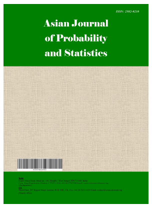


**Asian Journal of Probability and Statistics**

**7(4): 59-71, 2020; Article no.AJPAS.57771**

***ISSN: 2582-0230***

# Nonparametric Methods for the Nondecreasing Ordered Hypothesis in a Mixed Design

**Badr Alnssyan[1,2] and Rhonda C. Magel[2*]**

*[1]Department of Management and Humanities Sciences, Community College, Qassim University, Buraydah, KSA.*
*[2]Department of Statistics, North Dakota State University, Fargo, ND 58108, USA.*

***Authors' contributions***

*This work was carried out in collaboration between both authors. Author RCM designed and directed the study. Author BA wrote the computer programs and conducted the study. Both authors read and approved the final manuscript.*

***Article Information***

DOI: 10.9734/AJPAS/2020/v7i430193
Editor(s):
(1) Dr. Halim Zeghdoudi, Badji-Mokhtar University, Algeria.
Reviewers:
(1) B. Angelin, Bharathiar University, India.
(2) H. K. Shreedhar, Visveswaraya Technological University, India.
Complete Peer review History: http://www.sdiarticle4.com/review-history/57771

***Original Research Article***

***Received: 29 March 2020***
***Accepted: 02 June 2020***
***Published: 14 July 2020***

## Abstract

**Aims:** Introduce two new test statistics in testing for the nondecreasing alternative in a mixed design consisting of a Completely Randomized Portion and a Randomized Complete Block Portion.
**Study Design:** Simulation study comparing four test statistics for the nondecreasing alternative in a mixed design consisting of a CRD and an RCBD portion. The test statistics included two new test statistics and two existing test statistics. Random samples were taken from three different types of underlying distributions. Different percentages of the CRD portion will be considered as well as different sample sizes. Powers were estimated based on a variety location parameter shifts. Three, four, and five populations were considered.
**Place and Duration of Study:** The simulation study took place on the campus of North Dakota State University during the calendar year 2019.
**Methodology:** Levels of significance for each of the three types of underlying distributions, when the RCBD portion was larger than the CRD portion, when the CRD portion was larger than the RCBD portion, and when the CRD portion was equal to the RCBD portion, and when the number of populations were 3, 4, and 5.
**Results:** Regardless of the underlying population types, the proposed test statistics did better than the

*Corresponding author: E-mail: Rhonda.magel@ndsu.edu;*

existing test statistics when the difference between the last two parameters is large. This was true for 3, 4, and 5 populations.
**Conclusion:** When the differences between the last two parameters is large, the two new test statistics performed better. Otherwise, the existing test statistics are better. In both cases, it is better to use the combined test statistic that first standardizes the individual test statistics for the CRD and RCBD portions before adding them together.

*Keywords: Nonparametric; order-restricted inference; Jonckheere Terpstra; completely randomized design; randomized complete block design; mixed design.*

# 1 Introduction

In some experimental studies, researchers may wish to test the null hypothesis that there are no differences among treatment effects against an ordered alternative hypothesis. Researchers may be able to assume the treatments follow an a priori ordering, if they are different. The scope of this paper focusses on the nondecreasing ordered alternative hypothesis. That is,

$$H_0\colon\ \mu_1 = \mu_2 = \cdots = \mu_k \quad \text{versus} \quad H_1\colon\ \mu_1 \le \mu_2 \le \cdots \le \mu_k$$

where at least one inequality is strict and $\mu_i$ denotes the location parameter of the $i^{\text{th}}$ treatment. It is also possible that researchers may start out conducting an experiment using a randomized complete block design, but realize later that the design becomes too expensive and too hard to continue without several observations missing on treatments within a block. For instance, let us suppose a large company is interested in controlling the increasing cost of insurance of its employees. Hence, the company introduces a wellness program in an attempt to improve the overall health by reducing the average cholesterol level ($LDL$) of its employees. To test the competence of the program, the company starts measuring the cholesterol levels of random employees' samples – after their consent – three times in two years: at the beginning of the program and two times, annually, afterward. However, because the company is large and has an annual turnover of nearly 18% in its employees, many observations become obsolete because their donor employees left the company before completing them; therefore, their cholesterol level cannot be obtained anymore. To counter this problem, the company decides to discard observations that were incomplete for at least one year and perform a test using only a randomized complete block design (RCBD) to get accurate results on cholesterol levels over two years.

The company realizes that it loses a lot of data by sticking to a randomized complete block design (RCBD) which, in turn, hampers its effort to enhance its employees' general health. Thus, the company comes up with an idea to take advantage of the leftover observations that do not constitute a complete block and collect more observations over the next two years using a completely randomized design (CRD). Therefore, statistical test is required which combines the observations from the randomized complete block with those from the completely randomized design.

Page [1] proposed a nonparametric procedure that is applicable for testing nondecreasing ordered alternative when the data fit the two-way analysis of variance structure. Daniel [2] mentioned several assumptions for the validity of this test, including the independency of blocks, and no interaction between the blocks and the treatments. The test statistic is defined as

$$L = \sum_{j=1}^{k} jR_j \tag{1}$$

where $R_j$ is the $j^{th}$ treatment rank sum, based on the within block ranks of the original observations. Under $H_0$, the statistic $L$ has an asymptotic normal distribution with mean and variance, $bk(k+1)^2/4$ , $b(k^3 - k)^2/144(k-1)$, respectively. The standardized version of the statistic $L$ is defined as

$$Z_P = \frac{L - [bk(k+1)^2/4]}{\sqrt{b(k^3-k)^2/144(k-1)}} \tag{2}$$

where $b$ denotes the number of the blocks and $k$ denotes the number of treatments. Under $H_0$, the statistic $Z_P$ follows an asymptotic standard normal distribution (i.e., $N(0,1)$) and so the standard normal table can be used to get the critical values.

