Rightarrow (c)$: We obtain
$$\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}(f_{e_2}, f_{e_2}, f_{e_1}) + \tilde{G}(f_{e_2}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}(f_{e_1}, f_{e_2}, f_e) + \tilde{G}(f_{e_3}, f_{e_2}, f_{\acute{e}})$$
by using Proposition 3.1(b) and the hypothesis.

$(c) \Rightarrow (a)$: By hypothesis and $(FS\tilde{G}_4)$, we have
$\tilde{G}(f_{e_1}, f_{e_2}, f_{e_2}) \tilde{\leq} \tilde{G}(f_{e_1}, f_{e_2}, f_{e_1}) + \tilde{G}(f_{e_2}, f_{e_2}, f_{e_3}) = \tilde{G}(f_{e_1}, f_{e_2}, f_{e_1}) = \tilde{G}(f_{e_1}, f_{e_1}, f_{e_2})$.

In a similar way, we get $\tilde{G}(f_{e_2}, f_{e_1}, f_{e_1}) \tilde{\leq} \tilde{G}(f_{e_1}, f_{e_2}, f_{e_2})$. □

Proposition 3.3. *For any fuzzy soft metric $\tilde{d}$ on $\tilde{E}$ we can construct a fuzzy soft G-metric by the following mappings $\tilde{G}_s$ and $\tilde{G}_m$.*
(a) $\tilde{G}_s(\tilde{d})(f_{e_1}, f_{e_2}, f_{e_3}) = \tilde{\frac{\tilde{1}}{3}}[\tilde{d}(f_{e_1}, f_{e_2}) + \tilde{d}(f_{e_2}, f_{e_3}) + \tilde{d}(f_{e_1}, f_{e_3})]$.
(b) $\tilde{G}_m(\tilde{d})(f_{e_1}, f_{e_2}, f_{e_3}) = max\{\tilde{d}(f_{e_1}, f_{e_2}), \tilde{d}(f_{e_2}, f_{e_3}), \tilde{d}(f_{e_1}, f_{e_3})\}$.

Proof. (a) Since $\tilde{d}$ is a fuzzy soft metric, the proofs of $(FS\tilde{G}_1), (FS\tilde{G}_2)$, and $(FS\tilde{G}_4)$ are obvious. The proof of $(FS\tilde{G}_5)$ follows from (FSM_4).

$(FS\tilde{G}_3)$ Case 1: Let $f_{e_2} \neq f_{e_3}$, and $f_{e_1} \neq f_{e_2}$. Since $\tilde{G}_s(\tilde{d})(f_{e_1}, f_{e_1}, f_{e_2}) = \tilde{0}$, the assertion is clear.

Case 2: Let $f_{e_1} = f_{e_3}, f_{e_3} \neq f_{e_2}$ and $f_{e_1} \neq f_{e_2}$. Then,
$\tilde{G}_s(\tilde{d})(f_{e_1}, f_{e_1}, f_{e_2}) = \tilde{G}_s(\tilde{d})(f_{e_1}, f_{e_2}, f_{e_3})$.

Case 3: Let $f_{e_1} \neq f_{e_3}, f_{e_3} \neq f_{e_2}$, and $f_{e_1} \neq f_{e_2}$. From (FSM_4), we have
$\tilde{\tilde{2}}\tilde{d}(f_{e_1}, f_{e_2}) \tilde{\leq} \tilde{d}(f_{e_1}, f_{e_2}) + \tilde{d}(f_{e_1}, f_{e_3}) + \tilde{d}(f_{e_3}, f_{e_2})$. Then
$$\tilde{G}_s(\tilde{d})(f_{e_1}, f_{e_1}, f_{e_2}) = \tilde{\frac{\tilde{2}}{3}}\tilde{d}(f_{e_1}, f_{e_2}) \tilde{\leq} \tilde{\frac{\tilde{1}}{3}}[\tilde{d}(f_{e_1}, f_{e_2}) + \tilde{d}(f_{e_1}, f_{e_3}) + \tilde{d}(f_{e_3}, f_{e_2})] = \tilde{G}_s(\tilde{d})(f_{e_1}, f_{e_2}, f_{e_3}).$$

(a) The proofs of $(FS\tilde{G}_1), (FS\tilde{G}_2)$, and $(FS\tilde{G}_4)$ are obvious.

$(FS\tilde{G}_3)$ Let $f_{e_2} \neq f_{e_3}$.
$$\tilde{G}_s(\tilde{d})(f_{e_1}, f_{e_1}, f_{e_2}) = \tilde{d}(f_{e_1}, f_{e_2}) \tilde{\leq} max\{\tilde{d}(f_{e_1}, f_{e_2}) + \tilde{d}(f_{e_1}, f_{e_3}) + \tilde{d}(f_{e_2}, f_{e_3})\} = \tilde{G}_m(\tilde{d})(f_{e_1}, f_{e_2}, f_{e_3}).$$

$(FS\tilde{G}_5)$ Case 1: $\tilde{G}_m(\tilde{d})(f_{e_1}, f_{e_2}, f_{e_3}) = \tilde{d}(f_{e_1}, f_{e_2})$.

