Asian Journal of Probability and Statistics

7(4): 1-14, 2020; Article no.AJPAS.58031

ISSN: 2582-0230

Ameliorated Class of Estimators of Finite Population Variance

J. O. Muili[1*], E. N. Agwamba[1], A. B. Odeyale[2] and A. Adebiyi[2]

[1]Department of Mathematics, Kebbi State University of Science and Technology, Aliero, Nigeria.
[2]Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria.

Authors' contributions

This work was carried out in collaboration among all authors. Author JOM designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors ENA and ABO managed the analyses of the study. Author AA managed the literature searches. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJPAS/2020/v7i430189
Editor(s):
(1) Dr. Belkacem Chaouchi, Khemis Miliana University, Algeria.
Reviewers:
(1) Waqar Hafeez, Universiti Utara, Malaysia.
(2) Abhiram Dash, Odisha University of Agriculture and Technology, India.
(3) Heiber Cárdenas Henao, Universidad del Valle, Colombia.
Complete Peer review History: http://www.sdiarticle4.com/review-history/58031

Received: 17 April 2020
Accepted: 23 June 2020
Published: 29 June 2020

Original Research Article

Abstract

A class of ratio estimators of finite population variance is proposed in this study. The properties of the proposed estimators have been derived using Taylor's Series method up to first order of approximation. The efficiency conditions which are the mean square errors (MSEs) and percentage relative efficiency (PRE) of the proposed estimators over existing estimators have been established. The analytical illustration was also conducted to affirm the theoretical results. The results of the empirical study revealed that the proposed estimators are more efficient than the existing estimators considered in the study.

Keywords: Mid-range; quartiles; mean square error; estimator; finite population variance.

Abbreviations

MSE : *Mean Square Error*
PRE : *Percentage Relative Efficiency*
N : *Population size*

**Corresponding author: E-mail: jamiunice@yahoo.com;*

n : *Sample size*
Y : *Study variable*
X : *Auxiliary variable*
$\bar{y}, \bar{x}$: *Sample means of study and auxiliary variables*
$\bar{Y}, \bar{X}$: *Population means of study and auxiliary variables*
ρ : *Coefficient of correlation*
C_y, C_x : *Coefficient of variations of study and auxiliary variables*
$\beta_{1(y)}$: *Coefficient of skewness of study variable*
$\beta_{1(x)}$: *Coefficient of skewness of auxiliary variable*
$\beta_{2(y)}$: *Coefficient of kurtosis of study variable*
$\beta_{2(x)}$: *Coefficient of kurtosis of auxiliary variable*
TM : *Tri-Mean*
Q_3 : *The upper quartile*
Q_c : *Coefficient of quartile deviation*
M_d : *Median of the auxiliary variable*
QD : *Quartile Deviation*
G : *Gini's Mean Difference*
D : *Downtown's method*
S_{pw} : *Probability Weighted moments*
D_i : *Deciles i=1,2,3,…,10*
HL : *Hodges Lemma*
MR : *Mid-range*
$Q_{M.A}$: *Quartile mean average*
$D_{M.A}$: *Decile mean average*

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i,\ \bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i,\ \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i,\ \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i,\ \gamma = \frac{1-f}{n},\ f = \frac{n}{N},\ QD = \frac{(Q_3 - Q_1)}{2},$$

$$Q_c = \frac{(Q_3 - Q_1)}{(Q_3 + Q_1)},\ s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2,\ s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2,\ S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2,$$

$$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2,\ HL = Median\left(\frac{(X_i + X_j)}{2}, 1 \le i \le j \le N\right),\ MR = \frac{(X_1 + X_N)}{2},$$

$$TM = \frac{(Q_1 + 2Q_2 + Q_3)}{4}, S_{pw} = \frac{\sqrt{\lambda}}{N^2}\sum_{1}^{N}(2i - N - 1)X_i$$

Percentage Relative Efficiency (PRE) is given as:

$PRE = \frac{MSE\left(Var(\hat{t})\right)}{MSE\left(\hat{S}_i^2\right)}\ X\ 100$, where $\hat{S}_i^2$ are the existing and proposed estimators.

1 Introduction

In sample survey, it is well known that the efficiency of an estimator can be increased by the proper use of auxiliary information associated to auxiliary variable that is related with the study variable. Auxiliary information is used in various forms and at different stages of sampling in the form of population parameters of auxiliary variable for more efficiently estimating population parameters of the study variable under investigation. Whenever the information on an auxiliary variable X is known, a number of estimators come in like ratio: Kadilar and Cingi [1], exponential: Bahl and Tuteja, [2], product: Singh, [3] and linear regression: Singh and Ruiz, [4] estimators are available in the literature. When the correlation between the study variable and the auxiliary variable is positive, ratio method of estimation is quite effective. On the other hand, when the correlation is negative, Product method of estimation can be employed effectively. The estimation of the population variance of the study variable Y has received a considerable attention from experts engaged in survey statistics. For example, in agriculture, medical, and economical fields, etc. Upper quartile (Q_3) is of the quartiles which is not influenced by outliers or a skewed data set in observation of the study variable. The early work on the estimation of population variance was initiated from the work of Evans [5], Hansen, et al. [6] and Isaki [7]. Several authors have used auxiliary information in the estimation of population variance like Das and Triphati [8], Kadilar and Cingi [1], Gupta and Shabbir [9], Tailor and Sharma [10], Audu et al. [11], etc.