Jonckheere-Terpstra test (referred to as $JT$) is a widely known nonparametric test for the nondecreasing ordered alternatives in the k-sample case when the design is a completely randomized. The test was proposed independently by Terpstra [3] and Jonckheere [4]. For this test, the samples must be independent, and each sample is assumed to be drawn from a continuous population in which the distribution is the same for each population and may only differ in the location parameters. The test statistic is based on the summation of $k(k-1)/2$ Mann-Whitney statistics. Namely, it can be expressed as

$$JT = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} U_{ij} \tag{3}$$

where $U_{ij}$ is the U-statistics of Mann-Whitney and is defined as the number of pairs of observations ($X_{ia}$,$X_{ib}$) in which $X_{ia}$ is less than $X_{ib}$, once more, $X_{ia}$ is the $a^{th}$ observation in $i^{th}$ treatment sample, $a = 1, 2, \dots, n_i$ and $X_{jb}$ is the $b^{th}$ observation in $j^{th}$ treatment sample, $b = 1, 2, \dots, n_j$. Under the null hypothesis, $H_0$, the $JT$ statistic follows an asymptotic standard normal distribution with mean

$$E_0(JT) = \frac{N^2 - \sum_{i=1}^{k} n_i^2}{4} \tag{4}$$

and variance

$$Var_0(JT) = \frac{N^2(2N+3) - \sum_{i=1}^{k} n_i^2(2n_i+3)}{72} \tag{5}$$

where $N = \sum_{i=1}^{k} n_i$, and $n_i$ denotes the sample size of the $i^{th}$ treatment. However, the asymptotic normality of $JT$ depends on the number of samples. Jonckheere [4] mentioned that the normality approximation might be inaccurate if only one $n_i$ tends to infinity as $N$ increases. Therefore, to achieve the normality approximation, at least two samples increase as $N$ goes to infinity. Therefore, the standardized version of the test statistic, $JT$, is defined as

$$Z_{JT} = \frac{JT - E_0(JT)}{\sqrt{Var_0(JT)}} \tag{6}$$

Tryon and Hettmansperger [5] introduced a modified version of the *JT* test, *MJT*. Both tests, *JT* and *MJT*, are dealing with the nondecreasing ordered alternatives. However, the *MJT* test assigns more weights for each Mann-Whitney statistic based on the distance between the $i^{th}$ and $j^{th}$ populations. Thus, if the distance between the two populations is considered to be large, more amount of weight will be assigned to each Mann-Whitney statistic. The test statistic can be written as follows:

$$MJT = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j-i)U_{ij} \tag{7}$$

Under $H_0$, the asymptotic distribution for the test statistic follows a normal distribution with mean

$$E_0(MJT) = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j-i)\frac{n_i n_j}{2} \tag{8}$$

and variance

$$\begin{aligned} Var_0(MJT) &= Var\left\{\sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j-i)U_{ij}\right\} \\ &= \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j-i)^2 \ Var(U_{ij}) + 2\sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \sum_{s=1}^{k-1} \sum_{t=s+1}^{k} (j-i)(t-s) \ Cov(U_{ij}, U_{st}) \end{aligned} \tag{9}$$

where the values of the variance and the covariance terms can be defined as in Hollander and Wolfe [6] where

$$Var(U_{ij}) = \frac{n_i n_j (n_i + n_j + 1)}{12} \quad \text{for } 1 \le i \le j \le k \tag{10}$$

$$Cov(U_{ij}, U_{pq}) = \begin{cases} \dfrac{n_i n_j n_q}{12} & if \quad 1 \le i, p \le j \le k, j \ne q \\ -\dfrac{n_i n_j n_p}{12} & if \quad 1 \le p < j \le k, i = q \\ -\dfrac{n_i n_j n_q}{12} & if \quad 1 \le i < j < q \le k, j = p \\ \dfrac{n_i n_j n_p}{12} & if \quad 1 \le i, p \le j \le k, i \ne p \\ 0 & if \quad i, j, p, q \text{ are different} \end{cases} \tag{11}$$

Neuhäuser et al. [7] illustrated that in situations when the sample sizes are relatively small, the modified Jonckheere-Terpstra ($MJT$) has higher power than the original Jonckheere-Terpstra ( $JT$).

Terpstra and Magel [8] introduced a new nonparametric test for testing the nondecreasing ordered alternative that does not depend on pairwise information. Instead, it depends on the information that is obtained across all samples at the same time. The test statistic is given as follows

$$TM = \sum_{i_1=1}^{n_i} \cdots \sum_{i_k=1}^{n_k} I(X_{1i_1} \le X_{2i_2} \le \cdots \le X_{ki_k}) \tag{12}$$

where $I(X_{1i_1} \le X_{2i_2} \le \cdots \le X_{ki_k})$ is the indicator function that is equal to one only when there is at least one strict inequality and zero otherwise. Terpstra and Magel [8] compared their test to the *JT,* and the *MJT* tests. The results indicated that the proposed test has fairly higher power when the priori ordering is correct. However, if it is the other way around, the proposed test can have smaller power than the *JT* and the *MJT* tests.