From $(FS\tilde{G}_4)$, we have $\tilde{d}(f_{e_1}, f_{e_2}) \tilde{\leq} \tilde{d}(f_{e_1}, f_e) + \tilde{d}(f_e, f_{e_2}) = \tilde{G}_m(\tilde{d})(f_{e_1}, f_e, f_e) + \tilde{d}(f_e, f_{e_2}) \tilde{\leq}$
$\tilde{G}_m(\tilde{d})(f_{e_1}, f_e, f_e) + \tilde{G}_m(\tilde{d})(f_e, f_{e_2}, f_{e_3})$.

The other cases can be proved in a similar way. □

Proposition 3.4. *For any fuzzy soft G-metric $\tilde{G}$ on $\tilde{E}$, we can construct a fuzzy soft $\tilde{d}_{\tilde{G}}$ on $\tilde{E}$ defined by*

$$\tilde{d}_{\tilde{G}}(f_{e_1}, f_{e_2}) = \tilde{G}(f_{e_1}, f_{e_2}, f_{e_2}) + \tilde{G}(f_{e_1}, f_{e_1}, f_{e_2})$$

Proof. The proofs of $(FSM_1), (FSM_3)$, and (FSM_4) are obviously follows from
$(FS\tilde{G}_1) and (FS\tilde{G}_2), (FS\tilde{G}_4)$ and $(FS\tilde{G}_5)$, respectively.

(FSM_2) Let $\tilde{d}_{\tilde{G}}(f_{e_1}, f_{e_2}) = \tilde{0}$. Assume $f_{e_1} \neq f_{e_2}$. Since $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_2}) + \tilde{G}(f_{e_1}, f_{e_1}, f_{e_2}) = \tilde{0}$, we would have $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_2}) \tilde{\leq} \tilde{0}$ by Proposition 3.1(c). This contradicts to $(FS\tilde{G}_2)$.

Thus, $f_{e_1} = f_{e_2}$. The converse is clear. □

Proposition 3.5. *Let $\tilde{d}$ be a fuzzy soft metric on $\tilde{E}$. Then the following hold.*
(a) $\tilde{d}_{\tilde{G}_s(\tilde{d})}(f_{e_1}, f_{e_2}) = \tilde{\tilde{\frac{4}{3}}}\tilde{d}(f_{e_1}, f_{e_2})$.
(b) $\tilde{d}_{\tilde{G}_m(\tilde{d})}(f_{e_1}, f_{e_2}) = \tilde{\tilde{2}}\tilde{d}(f_{e_1}, f_{e_2})$.

Proof. The proofs follows from the definition of $\tilde{d}_{\tilde{G}}, \tilde{G}_m$ and $\tilde{G}_s$. □

Proposition 3.6. *Let $\tilde{G}$ be a fuzzy soft G-metric $\tilde{G}$ on $\tilde{E}$. Then the following hold.*
(a) $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}_s(\tilde{d}_{\tilde{G}})(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{\tilde{2}}\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})$.
(b) $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}_m(\tilde{d}_{\tilde{G}})(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{\tilde{3}}\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})$.

Proof. (a) By Proposition 3.1(b), we have
$\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{\tilde{\frac{1}{3}}}[\tilde{d}_{\tilde{G}}(f_{e_1}, f_{e_2}) + \tilde{d}_{\tilde{G}}(f_{e_2}, f_{e_3}) + \tilde{d}_{\tilde{G}}(f_{e_1}, f_{e_3})]$.
On the other hand, By Proposition 3.1(c) and $(FS\tilde{G}_3)$, we obtain
$\tilde{\tilde{\frac{1}{3}}}[\tilde{d}_{\tilde{G}}(f_{e_1}, f_{e_2}) + \tilde{d}_{\tilde{G}}(f_{e_2}, f_{e_3}) + \tilde{d}_{\tilde{G}}(f_{e_1}, f_{e_3})] \tilde{\leq} \tilde{\tilde{2}}\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})$.

(b) By Proposition 3.1(b), we obtain $\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}_m(\tilde{d}_{\tilde{G}})(f_{e_1}, f_{e_2}, f_{e_3})$.
Also, from Proposition 3.1(c) and $(FS\tilde{G}_3)$, we get $\tilde{G}_m(\tilde{d}_{\tilde{G}})(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{\tilde{3}}\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})$. □

Definition 3.3. Let $(\tilde{E}, \tilde{G})$ be a fuzzy soft G-metric space and $\tilde{\tilde{t}}$ be a fuzzy soft real number and $\tilde{\epsilon} \in (0, 1)$. A fuzzy soft open $\tilde{G}$-ball centered at the fuzzy soft element $f_e \tilde{\in} FSC(\tilde{E})$ and radius $\tilde{\tilde{t}}$ is a collection of all fuzzy soft elements g_e of f_e such that $\tilde{G}(f_e, g_e, g_e) \tilde{<} \tilde{\tilde{t}}$.
It is denoted by $\tilde{\tilde{B}}(f_e, \tilde{\tilde{t}}, \tilde{\epsilon})$ where $\tilde{\tilde{B}}(f_e, \tilde{\tilde{t}}, \tilde{\epsilon}) = \{g_e \tilde{\in} FSC(\tilde{E}) | \tilde{G}(f_e, g_e, g_e) \tilde{<} \tilde{\tilde{t}}\}$ with $|\mu^a_{f_e}(s) - \mu^a_{g_e}(s)| < \tilde{\epsilon}$.