In this study, improvement of ratio type estimators in simple random sampling for estimating finite population variance has been proposed with the linear combination of upper quartile and mid-range.

Let $\Delta = (1, 2, 3, \ldots, N)$ be a population of size N and Y, X are two real valued functions having values $(Y_i,\ X_i) \in \mathbb{R}^+ > 0$ on the i^{th} unit of $U(1 \le i \le N)$. Let S_y^2 and S_x^2 be the finite population variance of Y and X respectively and s_y^2 and s_x^2 be respective sample variances based on the random sample of size n drawn without replacement.

2 Literature Review

The sample variance estimator of the finite population variance is defined as

$$\hat{t} = s_y^2 \tag{2.1}$$

which is an unbiased estimator and its variance is given as:

$$Var\left(\hat{t}\right) = \gamma S_y^4\left(\beta_{2(y)} - 1\right) \tag{2.2}$$

Isaki [7] proposed a ratio type variance estimator for the finite population variance when the population mean square of auxiliary variable X is known. The bias and its mean squared error are given below:

$$\hat{S}_R^2 = s_y^2 \frac{S_x^2}{s_x^2} \tag{2.3}$$

$$Bias\left(\hat{S}_R^2\right) = \gamma S_y^4\left[\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right] \tag{2.4}$$

$$MSE\left(\hat{S}_R^2\right) = \gamma S_y^4\left[\left(\beta_{2(y)} - 1\right) + \left(\beta_{2(x)} - 1\right) - 2\left(\lambda_{22} - 1\right)\right] \tag{2.5}$$

Kadilar and Cingi [1] proposed a class of ratio type estimators for finite population variance by imposing Coefficient of variation and Coefficient of kurtosis on the work of Isaki [7] as:

$$\hat{S}_{kc_1}^2 = s_y^2\left(\frac{S_x^2 + C_x}{s_x^2 + C_x}\right) \tag{2.6}$$

$$\hat{S}_{kc_2}^2 = s_y^2\left(\frac{S_x^2 + \beta_{x(2)}}{s_x^2 + \beta_{x(2)}}\right) \tag{2.7}$$

$$\hat{S}_{kc_3}^2 = s_y^2\left(\frac{S_x^2\beta_{x(2)} + C_x}{s_x^2\beta_{x(2)} + C_x}\right) \tag{2.8}$$

$$\hat{S}_{kc_4}^2 = s_y^2\left(\frac{S_x^2 C_x + \beta_{x(2)}}{s_x^2 C_x + \beta_{x(2)}}\right) \tag{2.9}$$

$$Bias\left(\hat{S}_{kc_i}^2\right) = \gamma A_i S_y^2\left[A_i\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right], \text{ where i= 1,2,3,4} \tag{2.11}$$

$$MSE\left(\hat{S}_{kc_i}^2\right) = \gamma S_y^4\left[\left(\beta_{2(y)} - 1\right) + A_i^2\left(\beta_{2(x)} - 1\right) - 2A_i\left(\lambda_{22} - 1\right)\right], \text{ where i= 1,2,3,4} \tag{2.12}$$

where $A_1 = \frac{S_x^2}{s_x^2 + c_x}, A_2 = \frac{S_x^2}{s_x^2 + \beta_{2(x)}}, A_3 = \frac{S_x^2\beta_{2(x)}}{s_x^2\beta_{2(x)} + c_x}, A_4 = \frac{S_x^2 C_x}{s_x^2\beta_{2(x)} + \beta_{2(x)}}$

Subramani and Kumarpandiyan [12] proposed a generalized modified ratio type estimator for finite population variance using the known parameters of the auxiliary variable as:

$$\hat{S}_{jG}^2 = s_y^2\left(\frac{S_x^2 + \alpha w_i}{s_x^2 + \alpha w_i}\right) \tag{2.13}$$

$$Bias\left(\hat{S}_{jG}^2\right) = \gamma A_{jG} S_y^2\left[A_{jG}\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right] \tag{2.14}$$

$$MSE\left(\hat{S}_{jG}^2\right) = \gamma S_y^4\left[\left(\beta_{2(y)} - 1\right) + A_{jG}^2\left(\beta_{2(x)} - 1\right) - 2A_{jG}\left(\lambda_{22} - 1\right)\right] \tag{2.15}$$

where $A_{jG} = \frac{S_x^2}{s_x^2 + \alpha w_i}$

Bhat et al. [13] proposed a class of ratio type variance estimators for estimating finite population variance using linear combination of non-conventional and conventional measures of auxiliary variable as:

$$\hat{S}^2_{MS1} = s^2_y \left(\frac{S^2_x + Q_{M.A}}{s^2_x + Q_{M.A}} \right) \tag{2.16}$$

$$\hat{S}^2_{MS2} = s^2_y \left(\frac{S^2_x + \left(MR + Q_{M.A}\right)}{s^2_x + \left(MR + Q_{M.A}\right)} \right) \tag{2.17}$$

$$\hat{S}^2_{MS3} = s^2_y \left(\frac{S^2_x + \left(HL + Q_{M.A}\right)}{s^2_x + \left(HL + Q_{M.A}\right)} \right) \tag{2.18}$$

$$\hat{S}^2_{MS4} = s^2_y \left(\frac{S^2_x + \left(TM + Q_{M.A}\right)}{s^2_x + \left(TM + Q_{M.A}\right)} \right) \tag{2.19}$$

$$\hat{S}^2_{MS5} = s^2_y \left(\frac{S^2_x + \left(G + Q_{M.A}\right)}{s^2_x + \left(G + Q_{M.A}\right)} \right) \tag{2.21}$$

$$\hat{S}^2_{MS6} = s^2_y \left(\frac{S^2_x + \left(D + Q_{M.A}\right)}{s^2_x + \left(D + Q_{M.A}\right)} \right) \tag{2.22}$$

$$\hat{S}^2_{MS7} = s^2_y \left(\frac{S^2_x + \left(S_{pw} + Q_{M.A}\right)}{s^2_x + \left(S_{pw} + Q_{M.A}\right)} \right) \tag{2.23}$$

$$\hat{S}^2_{MS8} = s^2_y \left(\frac{S^2_x + D_{M.A}}{s^2_x + D_{M.A}} \right) \tag{2.24}$$

$$\hat{S}^2_{MS9} = s^2_y \left(\frac{S^2_x + \left(MR + D_{M.A}\right)}{s^2_x + \left(MR + D_{M.A}\right)} \right) \tag{2.25}$$

$$\hat{S}^2_{MS10} = s^2_y \left(\frac{S^2_x + \left(HL + D_{M.A}\right)}{s^2_x + \left(HL + D_{M.A}\right)} \right) \tag{2.26}$$

$$\hat{S}^2_{MS11} = s^2_y \left(\frac{S^2_x + \left(TM + D_{M.A}\right)}{s^2_x + \left(TM + D_{M.A}\right)} \right) \tag{2.27}$$

$$\hat{S}^2_{MS12} = s^2_y \left(\frac{S^2_x + \left(G + D_{M.A}\right)}{s^2_x + \left(G + D_{M.A}\right)} \right) \tag{2.28}$$

$$\hat{S}^2_{MS13} = s^2_y \left(\frac{S^2_x + \left(D + D_{M.A}\right)}{s^2_x + \left(D + D_{M.A}\right)} \right) \tag{2.29}$$

$$\hat{S}^2_{MS14} = s^2_y \left(\frac{S^2_x + \left(S_{pw} + D_{M.A}\right)}{s^2_x + \left(S_{pw} + D_{M.A}\right)} \right) \tag{2.31}$$

$$Bias\left(\hat{S}^2_{MSi}\right) = \gamma A_{MSi} S^2_y \left[A_{MSi}\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right) \right], \qquad i = 1, 2, ..., 14 \tag{2.32}$$

$$MSE\left(\hat{S}^2_{MSi}\right) = \gamma S^4_y \left[\left(\beta_{2(y)} - 1\right) + A^2_{MSi}\left(\beta_{2(x)} - 1\right) - 2A_{MSi}\left(\lambda_{22} - 1\right) \right] \qquad i = 1, 2, ..., 14 \tag{2.33}$$

where $A_{MSi} = \dfrac{S^2_x}{S^2_x + a_i}$,

$$a_i;\; i = \left(Q_{M.A}\right), \left(MR + Q_{M.A}\right), \left(HL + Q_{M.A}\right), \left(TM + Q_{M.A}\right), \left(G + Q_{M.A}\right), \left(D + Q_{M.A}\right), \left(S_{pw} + Q_{M.A}\right),$$
$$\left(D_{M.A}\right), \left(MR + D_{M.A}\right), \left(HL + D_{M.A}\right), \left(TM + D_{M.A}\right), \left(G + D_{M.A}\right), \left(D + D_{M.A}\right), \left(S_{pw} + D_{M.A}\right).$$