Ferdhiana et al. [9] proposed a test that is similar to the test proposed by Terpstra and Magel [8], with the exception of function. Ferdhiana et al. [9] used the Kendall's Tau correlation coefficient instead of the indicator function used in the *TM* test.

Moreover, another nonparametric test is given by Terpstra et al. [10] that is analogous test to the *TM*; however, in this test the indicator function in Equation (12) was replaced by Spearman's correlation coefficient. The result of both tests indicate that the proposed test has higher power than the *JT,* the *MJT* and the *TM* when the sample sizes are small with large shift between the two adjacent location parameters.

Magel et al. [11] developed two tests for the nondecreasing ordered alternatives in mixed design. Part of the design is considered a data from a randomized complete block design (RCBD) and the other part is considered a data from a complete randomized design (CRD). The tests in [11] are a mixture of Page's test and Jonckheere-Terpstra test. The first test in [11], can be written as follows:

$$C_1 = \frac{Z_p + Z_{JT}}{\sqrt{2}} \tag{13}$$

Under $H_0$, $C_1$ follows a standard normal distribution since the standardized version of Page's test, and the standardized version of Jonckheere-Terpstra, $Z_p, Z_{JT}$, respectively, follow a standard normal distribution. Thus, the null hypothesis will be rejected when $C_1 \geq Z_\alpha$ where $Z_\alpha$ is the upper $\alpha$ quantile of the standard normal distribution.

The second test in Magel et al. [11] is based on the idea that the Page's test and Jonckheere-Terpstra test were added together first and then standardized. That is,

$$C_2 = \frac{L + JT - E(0)}{\sqrt{Var(0)}} \tag{14}$$

where

$$E(0) = \frac{bk(k+1)^2 + (N^2 - \sum_{i=1}^{k} n_i^2)}{4} \tag{15}$$

and

$$Var(0) = \frac{b(k^3 - k)^2}{144(k-1)} + \frac{N^2(2N+3) - \sum_{i=1}^{k} n_i^2(2n_i + 3)}{72} \tag{16}$$

It can be noted that the mean and variance are the sum of Page's test and the Jonckheere-Terpstra test. More specifically, $\frac{bk(k+1)^2}{4}$ denotes the mean of Page's test and $\frac{(N^2-\sum_{i=1}^{k} n_i^2)}{4}$ denotes the mean of the Jonckheere-Terpstra. Similarly, $\frac{b\left(k^3-k\right)^2}{144(k-1)}$ and $\frac{N^2(2N+3)-\sum_{i=1}^{k} n_i^2(2n_i+3)}{72}$ denote the variance of Page test ($L$) and the variance of the Jonckheere-Terpstra test (*JT*), respectively. The null hypothesis would again be rejected if $C_2 \geq Z_\alpha$ .

# 2 Proposed Methods and Simulation Study

The Jonckheere-Terpstra (*JT*) test and some modifications of the *JT* test already exist for the completely randomized design (CRD). However, we need to propose new versions of the *JT* test for the randomized complete block design (RCBD) portion of the design.

## 2.1 First proposed method

In this proposed method, we are applying the idea discussed by Tryon and Hettmansperger [5] where the distance between the two populations is multiplied by the $i^{th}$ population. That is,

$$NMJT = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} i(j-i)U_{ij} \tag{17}$$

Since we are dealing with a mixed design, the standardized version of the test that will be applied to the CRD portion is written as

$$Z_{NMJT} = \frac{NMJT - E(NMJT)}{\sqrt{Var(NMJT)}} \tag{18}$$

where $NMJT$ is defined as the unstandardized version of the new modified Jonckheere-Terpstra test with mean

$$E(NMJT) = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} i(j-i)\frac{n_i n_j}{2} \tag{19}$$

and variance

$$\begin{aligned} Var\ (NMJT) &= Var\left(\sum_{i=1}^{k-1} \sum_{j=i+1}^{k} i(j-i)U_{ij}\right) \\ &= \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} i^2(j-i)^2\ Var\big(U_{ij}\big) + 2\sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \sum_{s=1}^{k-1} \sum_{t=s+1}^{k} (i.s)(j-i)(t-s)\ Cov\big(U_{ij}, U_{st}\big) \end{aligned} \tag{20}$$

Moreover, the values of the variance and the covariance terms can be obtained using Equation (10) and (11). Under $H_0$, as $\min(n_1, n_2, \dots, n_k)$ tends to infinity,

$$Z_{NMJT} \xrightarrow{D} N(0,1)$$

As for the RCBD portion, a test is designed by applying the $NMJT$ test to each block (i.e., $NMJT_1$, $NMJT_2$,..., $NMJT_l$ ; $l = 1, 2,\dots, b$) where $NMJT_1$ denotes the new version of the modified Jonckheere-Terpstra test for the first block, $NMJT_2$ denotes the new version of the modified Jonckheere-Terpstra test for the second block, and so on. Then, we sum up all the $NMJT$ tests together to form the $BNMJT$ test. That is,

$$BNMJT = \sum_{l=1}^{b} NMJT_l \tag{21}$$

where $b$ is the number of blocks. Corresponding to Equation (19) and since we are considering *one* observation per block-treatment, the mean of $BNMJT$ test can then be written as

$$E(BNMJT) = \sum_{l=1}^{b} E(NMJT_l) = \sum_{l=1}^{b} \left(\sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \frac{i(j-i)}{2}\right) \tag{22}$$