The fuzzy soft closed $\tilde{G}$-ball is denoted by $\tilde{\tilde{B}}[f_e, \tilde{\tilde{t}}, \tilde{\epsilon}] = \{g_e \tilde{\in} FSC(\tilde{E}) | \tilde{G}(f_e, g_e, g_e) \tilde{\leq} \tilde{\tilde{t}}\}$ with $|\mu^a_{f_e}(s) - \mu^a_{g_e}(s)| \leq \tilde{\epsilon}$.

Example 3.2. *Consider the fuzzy soft G-metric space $(\tilde{E}, \tilde{G})$ given in Example 3.2.*
We have

$$\tilde{\tilde{B}}(f_e, \tilde{\tilde{t}}, \tilde{\epsilon}) = \begin{cases} FSC(\tilde{E}), & \tilde{\tilde{t}} > \tilde{1}; \\ \{f_e\}, & \tilde{\tilde{t}} \leq \tilde{1}. \end{cases}$$

for any $f_e \tilde{\in} FSC(\tilde{E})$

Proposition 3.7. *Let $(\tilde{E}, \tilde{G})$ be a fuzzy soft G-metric space. For $f_e \tilde{\in} FSC(\tilde{E})$ and $\tilde{\tilde{t}} > \tilde{0}$, the following hold:*
(a) If $\tilde{G}_s(\tilde{d})(f_e, f_{e_1}, f_{e_2}) \tilde{<} \tilde{\tilde{r}}$, then $f_{e_1}, f_{e_2} \tilde{\in} \tilde{\tilde{B}}(f_e, \tilde{\tilde{t}}, \tilde{\epsilon})$.
(b) If $f_{e_1} \tilde{\in} \tilde{\tilde{B}}(f_e, \tilde{\tilde{t}}, \tilde{\epsilon})$, then there exists a $\tilde{\tilde{s}} \tilde{>} \tilde{0}$ such that $\tilde{\tilde{B}}(f_{e_1}, \tilde{\tilde{s}}, \tilde{\epsilon}) \tilde{\subseteq} \tilde{\tilde{B}}(f_e, \tilde{\tilde{t}}, \tilde{\epsilon})$.

Proof. (a) It obviously follows from $(FS\tilde{G}_3)$.
(b) Let $f_{e_1} \tilde{\in} \tilde{\tilde{B}}(f_e, \tilde{\tilde{t}}, \tilde{\epsilon})$. Assume that $f_{e_2} \tilde{\in} \tilde{\tilde{B}}(f_{e_1}, \tilde{\tilde{s}}, \tilde{\epsilon})$. Then, we have
$\tilde{G}(f_e, f_{e_2}, f_{e_2}) \tilde{\leq} \tilde{G}(f_e, f_{e_1}, f_{e_1}) + \tilde{G}(f_{e_1}, f_{e_2}, f_{e_2}) \tilde{<} \tilde{G}(f_e, f_{e_1}, f_{e_1}) + \tilde{\tilde{s}}$ by $(FS\tilde{G}_5)$.
Say $\tilde{\tilde{s}} = \tilde{\tilde{t}} - \tilde{G}(f_e, f_{e_1}, f_{e_1})$. Thus, we get $\tilde{G}(f_e, f_{e_2}, f_{e_2}) \tilde{<} \tilde{\tilde{t}}$. Hence $f_{e_2} \tilde{\in} \tilde{\tilde{B}}(f_e, \tilde{\tilde{t}}, \tilde{\epsilon})$. □

Proposition 3.8. *Let $(\tilde{E}, \tilde{G})$ be a fuzzy soft G-metric space. For $f_{e_0} \tilde{\in} FSC(\tilde{E})$ and $\tilde{\tilde{t}} \tilde{>} \tilde{0}$, we have*

$$\tilde{\tilde{B}}(f_{e_0}, \frac{\tilde{\tilde{t}}}{3}, \tilde{\epsilon}) \tilde{\subseteq} \tilde{\tilde{B}}_{\tilde{d}_{\tilde{G}}}(f_{e_0}, \tilde{\tilde{t}}, \tilde{\epsilon}) \tilde{\subseteq} \tilde{\tilde{B}}(f_{e_0}, \tilde{\tilde{t}}, \tilde{\epsilon}).$$

Proof. It obviously follows from Proposition 3.1(c). □

4 Fuzzy Soft G-convergence

Definition 4.1. Let $(\tilde{E}, \tilde{G})$ be a fuzzy soft G-metric space and $\{f_{e_n}\}$ a sequence of fuzzy soft elements in $\tilde{E}$. The sequence $\{f_{e_n}\}$ is said to be fuzzy soft G-convergent to f_e in $\tilde{E}$ if for every $\tilde{\epsilon}\tilde{\geq}\tilde{0}$, chosen arbitrary, there exists a natural number $N = N(\tilde{\epsilon})$ such that $\tilde{0}\tilde{\leq}\tilde{G}(f_{e_n}, f_{e_n}, f_e)\tilde{<}\tilde{\epsilon}$ whenever $n \geq N$ i.e $n \geq N \Rightarrow \{f_{e_n}\}\tilde{\in}\tilde{\tilde{B}}(f_e, \tilde{\tilde{t}}, \tilde{\epsilon})$. We denote this by $f_{e_n} \to f_e$ as $n \to \infty$ or $\text{Lim}_{n\to\infty}\{f_{e_n}\} = f_e$.