2.1 Proposed estimators

Motivated by the work of Bhat et al. [13], we proposed a class of new modified ratio estimators for estimating population variance using information of the linear combination of the value of upper quartile $\left(Q_3\right)$, coefficient of quartile deviation $\left(Q_c\right)$, coefficient of variation $\left(C_x\right)$ and Mid-range $\left(MR\right)$ of auxiliary variable as:

$$\hat{S}^2_{MJ1} = s^2_y \left(\frac{S^2_x Q_c + C_x}{s^2_x Q_c + C_x} \right) \tag{2.34}$$

$$\hat{S}^2_{MJ2} = s^2_y \left(\frac{S^2_x + \left(MR + Q_3\right)}{s^2_x + \left(MR + Q_3\right)} \right) \tag{2.35}$$

$$\hat{S}^2_{MJ3} = s^2_y \left(\frac{S^2_x + \left(HL + Q_3\right)}{s^2_x + \left(HL + Q_3\right)} \right) \tag{2.36}$$

$$\hat{S}^2_{MJ4} = s^2_y \left(\frac{S^2_x + \left(TM + Q_3\right)}{s^2_x + \left(TM + Q_3\right)} \right) \tag{2.37}$$

$$\hat{S}^2_{MJ5} = s_y^2\left(\frac{S_x^2 + (G + Q_3)}{s_x^2 + (G + Q_3)}\right) \tag{2.38}$$

$$\hat{S}^2_{MJ6} = s_y^2\left(\frac{S_x^2 + (D + Q_3)}{s_x^2 + (D + Q_3)}\right) \tag{2.39}$$

$$\hat{S}^2_{MJ7} = s_y^2\left(\frac{S_x^2 + (S_{pw} + Q_3)}{s_x^2 + (S_{pw} + Q_3)}\right) \tag{2.41}$$

$$\hat{S}^2_{MJ8} = s_y^2\left(\frac{S_x^2 + Q_3}{s_x^2 + Q_3}\right) \tag{2.42}$$

$$\hat{S}^2_{MJ9} = s_y^2\left(\frac{S_x^2 + (MR + MR)}{s_x^2 + (MR + MR)}\right) \tag{2.43}$$

$$\hat{S}^2_{MJ10} = s_y^2\left(\frac{S_x^2 + (HL + MR)}{s_x^2 + (HL + MR)}\right) \tag{2.44}$$

$$\hat{S}^2_{MJ11} = s_y^2\left(\frac{S_x^2 + (TM + MR)}{s_x^2 + (TM + MR)}\right) \tag{2.45}$$

$$\hat{S}^2_{MJ12} = s_y^2\left(\frac{S_x^2 + (G + MR)}{s_x^2 + (G + MR)}\right) \tag{2.46}$$

$$\hat{S}^2_{MJ13} = s_y^2\left(\frac{S_x^2 + (D + MR)}{s_x^2 + (D + MR)}\right) \tag{2.47}$$

$$\hat{S}^2_{MJ14} = s_y^2\left(\frac{S_x^2 + (S_{pw} + MR)}{s_x^2 + (S_{pw} + MR)}\right) \tag{2.48}$$

2.1.1 Properties of the proposed estimators

In order to obtain the bias and mean square error (MSE), we define $e_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \frac{s_x^2 - S_x^2}{S_x^2}$ such that $s_y^2 = S_y^2(1 + e_0)$ *and* $s_x^2 = S_x^2(1 + e_1)$, from the definitions of e_0 and e_1, we obtain

$$\left.\begin{array}{l} E\left(e_0\right)=E\left(e_1\right)=0,\ E\left(e_0^2\right)=\gamma\left(\beta_{2(y)}-1\right) \\ E\left(e_1^2\right)=\gamma\left(\beta_{2(x)}-1\right),\ E\left(e_0e_1\right)=\gamma\left(\lambda_{22}-1\right) \end{array}\right\} \quad (2.49)$$

$$\hat{S}_{MJ1}^2=s_y^2\left(\frac{S_x^2Q_c+C_x}{s_x^2Q_c+C_x}\right) \quad (2.51)$$

Expressing $\hat{S}_{MJ1}^2$ in terms of e_0 and e_1, we have

$$\hat{S}_{MJ1}^2=S_y^2\left(1+e_0\right)\left(\frac{S_x^2Q_c+C_x}{S_x^2Q_c\left(1+e_1\right)+C_x}\right) \quad (2.52)$$

$$\hat{S}_{MJ1}^2=S_y^2\left(1+e_0\right)\left(1+A_{MJ1}e_1\right)^{-1} \quad (2.53)$$

where $A_{MJ1}=\dfrac{S_x^2}{S_x^2Q_c+C_x}$

Simplifying (2.53) up to first order approximation, it reduces to (2.54) as:

$$\hat{S}_{MJ1}^2=S_y^2\left(1+e_0\right)\left(1-A_{MJ1}e_1+A_{MJ1}^2e_1^2-...\right) \quad (2.54)$$

Removing the brackets and subtracting both sides by S_y^2

$$\hat{S}_{MJ1}^2-S_y^2=S_y^2e_0-S_y^2A_{MJ1}e_1-S_y^2A_{MJ1}e_0e_1+S_y^2A_{MJ1}^2e_1^2 \quad (2.55)$$