In like manner, the variance of $BNMJT$ test can be defined as

$$Var(BNMJT) = \sum_{l=1}^{b} Var(NMJT_l) \tag{23}$$
$$= \sum_{l=1}^{b} \left( 3 \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \frac{i^2 (j-i)^2}{12} + 2 \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \sum_{s=1}^{k-1} \sum_{t=s+1}^{k} (i.s)(j-i)(t-s)\, Cov(U_{ij}, U_{st}) \right)$$

where $b$ is the number of blocks and $NMJT_l$ is the new (multiplied) modified Jonckheere-Terpstra test for the $l^{th}$ block. Moreover, the covariance term can be obtained from Equation (11). The standardized version of $BNMJT$ test is written as

$$Z_{BNMJT} = \frac{BNMJT - E(BNMJT)}{\sqrt{Var(BNMJT)}} \tag{24}$$

where $BNMJT$ is defined as the unstandardized version of the summation of the new modified Jonckheere-Terpstra test for the entire blocks. Under $H_0$, as $\min(n_1, n_2, \ldots n_k)$ tends to infinity,

$$Z_{BNMJT} \xrightarrow{D} N(0,1)$$

Therefore, the first proposed method for the nondecreasing ordered alternatives in a mixed design is written as follows:

$$T_1 = \frac{Z_{NMJT} + Z_{BNMJT}}{\sqrt{2}} \tag{25}$$

Here, we added the standardized version of $NMJT$ and $BNMJT$ together first, and then we standardized the two tests by subtracting the means and divided by the standard deviations.

Under $H_0$, for large sample sizes, this method will follow an asymptotic normal distribution since it is a combination of two tests which follow a standard normal distribution. Thus, the null hypothesis, $H_0$, will be rejected when $T_1 \geq Z_\alpha$ where $Z_\alpha$ is the upper $\alpha$ quantile of the standard normal distribution.

## 2.2 Second proposed method

The second method we are proposing for the nondecreasing ordered alternatives in a mixed design is designed as follows:

$$T_2 = \frac{T_2^* - [E(NMJT) + E(BNMJT)]}{\sqrt{V(NMJT) + V(BNMJT)}} \tag{26}$$

where

$$T_2^* = NMJT + BNMJT = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} i(j-i)U_{ij} + \sum_{l=1}^{b} NMJT_l \tag{27}$$

Here, $k$ is the number of treatments, and $b$ is the number of blocks. $NMJT$ is the new modified Jonckheere-Terpstra test for the $l^{th}$ block. Under $H_0$, for large sample sizes, this test will also follow an asymptotic normal distribution. The null hypothesis, $H_0$, will be rejected when $T_2 \geq Z_\alpha$ .

It is noted that versions of the Jonckheere-Terpstra test are being developed for other types of designs such as a two-stage nested design [12].

### 2.3 Simulation study

The aim of this section is to describe in detail the Monte Carlo simulation study that is used to investigate the performance of the proposed tests and to compare them to each other and with the tests proposed by Magel et al. [11]. The performance of the tests is evaluated by the estimated powers and whether or not they maintain the stated level of significance (α). The power of a test can be defined as the probability of rejecting a false $H_0$. Likewise, the level of significance (α) is defined as the probability of rejecting a true $H_0$.

In this study, three underlying distributions are used including, the standard normal distribution, the exponential distribution with mean one, and t-distribution with $DF = 3$. For each distribution, we consider three scenarios of proportions of the number of blocks in the RCBD portion to the sample size in the CRD portion, namely, assuming that the portion of the number of blocks in RCBD is *larger*, *equal*, and *smaller* than the portion of the sample size in the CRD. However, regarding the CRD portion, we consider cases where the sample sizes are equal.

Based on 5000 iterations, the study is conducted for each combination of distributions with nondecreasing location parameters at 0.05 level of significance. The level of significance is estimated for each test by generating 5,000 sets of samples from the populations when the null hypothesis is true (i.e., the location parameter arrangements are the same for all treatments) and counting the number of times the null hypothesis is rejected, dividing by the number of iterations. Similarly, the power for each test is estimated by generating 5,000 sets of samples from the populations when the alternative hypothesis is true (i.e., the location parameter arrangements are different for at least one treatment) and count the number of times the null hypothesis is rejected, divided by the number of iterations. All the simulations are performed using the statistical program SAS9.4.

Moreover, powers are estimated based on a variety of location parameter arrangements. In particular, we considered cases when there is an equal distance between the parameters; cases where the distances between the parameters are distinct; cases where the some parameters are equal and the rest were different; cases where the distance between the some parameters is twice as large as the distance of others; and cases where the distance between the first two parameters are chosen to be the same as the distance between the last parameters. The location parameters are denoted by $\mu_1$, $\mu_2$, $\mu_3$, $\mu_4$, and $\mu_5$ for treatment *one*, *two*, *three*, *four*, and *five*, respectively.

## 3 Results and Discussion

The results of the simulation study play a significant role in evaluating the performance of the proposed tests compared to each other and to those proposed by Magel et al. [11]. Thus, the aim of this section is to introduce the results of the simulation study described in the previous section. The proposed tests are designed to analyze data in a mixed design of a CRD and a RCBD.

In each table, the results of the simulation study are defined based on number of treatments ($k$), the distribution used to simulate the data, the sample size in the CRD portion, and the number of blocks in the RCBD portion. Besides, a variety of location parameter arrangements, shifts, are considered on $k = 3$, 4, and 5 treatments. The estimated level of significance (α) and the estimated power for each test are given so that the first entry of each table represents the estimated α-level; however, the rest represent the estimated powers.