Proposition 4.1. *Let $(\tilde{E}, \tilde{G})$ be a fuzzy soft G-metric space, for sequence $\{f_{e_n}\}$ in $\tilde{E}$ and a fuzzy soft element f_e, then the following are equivalent:*
(a) *$\{f_{e_n}\}$ is fuzzy soft G-convergent to f_e.*
(b) *$d_{\tilde{G}}(f_{e_n}, f_e) \to \tilde{0}$ as $n \to \infty$.*
(c) *$\tilde{G}(f_{e_n}, f_{e_n}, f_e) \to \tilde{0}$ as $n \to \infty$.*
(d) *$\tilde{G}(f_{e_n}, f_e, f_e) \to \tilde{0}$ as $n \to \infty$.*
(e) *$\tilde{G}(f_{e_n}, f_{e_m}, f_e) \to \tilde{0}$ as $n, m \to \infty$.*

Proof. $(a) \Rightarrow (b)$ follows from Proposition 3.8. The other implications can be proved by using Propositions 3.1 and 3.3. □

Definition 4.2. Let $(\tilde{E}, \tilde{G})$ be a fuzzy soft G-metric space and $\{f_{e_n}\}$ be sequence of fuzzy soft elements in $\tilde{E}$. Then the sequence $\{f_{e_n}\}$ is said to be fuzzy soft G-Cauchy if for every $\tilde{\epsilon}\tilde{\geq}\tilde{0}$, there exist $\tilde{\delta}\tilde{>}\tilde{0}$ and a positive integer $N = N(\tilde{\epsilon})$ such that $\tilde{G}(f_{e_n}, f_{e_m}, f_{e_l})\tilde{<}\tilde{\epsilon}$ for all $n, m, l \geq N$; that is $\tilde{G}(f_{e_n}, f_{e_m}, f_{e_l}) \to \tilde{0}$ as $n, m, l \to \infty$.

Definition 4.3. A fuzzy soft G-metric space $(\tilde{E}, \tilde{G})$ is said to be fuzzy soft G-complete if every fuzzy soft G-Cauchy sequence in $(\tilde{E}, \tilde{G})$ is fuzzy soft G-convergent in $(\tilde{E}, \tilde{G})$.

Proposition 4.2. *Let $(\tilde{E}, \tilde{G})$ be a fuzzy soft G-metric space and $\{f_{e_n}\}$ be sequence of fuzzy soft elements in $\tilde{E}$. Then the following are equivalent:*
(1) *The sequence $\{f_{e_n}\}$ is fuzzy soft G-Cauchy sequence.*
(2) *For every $\tilde{\epsilon}\tilde{\geq}\tilde{0}$, there exist a natural number N such that $\tilde{G}(f_{e_n}, f_{e_m}, f_{e_m})\tilde{<}\tilde{\epsilon}$ for all $n, m \geq N$.*
(3) *The sequence $\{f_{e_n}\}$ is a Cauchy sequence in the fuzzy soft metric space $(\tilde{E}, d_{\tilde{G}})$.*

Proof. (1) ⇒ (2) It is obvious by axiom $(FS\tilde{G}_3)$.
(2) ⇔ (3) It is clear by the definition of $d_{\tilde{G}}$.
(2) ⇒ (1) If we set $f_e = f_{e_m}$, then it is obvious by axiom $(FS\tilde{G}_5)$. □

Corollary 4.1. *Every fuzzy soft G-convergent sequence in any fuzzy soft G-metric space $(\tilde{E}, \tilde{G})$ is fuzzy soft G-Cauchy sequence.*

Proposition 4.3. *A fuzzy soft G-metric space $(\tilde{E}, \tilde{G})$ is fuzzy soft G-complete if and only if $(\tilde{E}, d_{\tilde{G}})$ is a complete fuzzy soft metric space.*

Proof. It follows from Propositions 4.2 and 3.4. □

Definition 4.4. Let $(\tilde{E}, \tilde{G}), (\tilde{\tilde{E}}, \tilde{\tilde{G}})$ be two fuzzy soft G-metric spaces. Then a function $T : \tilde{E} \to \tilde{\tilde{E}}$ is fuzzy soft G-continuous at a fuzzy soft element $f_e\tilde{\in}FSC(\tilde{E})$ if and only if for every $\tilde{\epsilon}\tilde{>}\tilde{0}$, there exists $\tilde{\delta}\tilde{\geq}\tilde{0}$ such that $f_{e_1}, f_{e_2}\tilde{\in}FSC(\tilde{E})$ and $\tilde{G}(f_e, f_{e_1}, f_{e_2})\tilde{<}\tilde{\delta}$ implies that $\tilde{\tilde{G}}(Tf_e, Tf_{e_1}, Tf_{e_2})\tilde{<}\tilde{\epsilon}$.

A function T is a fuzzy soft G-continuous if and only if it is fuzzy soft G-continuous at all fuzzy soft elements $f_e\tilde{\in}FSC(\tilde{E})$

Proposition 4.4. *Let $(\tilde{E},\tilde{G}),(\tilde{\tilde{E}},\tilde{\tilde{G}})$ be two fuzzy soft G-metric spaces. Then a function $T:\tilde{E}\to\tilde{\tilde{E}}$ is fuzzy soft G-continuous at a fuzzy soft element $f_e\tilde{\in}FSC(\tilde{E})$ if and only if it is a fuzzy soft G-sequentially continuous at a fuzzy soft element $f_e\tilde{\in}FSC(\tilde{E})$, i.e. whenever $\{f_{e_n}\}$ is fuzzy soft G-convergent to f_e, $\{Tf_{e_n}\}$ is fuzzy soft G-convergent to Tf_e.*

Proof. Necessity: Assume that T is a fuzzy soft G-continuous. Given $f_n\to f_e$, we wish to show that $Tf_n\to Tf_e$.