Taking Expectation of both sides of (2.55)

$$E\left(\hat{S}_{MJ1}^2-S_y^2\right)=S_y^2E\left(e_0\right)-S_y^2A_{MJ1}E\left(e_1\right)-S_y^2A_{MJ1}E\left(e_0e_1\right)+S_y^2A_{MJ1}^2E\left(e_1^2\right) \quad (2.56)$$

Applying the results of (2.49) obtaining the bias as

$$Bias\left(\hat{S}_{MJ1}^2\right)=\gamma A_{MJ1}S_y^2\left[A_{MJ1}\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right] \quad (2.57)$$

To get the MSE, Squaring both sides of (2.55) and ignoring e of power two and above as:

$$E\left(\hat{S}_{MJ1}^2-S_y^2\right)^2=S_y^4E\left(e_0-A_{MJ1}e_1\right)^2 \quad (2.58)$$

Expanding, and taking expectation of (2.58)

$$MSE\left(\hat{S}_{MJ1}^2\right)=S_y^4E\left(e_0^2+A_{MJ1}^2e_1^2-2A_{MJ1}e_0e_1\right) \quad (2.59)$$

Applying the results of (2.49), obtaining $MSE\left(\hat{S}^2_{MJ1}\right)$ as:

$$MSE\left(\hat{S}^2_{MJ1}\right)=\gamma A_{MJ1}S_y^4\left[\left(\beta_{2(y)}-1\right)+A^2_{MJ1}\left(\beta_{2(x)}-1\right)-2A_{MJ1}\left(\lambda_{22}-1\right)\right] \tag{2.61}$$

The proposed estimators $\hat{S}^2_{MJ_i}; i=2,3,4,...,14$ can be written in general from as

$$\hat{S}^2_{MJ_i}=s_y^2\left(\frac{S_x^2+u_i}{s_x^2+u_i}\right) \tag{2.62}$$

Expressing $\hat{S}^2_{MJ_i}$ in terms of e_0 and e_1, we have

$$\hat{S}^2_{MJ_i}=S_y^2\left(1+e_0\right)\left(\frac{S_x^2+u_i}{S_x^2\left(1+e_1\right)+u_i}\right) \tag{2.63}$$

$$\hat{S}^2_{MJ_i}=S_y^2\left(1+e_0\right)\left(1+A_{MJ_i}e_1\right)^{-1}, \quad \text{where } A_{MJ_i}=\frac{S_x^2}{S_x^2+u_i} \tag{2.64}$$

$$\begin{aligned}u_i;\quad u_i=&\left(MR+Q_3\right),\left(HL+Q_3\right),\left(TM+Q_3\right),\left(G+Q_3\right),\left(D+Q_3\right),\left(S_{pw}+Q_3\right),\\&\left(Q_3\right),\left(MR+MR\right),\left(HL+MR\right),\left(TM+MR\right),\left(G+MR\right),\left(D+MR\right),\left(S_{pw}+MR\right).\end{aligned} \tag{2.65}$$

Simplifying (2.64) up to first order approximation, it reduces to (2.66) as:

$$\hat{S}^2_{MJ_i}=S_y^2\left(1+e_0\right)\left(1-A_{MJ_i}e_1+A^2_{MJ_i}e_1^2-...\right) \tag{2.66}$$

$$\hat{S}^2_{MJ_i}=S_y^2+S_y^2e_0-S_y^2A_{MJ_i}e_1-S_y^2A_{MJ_i}e_0e_1+S_y^2A^2_{MJ_i}e_1^2 \tag{2.67}$$

Subtracting S_y^2 from both sides and taking Expectation of both sides of (2.67)

$$E\left(\hat{S}^2_{MJ_i}-S_y^2\right)=S_y^2E\left(e_0\right)-S_y^2A_{MJ_i}E\left(e_1\right)-S_y^2A_{MJ_i}E\left(e_0e_1\right)+S_y^2A^2_{MJ_i}E\left(e_1^2\right) \tag{2.68}$$

Applying the results of (2.49), obtaining $Bias\left(\hat{S}^2_{MJi}\right)$ as:

$$Bias\left(\hat{S}^2_{MJ_i}\right)=\gamma A_{MJi}S_y^2\left[A_{MJi}\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right] \tag{2.69}$$

Squaring both sides of (2.68) and applying the results of (2.49), we have (2.71), (2.72) and (2.73)

$$E\left(\hat{S}^2_{MJ_i}-S_y^2\right)^2=S_y^4E\left(e_0-A_{MJ_i}e_1\right)^2 \tag{2.71}$$

$$MSE\left(\hat{S}^2_{MJ_i}\right) = S_y^4 E\left(e_0^2 + A^2_{MJ_i} e_1^2 - 2A_{MJ_i} e_0 e_1\right) \quad (2.72)$$

$$MSE\left(\hat{S}^2_{MJ_i}\right) = \gamma A_{MJ_i} S_y^4 \left[\left(\beta_{2(y)} - 1\right) + A^2_{MJ_i}\left(\beta_{2(x)} - 1\right) - 2A_{MJ_i}\left(\lambda_{22} - 1\right)\right] \quad (2.73)$$

2.2 Efficiency comparisons

Comparison of the proposed estimators with other existing estimators considered in the study with certain conditions to determine the estimators with high precision.