Results as to which test did better in particular situation are consistent regardless of the number of treatments (namely, 3, 4, or 5) and the different distributions considered. Results are shown in Tables 1 and 2 for cases when we have four treatments ($k = 4$) and the number of blocks in the randomized complete block design

(RCBD) portion is *larger* than the sample sizes in the completely randomized design (CRD) portion. These results are for the standard normal and standard exponential distributions. It can be seen that the first proposed test ($T_1$) has higher powers in cases where the arrangements of location parameters follow the pattern that the first two parameters are equal and the last two parameters are different such as (0, 0, 0.05, 0.3), cases where the arrangements of location parameters following the pattern that distance between the first two parameter is equal to the distance between the last two parameters such as (0, 0.5, 0.5, 1), and lastly cases where the arrangements of location parameters follow the pattern that the distance between the fourth and third parameters is twice as large as the distance between the third and the second, while the distance between the third and the second parameters is twice as large as the distance between the second and the first such as (0, 0.1, 0.3, 0.7).

**Table 1. Percentage of rejection for $k$ = 4; Exponential distribution: Block = 16 and $n$ = 8**

| $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $C_1$ | $C_2$ | $T_1$ | $T_2$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 5.46 | 4.96 | 5.32 | 5.04 |
| 0 | 0.1 | 0.2 | 0.3 | **47.44** | 38.36 | 43.56 | 29.48 |
| 0 | 0 | 0.25 | 0.25 | **47.22** | 37.30 | 44.64 | 30.54 |
| 0 | 0.125 | 0.25 | 0.25 | **40.18** | 32.42 | 31.44 | 22.04 |
| 0 | 0 | 0 | 0.5 | 67.54 | 56.12 | **81.80** | 60.38 |
| 0.05 | 0.1 | 0.3 | 0.5 | 74.88 | 64.76 | **76.68** | 56.02 |
| 0 | 0.25 | 0.5 | 0.5 | 80.66 | 71.60 | 69.94 | 50.62 |
| 0 | 0.5 | 0.5 | 1 | 98.84 | 96.24 | **99.08** | 90.26 |
| 0.1 | 0.2 | 0.6 | 1 | 99.42 | 97.64 | **99.50** | 94.22 |
| 0.25 | 0.25 | 0.5 | 0.5 | **47.22** | 37.30 | 43.46 | 29.76 |
| 0 | 0.1 | 0.3 | 0.7 | 94.54 | 88.52 | **96.24** | 83.78 |
| 0 | 0.05 | 0.15 | 0.35 | 55.40 | 44.96 | **57.38** | 38.74 |
| 0 | 0.15 | 0.2 | 0.5 | 74.84 | 64.26 | **76.14** | 54.54 |
| 0 | 0 | 0.05 | 0.3 | 42.26 | 33.54 | **48.24** | 33.06 |
| 0 | 0 | 0.1 | 0.6 | 85.60 | 75.74 | **92.66** | 74.76 |

**Table 2. Percentage of rejection for $k$ = 4; Normal distribution: Block = 32 and $n$ = 8**

| $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $C_1$ | $C_2$ | $T_1$ | $T_2$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 4.78 | 5.32 | 5.00 | 4.86 |
| 0 | 0.1 | 0.2 | 0.3 | **33.84** | 28.40 | 31.24 | 20.68 |
| 0 | 0 | 0.25 | 0.25 | **33.58** | 28.18 | 31.88 | 21.08 |
| 0 | 0.125 | 0.25 | 0.25 | **28.84** | 24.18 | 22.54 | 15.86 |
| 0 | 0 | 0 | 0.5 | 55.48 | 47.06 | **68.00** | 45.72 |
| 0.05 | 0.1 | 0.3 | 0.5 | 59.14 | 49.76 | **59.34** | 38.16 |
| 0 | 0.25 | 0.5 | 0.5 | **67.10** | 58.12 | 55.18 | 36.58 |
| 0 | 0.5 | 0.5 | 1 | **97.14** | 93.06 | 95.26 | 77.86 |
| 0.1 | 0.2 | 0.6 | 1 | 97.80 | 94.56 | **98.12** | 86.34 |
| 0.25 | 0.25 | 0.5 | 0.5 | **33.58** | 28.18 | 31.70 | 21.60 |
| 0 | 0.1 | 0.3 | 0.7 | 85.56 | 77.40 | **89.26** | 67.12 |
| 0 | 0.05 | 0.15 | 0.35 | 40.08 | 33.68 | **42.32** | 27.36 |
| 0 | 0.15 | 0.2 | 0.5 | **59.00** | 50.32 | 58.60 | 39.26 |
| 0 | 0 | 0.05 | 0.3 | 31.28 | 26.88 | **35.98** | 24.52 |
| 0 | 0 | 0.1 | 0.6 | 72.80 | 63.10 | **83.18** | 58.68 |

Tables 3 and 4 show the results of the simulation study for cases when $k$ = 4, and the number of blocks in the RCBD portion is equal relative to the sample sizes in the CRD portion. When one of the parameters is quite a bit larger relative to the other parameters, the first proposed test ($T_1$) has the highest powers. As an example of this, consider the case where the first two parameters are equal and the last two are distinct such as (0, 0, 0.1, 0.6). When the parameters are equally spaced, or the last two parameters are equal to each other, $C_1$ has the highest powers. The test $C_1$ was one of the tests proposed by Magel et al. [11] and given in Equation (13).