Let $\tilde{\epsilon}\tilde{>}\tilde{0}$. By hypothesis, there exists a $\tilde{\delta}\tilde{\geq}\tilde{0}$ such that $f_{e_1},f_{e_2}\tilde{\in}FSC(\tilde{E})$ and

$$\tilde{G}(f_e,f_{e_1},f_{e_2})\tilde{<}\tilde{\delta}\Longrightarrow\tilde{\tilde{G}}(Tf_e,Tf_{e_1},Tf_{e_2})\tilde{<}\tilde{\epsilon}.$$

Since $f_n\to f_e$ corresponds to $\tilde{\delta}\tilde{>}\tilde{0}$ where $\tilde{\delta}(\lambda)=\delta_\lambda$ there exists a natural number N such that

$$n\geq N\Longrightarrow\tilde{G}(f_e,f_{e_n},f_{e_n})\tilde{<}\tilde{\delta}\Longrightarrow\tilde{G}(f_e,f_{e_n},f_{e_n})(\lambda)<\delta_\lambda.$$

Hence, we have

$$n\geq N\Longrightarrow\tilde{\tilde{G}}(Tf_e,Tf_{e_n},Tf_{e_n})\tilde{<}\tilde{\epsilon}\Longrightarrow\tilde{\tilde{G}}(Tf_e,Tf_{e_n},Tf_{e_n})(\lambda)<\epsilon_\lambda.$$

This implies that $Tf_{e_n}\to Tf_e$.

Sufficiency: This can be clearly proved assuming that T is not fuzzy soft G-continuous at a fuzzy soft element f_e. □

Theorem 4.2. *Let $(\tilde{E},\tilde{G})$ be a fuzzy soft G-complete and $T:(\tilde{E},\tilde{G})\to(\tilde{E},\tilde{G})$ be a mapping that satisfies the following condition for all $f_{e_1},f_{e_2},f_{e_3}\tilde{\in}FSC(\tilde{E})$,*

$$\begin{aligned}\tilde{G}(Tf_{e_1},Tf_{e_2},Tf_{e_3})\tilde{\leq}\bar{\bar{a}}\tilde{G}(f_{e_1},Tf_{e_1},Tf_{e_1})+\bar{\bar{b}}\tilde{G}(f_{e_2},Tf_{e_2},Tf_{e_2})+\\ \bar{\bar{c}}\tilde{G}(f_{e_3},Tf_{e_3},Tf_{e_3})+\bar{\bar{d}}\tilde{G}(f_{e_1},f_{e_2},f_{e_3}).\end{aligned}\tag{4.1}$$

where $\tilde{0}\tilde{\leq}\bar{\bar{a}}+\bar{\bar{b}}+\bar{\bar{c}}+\bar{\bar{d}}\tilde{<}\tilde{1}$.

Then T has a unique fixed point, say f_e, and T is a fuzzy soft G-continuous at f_e.

Proof. Let $f_{e_0}\tilde{\in}FSC(\tilde{E})$ be an arbitrary fuzzy soft element and define the sequence $\{f_{e_n}\}_{n\in\mathbb{N}}$ by $f_{e_n}=T^n(f_{e_0})$. From (4.1), we get

$$\begin{aligned}\tilde{G}(f_{e_n},f_{e_{n+1}},f_{e_{n+1}})\tilde{\leq}\bar{\bar{a}}\tilde{G}(f_{e_{n-1}},f_{e_n},f_{e_n})+\bar{\bar{b}}\tilde{G}(f_{e_n},f_{e_{n+1}},f_{e_{n+1}})+\\ \bar{\bar{c}}\tilde{G}(f_{e_n},f_{e_{n+1}},f_{e_{n+1}})+\bar{\bar{d}}\tilde{G}(f_{e_{n-1}},f_{e_n},f_{e_n}).\end{aligned}$$

Then

$$\tilde{G}(f_{e_n},f_{e_{n+1}},f_{e_{n+1}})\tilde{\leq}(\bar{\bar{a}}+\bar{\bar{d}})\tilde{G}(f_{e_{n-1}},f_{e_n},f_{e_n})+(\bar{\bar{b}}+\bar{\bar{c}})\tilde{G}(f_{e_n},f_{e_{n+1}},f_{e_{n+1}}).$$

Thus we have

$$\tilde{G}(f_{e_n},f_{e_{n+1}},f_{e_{n+1}})\tilde{\leq}\tfrac{\bar{\bar{a}}+\bar{\bar{d}}}{\tilde{1}-(\bar{\bar{b}}+\bar{\bar{c}})}\tilde{G}(f_{e_{n-1}},f_{e_n},f_{e_n}).$$

Let $\bar{\bar{k}}=\frac{\bar{\bar{a}}+\bar{\bar{d}}}{\tilde{1}-(\bar{\bar{b}}+\bar{\bar{c}})}$.