The $\hat{S}^2_{MJi}$ - estimators of the finite population variance are more efficient than $Var(\hat{t})$ if,

$$MSE\left(\hat{S}^2_{MJi}\right) < Var\left(\hat{t}\right)$$

$$\left[\left(\beta_{2(y)} - 1\right) + A^2_{MJ_i}\left(\beta_{2(x)} - 1\right) - 2A_{MJ_i}\left(\lambda_{22} - 1\right)\right] < \left(\beta_{2(y)} - 1\right) \quad (2.74)$$

The $\hat{S}^2_{MJi}$ - estimators of the finite population variance are more efficient than $\hat{S}^2_R$ if,

$$MSE\left(\hat{S}^2_{MJi}\right) < MSE\left(\hat{S}^2_R\right)$$

$$\left[\left(\beta_{2(y)} - 1\right) + A^2_{MJ_i}\left(\beta_{2(x)} - 1\right) - 2A_{MJ_i}\left(\lambda_{22} - 1\right)\right] < \left[\left(\beta_{2(y)} - 1\right) + \left(\beta_{2(x)} - 1\right) - 2\left(\lambda_{22} - 1\right)\right] \quad (2.75)$$

The $\hat{S}^2_{MJi}$ - estimators of the finite population variance are more efficient than $\hat{S}^2_{kc_i}$ if,

$$MSE\left(\hat{S}^2_{MJi}\right) < MSE\left(\hat{S}^2_{kc_i}\right)$$

$$\left[A^2_{MJ_i}\left(\beta_{2(x)} - 1\right) - 2A_{MJ_i}\left(\lambda_{22} - 1\right)\right] < \left[A_i^2\left(\beta_{2(x)} - 1\right) - 2A_i\left(\lambda_{22} - 1\right)\right] \quad (2.76)$$

The $\hat{S}^2_{MJi}$ - estimators of the finite population variance are more efficient than $\hat{S}^2_{jG}$ if,

$$MSE\left(\hat{S}^2_{MJi}\right) < MSE\left(\hat{S}^2_{jG}\right)$$

$$\left[A^2_{MJ_i}\left(\beta_{2(x)} - 1\right) - 2A_{MJ_i}\left(\lambda_{22} - 1\right)\right] < \left[A^2_{jG}\left(\beta_{2(x)} - 1\right) - 2A_{jG}\left(\lambda_{22} - 1\right)\right] \quad (2.77)$$

The $\hat{S}^2_{MJi}$ - estimators of the finite population variance are more efficient than $\hat{S}^2_{MSi}$ if,

$$MSE\left(\hat{S}^2_{MJi}\right) < MSE\left(\hat{S}^2_{MSi}\right)$$

$$\left[A^2_{MJ_i}\left(\beta_{2(x)} - 1\right) - 2A_{MJ_i}\left(\lambda_{22} - 1\right)\right] < \left[A^2_{MSi}\left(\beta_{2(x)} - 1\right) - 2A_{MSi}\left(\lambda_{22} - 1\right)\right] \quad (2.78)$$

When conditions (2.74), (2.75), (2.76), (2.77), and (2.78) are satisfied, we conclude that the proposed estimators $\left(\hat{S}^2_{MJi}\right)$ are more efficient than existing estimators.

2.2.1 Numerical study

Empirical study is carried out to support the efficiency comparison stated above by considering a real life population as:

Data: Murthy [14]

Fixed capital (Auxiliary variable X)

Output of 80 factories (Study variable Y)

$N = 80,\ n = 20,\ S_x = 8.4542,\ S_y = 18.3569, HL = 10.405,\ C_x = 0.7507, Q_{MA} = 11.2893,\ \overline{X} = 11.2624,$
$\overline{Y} = 51.8264,\ \beta_{2(x)} = 2.8664,\ \beta_{2(y)} = 2.2667,\ \beta_{1(x)} = 1.05,\ G = 9.0408,\ \rho = 0.9413,\ \lambda_{22} = 2.2209,$
$Md = 7.5750,\ Q_1 = 9.318,\ C_y = 0.3542,\ D = 8.0138,\ Q_2 = 7.5750,\ Q_3 = 16.975,\ QD = 5.9125,$
$Q_c = 0.5345,\ D_{MA} = 12.95,\ TM = 9.318,\ S_{pw} = 7.9136,\ MR = 17.955.$

3 Results and Discussion

Table 1 shows the bias, MSE and PRE of the proposed and some existing estimators using the real life population. The result revealed that all the proposed estimators have minimum MSE and higher PRE compared to the conventional estimators under simple random sampling. This implies that the proposed estimators are more efficient than other existing estimators in the study.