**Table 3. Percentage of rejection for *k* = 4; Exponential distribution: Block = 10 and *n* = 10**

| $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $C_1$ | $C_2$ | $T_1$ | $T_2$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 5.26 | 5.00 | 4.88 | 5.04 |
| 0 | 0.1 | 0.2 | 0.3 | **42.36** | 35.24 | 38.36 | 28.80 |
| 0 | 0 | 0.25 | 0.25 | **41.70** | 35.80 | 40.60 | 31.66 |
| 0 | 0.125 | 0.25 | 0.25 | **36.12** | 31.04 | 29.50 | 23.26 |
| 0 | 0 | 0 | 0.5 | 62.86 | 52.34 | **76.82** | 60.74 |
| 0.05 | 0.1 | 0.3 | 0.5 | 69.52 | 60.74 | **72.60** | 57.36 |
| 0 | 0.25 | 0.5 | 0.5 | **76.84** | 67.42 | 66.26 | 51.62 |
| 0 | 0.5 | 0.5 | 1 | **98.08** | 94.84 | 97.32 | 90.84 |
| 0.1 | 0.2 | 0.6 | 1 | 98.76 | 96.44 | **99.20** | 95.48 |
| 0.25 | 0.25 | 0.5 | 0.5 | **41.64** | 34.92 | 40.44 | 31.12 |
| 0 | 0.1 | 0.3 | 0.7 | 92.10 | 85.46 | **94.96** | 84.32 |
| 0 | 0.05 | 0.15 | 0.35 | 51.64 | 43.26 | **53.28** | 40.66 |
| 0 | 0.15 | 0.2 | 0.5 | 69.10 | 59.52 | **70.60** | 54.58 |
| 0 | 0 | 0.05 | 0.3 | 39.72 | 31.58 | **46.56** | 33.36 |
| 0 | 0 | 0.1 | 0.6 | 80.86 | 70.32 | **89.16** | 74.68 |

**Table 4. Percentage of rejection for *k* = 4; Normal distribution: Block = 20 and *n* = 20**

| $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $C_1$ | $C_2$ | $T_1$ | $T_2$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 4.46 | 5.08 | 4.68 | 5.16 |
| 0 | 0.1 | 0.2 | 0.3 | **35.22** | 27.38 | 32.92 | 24.40 |
| 0 | 0 | 0.25 | 0.25 | **37.24** | 27.72 | 34.82 | 25.12 |
| 0 | 0.125 | 0.25 | 0.25 | **31.24** | 24.10 | 25.74 | 19.04 |
| 0 | 0 | 0 | 0.5 | 61.36 | 46.78 | **73.12** | 53.98 |
| 0.05 | 0.1 | 0.3 | 0.5 | 63.34 | 49.48 | **64.60** | 47.76 |
| 0 | 0.25 | 0.5 | 0.5 | **73.80** | 56.50 | 60.36 | 42.82 |
| 0 | 0.5 | 0.5 | 1 | **98.78** | 93.06 | 97.50 | 87.28 |
| 0.1 | 0.2 | 0.6 | 1 | 99.10 | 94.78 | **99.40** | 94.24 |
| 0.25 | 0.25 | 0.5 | 0.5 | **36.48** | 28.28 | 34.80 | 25.48 |
| 0 | 0.1 | 0.3 | 0.7 | 90.90 | 78.30 | **92.10** | 77.94 |
| 0 | 0.05 | 0.15 | 0.35 | 44.28 | 34.06 | **46.22** | 33.30 |
| 0 | 0.15 | 0.2 | 0.5 | 63.84 | 49.10 | **65.02** | 47.02 |
| 0 | 0 | 0.05 | 0.3 | 34.18 | 25.80 | **40.46** | 28.96 |
| 0 | 0 | 0.1 | 0.6 | 79.74 | 64.04 | **87.98** | 70.48 |

Tables 5 and 6 show the results of the proposed tests and the tests proposed by Magel et al. [11] in terms of the estimated level of significance and the estimated powers for the normal, exponential, and student's t distributions for four treatments ($k$ = 4) when the proportion of the RCBD portion is *smaller,* than the CRD portion. Results were the same as when the CRD and RCBD portions were equal as to which test performed better when the RCBD portion was 1/2 the CRD portion (Table 5).

When the proportion of the number of blocks in the RCBD is *one-eighth* the sample size in the CRD portion (Table 6), the proposed test $T_2$ tends to have higher estimated powers then the others under the following location parameters arrangements: (0, 0, 0, 0.5), (0.05, 0.1, 0.3, 0.5), (0, 0.1, 0.3, 0.7), (0, 0.05, 0.15, 0.35) and (0, 0, 0.05, 0.3). Note that in this case, it is the second versions of the test statistics that perform better.

In Tables 7 and 8, the RCBD portion is larger than the CRD portion. The results in this case are the same as the results when the RCBD portion is equal to the CRD portion. The first versions of the test statistics have larger powers. The first newly proposed test statistic has larger powers when there is a relatively large difference between the last two location parameters. When the parameters are equally spaced or fairly close to each other, $C_1$ (one of the tests in Magel et al. [11]) has higher powers. In Table 7, $k$ = 3 and in Table 8, $k$ = 5. The number of populations did not affect the result as to which test statistic had higher powers, but the spacing between the last two parameters did.