Since $\tilde{0}\tilde{\leq}\bar{\bar{a}}+\bar{\bar{b}}+\bar{\bar{c}}\tilde{<}\tilde{1},\tilde{0}\tilde{\leq}\bar{\bar{k}}\tilde{<}\tilde{1}$. So we get

$$\tilde{G}(f_{e_n},f_{e_{n+1}},f_{e_{n+1}})\tilde{\leq}\bar{\bar{k}}\tilde{G}(f_{e_{n-1}},f_{e_n},f_{e_n}).\tag{4.2}$$

Hence we have the following inequalities:

$$\begin{aligned} \tilde{G}(f_{e_{n-1}}, f_{e_n}, f_{e_n}) &\tilde{\leq} \bar{\bar{k}}\tilde{G}(f_{e_{n-2}}, f_{e_{n-1}}, f_{e_{n-1}}), \\ \tilde{G}(f_{e_{n-2}}, f_{e_{n-1}}, f_{e_{n-1}}) &\tilde{\leq} \bar{\bar{k}}\tilde{G}(f_{e_{n-3}}, f_{e_{n-2}}, f_{e_{n-2}}), \\ &. \\ &. \\ &. \end{aligned} \tag{4.3}$$

Combining (4.2) and (4.3) we obtain

$$\tilde{G}(f_{e_n}, f_{e_{n+1}}, f_{e_{n+1}}) \tilde{\leq} (\bar{\bar{k}})^n \tilde{G}(f_{e_0}, f_{e_1}, f_{e_1}). \tag{4.4}$$

For all $m, n \in \mathbb{N}$ such that $n < m$, we have

$$\begin{aligned} \tilde{G}(f_{e_n}, f_{e_m}, f_{e_m}) &\tilde{\leq} \tilde{G}(f_{e_n}, f_{e_{n+1}}, f_{e_{n+1}}), \tilde{G}(f_{e_{n+1}}, f_{e_{n+2}}, f_{e_{n+2}}) + ... + \tilde{G}(f_{e_{m-1}}, f_{e_m}, f_{e_m}) \\ &\tilde{\leq} ((\bar{\bar{k}})^n + (\bar{\bar{k}})^{n+1} + ... + (\bar{\bar{k}})^{m-1}) \tilde{G}(f_{e_0}, f_{e_1}, f_{e_1}) \\ &\tilde{\leq} \frac{(\bar{\bar{k}})^n}{\bar{1} - \bar{\bar{k}}} \tilde{G}(f_{e_0}, f_{e_1}, f_{e_1}), \end{aligned}$$

by $(FS\tilde{G}_5)$ and (4.4).

Thus $\tilde{G}(f_{e_n}, f_{e_m}, f_{e_m}) \to \tilde{0}$ as $m, n \to \infty$.

So $\{f_{e_n}\}$ is a fuzzy soft G-Cauchy sequence. Since $(\tilde{E}, \tilde{G})$ is a fuzzy soft G-complete, there exists $g_e \tilde{\in} FSC(\tilde{E})$ such that $\{f_{e_n}\}$ fuzzy soft G-convergence to g_e.

Assume that $Tg_e \neq g_e$, i.e., $Tg_e(\lambda_0) \neq g_e(\lambda_0)$ for some $\lambda_0 \in E$. Then by (4.1) we have $\tilde{G}(f_{e_n}, Tg_e, Tg_e) \tilde{\leq} \bar{\bar{a}}\tilde{G}(f_{e_{n-1}}, f_{e_n}, f_{e_n}) + \bar{\bar{b}}\tilde{G}(g_e, Tg_e, Tg_e) + \bar{\bar{c}}\tilde{G}(g_e, Tg_e, Tg_e) + \bar{\bar{d}}\tilde{G}(f_{e_{n-1}}, g_e, g_e)$.

Thus

$\tilde{G}(f_{e_n}, Tg_e, Tg_e) \tilde{\leq} \bar{\bar{a}}\tilde{G}(f_{e_{n-1}}, f_{e_n}, f_{e_n}) + (\bar{\bar{b}} + \bar{\bar{c}})\tilde{G}(g_e, Tg_e, Tg_e) + \bar{\bar{d}}\tilde{G}(f_{e_{n-1}}, g_e, g_e)$.

By taking the limlt as $n \to \infty$, we get

$$\tilde{G}(g_e, Tg_e, Tg_e) \tilde{\leq} (\bar{\bar{b}} + \bar{\bar{c}})\tilde{G}(g_e, Tg_e, Tg_e).$$

Since $\{f_{e_n}\} \to g_e$. This is a contradiction. Hence $Tg_e = g_e$.

Let us prove uniqueness. Suppose there exists a fuzzy soft element h_e such that $g_e \neq h_e$ and $Th_e = h_e$. Then by (4.1), we get

$$\begin{aligned} \tilde{G}(g_e, h_e, h_e) = \tilde{G}(Tg_e, Th_e, Th_e) &\tilde{\leq} \bar{\bar{a}}\tilde{G}(g_e, Tg_e, Tg_e) + (\bar{\bar{b}} + \bar{\bar{c}})\tilde{G}(h_e, Th_e, Th_e) + \bar{\bar{d}}\tilde{G}(g_e, h_e, h_e) \\ &= \bar{\bar{d}}\tilde{G}(g_e, h_e, h_e). \end{aligned}$$

Thus we find that $g_e = h_e$.