A class of ratio estimators of finite population variance has been proposed in the study. The performance of the proposed estimators over the usual ratio estimator and some selected existing estimators with real life population were examined. The Bias and Mean Square Error (MSE) of the proposed estimators were derived. Table 1 shows the bias values of the existing and proposed estimators. The results revealed that the proposed estimators has minimum values of MSE and Higher PRE in the population considered in the study.

Table 1. Bias, MSE and PRE of existing and proposed estimators

Estimator	Bias	MSE	PRE
Sample variance	0	5393.89	100
Isaki (1983)	10.88	3925.16	137.42
Kadilar & Cingi (2006) 1	10.44	3850.16	140.10
Kadilar & Cingi (2006) 2	9.29	3658.41	147.44
Kadilar & Cingi (2006) 3	10.72	3898.56	138.36
Kadilar & Cingi (2006) 4	8.81	3580.83	150.63
Subramani & Kumarapandiyan (2015)	6.12	3180.77	169.58
Bhat *et. al.* (2017) 1	5.68	3122.11	172.76
Bhat *et. al.* (2017) 2	1.24	2690.05	200.51
Bhat *et. al.* (2017) 3	2.72	2792.81	193.13
Bhat *et. al.* (2017) 4	2.97	2814.96	191.62
Bhat *et. al.* (2017) 5	3.03	2816.09	191.54
Bhat *et. al.* (2017) 6	3.28	2843.34	189.70
Bhat *et. al.* (2017) 7	3.31	2844.48	189.63
Bhat *et. al.* (2017) 8	5.10	3043.20	177.24

Estimator	Bias	MSE	PRE
Bhat *et. al.* (2017) 9	0.97	2678.13	201.41
Bhat *et. al.* (2017) 10	2.36	2762.72	195.24
Bhat *et. al.* (2017) 11	2.59	2782.03	193.88
Bhat *et. al.* (2017) 12	2.63	2785.03	193.67
Bhat *et. al.* (2017) 13	2.88	2804.74	192.31
Bhat *et. al.* (2017) 14	2.90	2804.50	192.33
Proposed Estimator (MJ1)	-1.562238	2117.72	254.70
Proposed Estimator (MJ2)	0.2802017	1995.56	270.29
Proposed Estimator (MJ3)	1.17639	2030.88	265.59
Proposed Estimator (MJ4)	1.328058	2040.21	264.38
Proposed Estimator (MJ5)	1.367772	2042.79	264.05
Proposed Estimator (MJ6)	1.502724	2053.08	262.72
Proposed Estimator (MJ7)	1.516044	2054.15	262.59
Proposed Estimator (MJ8)	2.663363	2181.58	247.25
Proposed Estimator (MJ9)	0.1998184	1994.13	270.49
Proposed Estimator (MJ10)	1.070133	2023.50	266.56
Proposed Estimator (MJ11)	1.206391	2031.75	265.48
Proposed Estimator (MJ12)	1.24162	2034.04	265.18
Proposed Estimator (MJ13)	1.373886	2043.23	263.99
Proposed Estimator (MJ14)	1.38694	2044.19	263.86