**Table 5. Percentage of rejection for $k$ = 4; Exponential distribution: Block = 8 and $n$ = 16**

| $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $C_1$ | $C_2$ | $T_1$ | $T_2$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 5.00 | 4.72 | 4.68 | 4.50 |
| 0 | 0.1 | 0.2 | 0.3 | **49.48** | 44.84 | 46.14 | 39.62 |
| 0 | 0 | 0.25 | 0.25 | **47.62** | 42.34 | 44.96 | 39.12 |
| 0 | 0.125 | 0.25 | 0.25 | **40.94** | 36.76 | 32.36 | 27.86 |
| 0 | 0 | 0 | 0.5 | 69.84 | 62.88 | **83.86** | 75.62 |
| 0.05 | 0.1 | 0.3 | 0.5 | 77.68 | 70.96 | **79.70** | 70.98 |
| 0 | 0.25 | 0.5 | 0.5 | **83.06** | 76.72 | 72.40 | 64.06 |
| 0 | 0.5 | 0.5 | 1 | **99.40** | 98.22 | 99.08 | 96.88 |
| 0.1 | 0.2 | 0.6 | 1 | 99.58 | 98.90 | **99.88** | 99.14 |
| 0.25 | 0.25 | 0.5 | 0.5 | **46.10** | 41.28 | 43.28 | 37.70 |
| 0 | 0.1 | 0.3 | 0.7 | 95.84 | 92.14 | **97.44** | 93.56 |
| 0 | 0.05 | 0.15 | 0.35 | 56.50 | 50.82 | **59.92** | 50.76 |
| 0 | 0.15 | 0.2 | 0.5 | 76.32 | 70.08 | **78.42** | 69.04 |
| 0 | 0 | 0.05 | 0.3 | 43.66 | 37.30 | **51.80** | 42.40 |
| 0 | 0 | 0.1 | 0.6 | 86.58 | 81.58 | **93.80** | 88.44 |

**Table 6. Percentage of rejection for $k$ = 4; Normal distribution: Block = 4 and $n$ = 32**

| $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $C_1$ | $C_2$ | $T_1$ | $T_2$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 4.76 | 4.84 | 5.26 | 5.16 |
| 0 | 0.1 | 0.2 | 0.3 | 31.10 | **34.60** | 29.88 | 32.32 |
| 0 | 0 | 0.25 | 0.25 | 31.20 | **34.04** | 29.90 | 32.24 |
| 0 | 0.125 | 0.25 | 0.25 | 25.58 | **27.62** | 21.04 | 23.26 |
| 0 | 0 | 0 | 0.5 | 52.04 | 56.78 | 63.36 | **68.78** |
| 0.05 | 0.1 | 0.3 | 0.5 | 54.36 | 59.78 | 55.38 | **61.34** |
| 0 | 0.25 | 0.5 | 0.5 | 64.84 | **69.18** | 52.80 | 56.84 |
| 0 | 0.5 | 0.5 | 1 | 96.50 | **97.80** | 93.68 | 96.46 |
| 0.1 | 0.2 | 0.6 | 1 | 97.08 | 98.46 | 97.34 | **98.60** |
| 0.25 | 0.25 | 0.5 | 0.5 | 30.42 | **34.58** | 29.54 | 32.88 |
| 0 | 0.1 | 0.3 | 0.7 | 84.74 | 88.54 | 86.80 | **90.86** |
| 0 | 0.05 | 0.15 | 0.35 | 36.52 | 41.76 | 39.52 | **43.92** |
| 0 | 0.15 | 0.2 | 0.5 | 54.80 | 61.02 | 55.30 | **61.34** |
| 0 | 0 | 0.05 | 0.3 | 29.58 | 31.78 | 35.20 | **37.54** |
| 0 | 0 | 0.1 | 0.6 | 69.60 | 75.26 | 78.96 | **83.08** |

**Table 7. Percentage of rejection for *k* = 3; Normal distribution: Block = 16 and *n* = 4**

| $\mu_1$ | $\mu_2$ | $\mu_3$ | $C_1$ | $C_2$ | $T_1$ | $T_2$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 4.88 | 4.98 | 4.74 | 4.36 |
| 0 | 0 | 0.5 | 36.52 | 35.58 | **41.28** | 33.70 |
| 0 | 0.5 | 0.5 | **36.32** | 35.68 | 27.48 | 23.18 |
| 0.05 | 0.25 | 0.5 | 31.92 | 30.96 | **32.14** | 26.18 |
| 0 | 0.3 | 0.5 | **37.10** | 36.14 | 33.76 | 27.90 |
| 0 | 0 | 1 | 80.92 | 78.96 | **88.84** | 80.40 |
| 0 | 1 | 1 | 80.86 | 78.92 | 66.56 | 56.16 |
| 0 | 0.5 | 1 | **81.86** | 80.32 | 80.62 | 70.96 |
| 0.5 | 0.5 | 1 | 36.52 | 35.58 | **41.70** | 34.44 |
| 0.5 | 1 | 1 | **36.22** | 35.68 | 29.34 | 23.98 |
| 0.1 | 0.5 | 1 | 74.12 | 72.38 | **75.36** | 64.90 |
| 0.1 | 0.3 | 0.7 | 46.30 | 44.92 | **47.60** | 38.78 |
| 0 | 0.25 | 0.5 | **36.88** | 35.94 | 35.32 | 29.52 |
| 0.2 | 0.5 | 0.8 | **46.12** | 44.96 | 43.98 | 36.38 |
| 0 | 0.1 | 0.8 | 65.68 | 63.34 | **71.64** | 61.24 |