Let us prove that T is fuzzy soft G-continuous at g_e. Let $\{g_{e_n}\}$ be a sequence of fuzzy soft elements in $\tilde{E}$ such that $\{g_{e_n}\} \to g_e$. Then by (4.1) we have

$$\begin{aligned} \tilde{G}(g_e, Tg_{e_n}, Tg_{e_n}) &\tilde{\leq} \bar{\bar{a}}\tilde{G}(g_e, Tg_e, Tg_e) + (\bar{\bar{b}} + \bar{\bar{c}})\tilde{G}(g_{e_n}, Tg_{e_n}, Tg_{e_n}) + \bar{\bar{d}}\tilde{G}(g_e, g_{e_n}, g_{e_n}) \\ &= (\bar{\bar{b}} + \bar{\bar{c}})\tilde{G}(g_{e_n}, Tg_{e_n}, Tg_{e_n}) + \bar{\bar{d}}\tilde{G}(g_e, g_{e_n}, g_{e_n}). \end{aligned} \tag{4.5}$$

Also, by $FS\tilde{G}_5$, we have

$$\tilde{G}(g_{e_n}, Tg_{e_n}, Tg_{e_n}) \tilde{\leq} \tilde{G}(g_{e_n}, g_e, g_e) + \tilde{G}(g_e, Tg_{e_n}, Tg_{e_n}). \tag{4.6}$$

Then we combine (4.5) and (4.6), to get

$$\tilde{G}(g_e, Tg_{e_n}, Tg_{e_n}) \tilde{\leq} (\bar{\bar{b}} + \bar{\bar{c}})\tilde{G}(g_{e_n}, g_e, g_e) + (\bar{\bar{b}} + \bar{\bar{c}})\tilde{G}(g_e, Tg_{e_n}, Tg_{e_n}) + \bar{\bar{d}}\tilde{G}(g_e, g_{e_n}, g_{e_n}).$$

Thus

$\tilde{G}(g_e, Tg_{e_n}, Tg_{e_n}) \tilde{\leq} \frac{\bar{\bar{b}}+\bar{\bar{c}}}{\tilde{1}-(\bar{\bar{b}}+\bar{\bar{c}})}\tilde{G}(g_{e_n}, g_e, g_e) + \frac{\bar{\bar{d}}}{\tilde{1}-(\bar{\bar{b}}+\bar{\bar{c}})}\tilde{G}(g_e, g_{e_n}, g_{e_n}).$

By taking the limit as $n \to \infty$, we obtain

$\tilde{G}(g_e, Tg_{e_n}, Tg_{e_n}) \to \tilde{0}$ since $\{g_{e_n}\} \to g_e$. So $\{Tg_{e_n}\} \to Tg_e = g_e$, from Proposition 4.1.

Hence T is fuzzy soft G-continuous at g_e by Proposition 4.4. □

Corollary 4.3. *Let $(\tilde{E}, \tilde{G})$ be a fuzzy soft G-complete metric space and $T : (\tilde{E}, \tilde{G}) \to (\tilde{E}, \tilde{G})$ be a mapping that satisfies the following condition for all $f_{e_1}, f_{e_2}, f_{e_3} \tilde{\in} FSC(\tilde{E})$,*

$$\tilde{G}(Tf_{e_1}, Tf_{e_2}, Tf_{e_3}) \tilde{\leq} \bar{\bar{a}}\tilde{G}(f_{e_1}, Tf_{e_1}, Tf_{e_1}) + \bar{\bar{b}}\tilde{G}(f_{e_2}, Tf_{e_2}, Tf_{e_2}) + \bar{\bar{c}}\tilde{G}(f_{e_3}, Tf_{e_3}, Tf_{e_3}),$$

where $\tilde{0} \tilde{<} \bar{\bar{a}} + \bar{\bar{b}} + \bar{\bar{c}} \tilde{<} \tilde{1}$.

Then T has a unique fixed point, say g_e, and T is a fuzzy soft G-continuous at g_e.

Proof. If we take $\bar{\bar{d}} = \tilde{0}$ in Theorem 4.2, it is obvious. □

Theorem 4.4. *Let $(\tilde{E}, \tilde{G})$ be a complete fuzzy soft G-metric space, and suppose $T : (\tilde{E}, \tilde{G}) \to (\tilde{E}, \tilde{G})$ satisfies for all $f_{e_i} \in FSC(\tilde{E})$ for $i = 1, 2, 3$, the following*

$$G(T^2 f_{e_1}, T^2 f_{e_2}, T^2 f_{e_3}) \tilde{\leq} \bar{\bar{a}}\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) + \bar{\bar{b}}\tilde{G}(Tf_{e_1}, Tf_{e_2}, Tf_{e_3}),$$

where
$\tilde{0} \tilde{\leq} \bar{\bar{a}}, \bar{\bar{b}} \tilde{<} (\tilde{\bar{\frac{1}{4}}})$ *and* $\tilde{0} \tilde{\leq} 3\bar{\bar{a}} + \bar{\bar{b}} \tilde{<} \tilde{1}$.
Then, T^2 has a unique fixed point for some unique $f_e \tilde{\in} FSC(\tilde{E})$ (i.e. $T^2 f_e = f_e$), and T^2 is fuzzy soft G-continuous at f_e.

Proof. Let $f_{e_n} = Tf_{e_{n-1}}$ and $f_{e_{n+1}} = Tf_{e_n} = T^2 f_{e_{n-1}}$. Now observe that

$$\begin{aligned}
\tilde{G}(f_{e_{n+1}}, f_{e_{n+2}}, f_{e_{n+3}}) &= \tilde{G}(T^2 f_{e_{n-1}}, T^2 f_{e_n}, T^2 f_{e_{n+1}}) \\
&\tilde{\leq} \bar{\bar{a}}\tilde{G}(f_{e_{n-1}}, f_{e_n}, f_{e_{n+1}}) + \bar{\bar{b}}\tilde{G}(Tf_{e_{n-1}}, Tf_{e_n}, Tf_{e_{n+1}}) \\
&= \bar{\bar{a}}\tilde{G}(f_{e_{n-1}}, f_{e_n}, f_{e_{n+1}}) + \bar{\bar{b}}\tilde{G}(f_{e_n}, f_{e_{n+1}}, f_{e_{n+2}}) \\
&\tilde{\leq} \bar{\bar{a}}\tilde{G}(f_{e_{n-1}}, f_{e_{n+3}}, f_{e_{n+3}}) + \bar{\bar{a}}\tilde{G}(f_{e_{n+3}}, f_{e_n}, f_{e_{n+1}}) + \bar{\bar{b}}\tilde{G}(f_{e_n}, f_{e_{n+1}}, f_{e_{n+2}}) \\
&\tilde{\leq} \bar{\bar{a}}\tilde{G}(f_{e_{n-1}}, f_{e_{n+3}}, f_{e_{n+3}}) + \bar{\bar{a}}\tilde{G}(f_{e_{n+3}}, f_{e_{n+2}}, f_{e_{n+2}}) \\
&\quad + \bar{\bar{a}}\tilde{G}(f_{e_{n+2}}, f_{e_n}, f_{e_{n+1}}) + \bar{\bar{b}}\tilde{G}(f_{e_n}, f_{e_{n+1}}, f_{e_{n+2}}) \\
&\tilde{\leq} \tilde{\bar{\bar{2}}}\bar{\bar{a}}\tilde{G}(f_{e_{n-1}}, f_{e_{n+3}}, f_{e_{n+3}}) + (\bar{\bar{a}} + \bar{\bar{b}})\tilde{G}(f_{e_n}, f_{e_{n+1}}, f_{e_{n+2}}).
\end{aligned} \tag{4.7}$$

From (4.7), we deduce that
$\tilde{G}(f_{e_{n+1}}, f_{e_{n+2}}, f_{e_{n+3}}) \tilde{\leq} \bar{\bar{\gamma}}\tilde{G}(f_{e_n}, f_{e_{n+1}}, f_{e_{n+2}})$,
where $\bar{\bar{\gamma}} = \frac{\bar{\bar{a}}+\bar{\bar{b}}}{\tilde{1}-\tilde{\bar{\bar{2}}}\bar{\bar{a}}} \tilde{<} \tilde{1}$, and by induction,
$\tilde{G}(f_{e_{n+1}}, f_{e_{n+2}}, f_{e_{n+3}}) \tilde{\leq} \bar{\bar{\gamma}}^n \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})$.

Consequently, the sequence $\{f_{e_n}\}$ is fuzzy soft G-Cauchy. By the fuzzy soft G-completeness of $(\tilde{E}, \tilde{G})$, there exists f_e such that $\lim_{n\to\infty} f_{e_n} = f_e$. By the fuzzy soft G-continuity of T^2 at f_e, that is, T is also fuzzy soft G-continuous at f_e and we have
$f_e = \lim_{n\to\infty} f_{e_{n+2}} = \lim_{n\to\infty} T^2(f_{e_n}) = T^2\ (\lim_{n\to\infty} f_{e_n}) = T^2(f_e)$.

For uniqueness, Suppose there exists a fuzzy soft element g_e such that $f_e \neq g_e$ and $Tg_e = g_e$, then

$$\begin{aligned}\tilde{G}(f_e, g_e, g_e) = \tilde{G}(T^2 f_e, T^2 g_e, T^2 g_e) &\tilde{\leq} \bar{\bar{a}}\tilde{G}(f_e, g_e, g_e) + \bar{\bar{b}}\tilde{G}(Tf_e, Tg_e, Tg_e) \\ &\tilde{\leq}(\bar{\bar{a}} + \bar{\bar{b}})\tilde{G}(f_e, g_e, g_e) \\ &\tilde{<}(\frac{\bar{\bar{1}}}{2})\tilde{G}(f_e, g_e, g_e).\end{aligned} \tag{4.8}$$

Clearly, the above inequality (4.8) implies that $\tilde{G}(f_e, g_e, g_e) = \tilde{0}$; hence $f_e = g_e$ and so, T^2 has a unique fixed point. □

5 Conclusions

It is well known that if X is a nonempty set and $G : X^3 \to \mathbb{R}^+$ satisfies Definition 3 [16], then G is called a G-metric and (X, G) is called a G-metric space. If we replace the set X with the set of all fuzzy soft points of an absolute fuzzy soft set, and $\mathbb{R}^+$ with the set of all nonnegative fuzzy soft real numbers, then we get a refinement of G-metric space called fuzzy soft G-metric space, which appears as a new concept is introduced in this paper. Then, fuzzy soft convergence and fuzzy soft continuity are studied in fuzzy soft G-metric space, Finally, some fixed points theorems are proved. In the future, we will try to improve the search performance further for some types of mappings.

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Competing Interests

Authors have declared that no competing interests exist.

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