**Table 8. Percentage of rejection for *k* = 5; Normal distribution: Block = 16 and *n* =** 8

| $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $\mu_5$ | $C_1$ | $C_2$ | $T_1$ | $T_2$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 5.38 | 4.86 | 5.06 | 4.78 |
| 0.05 | 0.15 | 0.25 | 0.35 | 0.45 | **40.76** | 35.06 | 37.22 | 26.12 |
| 0 | 0.025 | 0.075 | 0.175 | 0.375 | 34.90 | 30.04 | **39.42** | 27.02 |
| 0 | 0 | 0 | 0 | 0.5 | 39.86 | 33.24 | **51.70** | 35.72 |
| 0 | 0 | 0.125 | 0.25 | 0.25 | **27.70** | 23.74 | 25.48 | 19.02 |
| 0 | 0.05 | 0.05 | 0.3 | 0.3 | **32.62** | 27.78 | 31.02 | 21.82 |
| 0.05 | 0.2 | 0.3 | 0.4 | 0.5 | **45.74** | 39.62 | 41.16 | 28.78 |
| 0 | 0 | 0 | 0.25 | 0.5 | 53.34 | 46.82 | **62.08** | 43.60 |
| 0 | 0 | 0 | 0.35 | 0.35 | 42.90 | 37.00 | **46.80** | 32.04 |
| 0 | 0 | 0.25 | 0.25 | 0.5 | 53.88 | 46.86 | 53.66 | 36.66 |
| 0 | 0 | 0 | 0.1 | 0.3 | 25.42 | 22.06 | **30.28** | 20.30 |
| 0 | 0 | 0 | 0.2 | 0.7 | 71.18 | 63.20 | **81.62** | 61.48 |
| 0 | 0.1 | 0.1 | 0.6 | 0.6 | 76.70 | 69.10 | **76.76** | 56.04 |
| 0 | 0.1 | 0.3 | 0.4 | 0.4 | **45.82** | 39.80 | 38.80 | 25.88 |
| 0 | 0.05 | 0.2 | 0.4 | 0.4 | **48.64** | 42.20 | 43.76 | 29.74 |

# 4 Conclusion

In this paper, novel nonparametric methods for the nondecreasing ordered alternative are proposed for a mixed design consisting of a combination of a CRD and RCBD. Three cases were considered where the number of blocks were proportional to the sample sizes. In particular, the number of blocks in the RCBD portion are *larger*, *equal*, and *smaller* than the sample sizes in the CRD portion. In either case, from the findings of the simulation study, it was shown that both the proposed tests appear to maintain their type one errors. This was also true of the tests proposed in Magel et al. [11].

Moreover, the estimated powers for the method formed by standardized last idea ($T_2$) are less than the method formed by standardized first idea ($T_1$) under all distributions considered for all cases when the RCBD portion is at least 1/2 or greater of the CRD portion. When the number of blocks in the RCBD portion are *one-eighth* the sample sizes in the CRD portion, we found ($T_2$) had higher powers than ($T_1$).

The simulation study has also shown that the estimated power for the proposed test ($T_1$) is better than the test statistics proposed by Magel et al. [11] under the nondecreasing ordered alternative as long as a large jump is present between the last two adjacent location parameters such as (0, 0, 0.1, 0.6) and (0, 0, 0, 0.5) when the

RCBD portion was 1/2 or greater of the CRD portion. If the RCBD portion was only 1/8 that of the CRD portion, then $T_2$ had the largest powers of all the test statistics. If there was not a large jump between the last two parameters, $C_1$ had the largest powers when the RCBD portion was at least 1/2 of the CRD portion. If the RCBD portion was only 1/8 the CRD portion under this circumstance, then $C_2$ had the larger powers.

## Competing Interests

Authors have declared that no competing interests exist.

## References

[1] Page EB. Ordered hypotheses for multiple treatments: A significance test for linear ranks. Journal of the American Statistical Association. 1963;58:216-230.

[2] Daniel WW. Applied nonparametric statistics. 2nd Edition. PWS-Kent Publishing Company, Boston; 1990.

[3] Terpstra TJ. The asymptotic normality and consistency of Kendall's test against trend, when ties are present in one ranking. Indagationes. Mathematicae. 1952;14:327-333.

[4] Jonckheere AR. A distribution-free k-sample test against ordered alternatives. Biometrika. 1951;41:133-145.

[5] Tryon PV, Hettmansperger TP. A class of nonparametric tests for homogeneity against ordered alternatives. Annals of Statistics. 1973;1:1061-1070.

[6] Hollander M, Wolfe D. Nonparametric statistical methods. (2nd Edition), John Wiley and Sons, New York, USA; 1999.

[7] Neuhäuser M, Liu PY, Hothorn L. Nonparametric tests for trend: Jonckheere's test, a modification and maximum test. Biometrical Journal. 1998;40(8):899-909.

[8] Terpstra J, Magel R. A new nonparametric test for the ordered alternative problem. Nonparametric Statistics. 2003;15(3):289-301.

[9] Ferdhiana R, Terpstra J, Magel R. A nonparametric test for the ordered alternative based on Kendall's correlation coefficient. Communication in Statistics-Simulation and Computation®. 2008;37(6):1117-1128.

[10] Terpstra J, Chang C, Magel R. On the use of Spearman's correlation coefficient for testing ordered alternatives. Journal of Statistical Computation and Simulation. 2011;81(11):1381-1392.

[11] Magel R, Terpstra J, Wen J. Proposed tests for the nondecreasing alternative in a mixed design. Journal of Statistics and Management Systems. 2009;12(5):963-977.

[12] Le Bang. Nonparametric tests for ordered alternatives in a two-stage nested design. Dissertations. 2019;3470.
Available:https://scholarworks.wmich.edu/dissertations/3470



***Peer-review history:***
*The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)*
*http://www.sdiarticle4.com/review-history/57771*