**Values of bias, MSE, and PRE of the estimators*

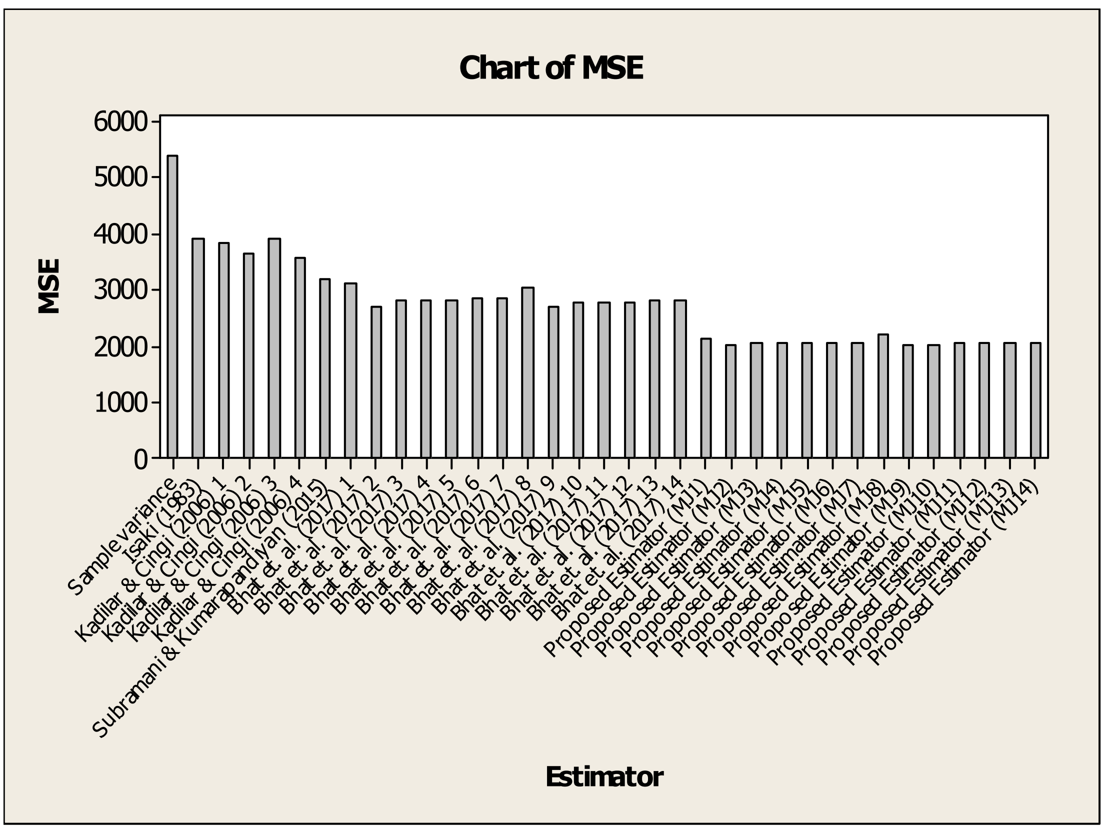

Fig. 1. Show the MSE of the existing and proposed Estimators. The shorter the bar, the better and efficient the estimator

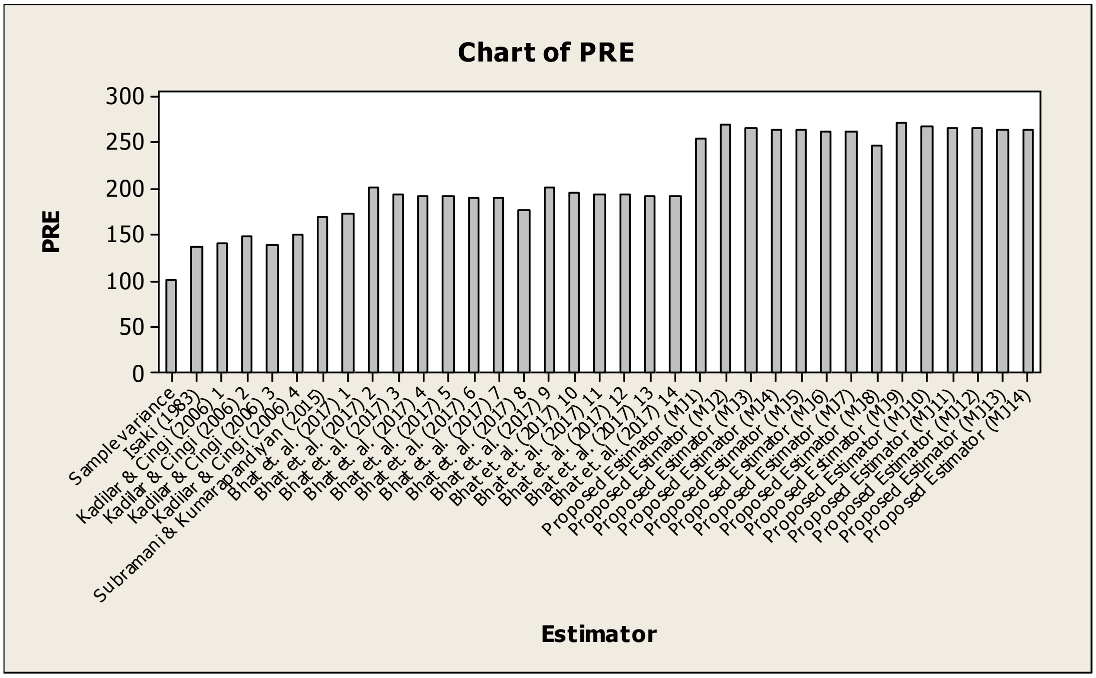

Fig. 2. Show the PRE of the existing and proposed Estimators. The longer the bar, the higher precision of the estimate

4 Conclusion

The results (Table 1, Fig. 1 and Fig. 2) of the study have revealed that the proposed estimators, having minimum mean square error (MSE) and highest percentage relative efficiency (PRE) among the existing estimators. With this fact, we conclude that the proposed estimators are proficient and more efficient than the existing estimators and be use in practical applications in estimating finite population variance.

Acknowledgements

The authors are thankful to the Editor, Journal editorial office and anonymous referees for their fruitful comments for the success of this work.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Kadilar C, Cingi H. Improvement in variance estimation using auxiliary information. Hacettepe Journal of Mathematics and Statistics. 2006;35:111-115.

[2] Bahl S, Tuteja RK. Ratio and product type exponential estimator. Information and Optimization Sciences. 1991;12:159-163.

[3] Singh GN. On the improvement of product method of estimation in sample surveys. Journal of Indian Society of Agricultural Statistics. 2003;56(3):267-275.

[4] Singh HP, Ruiz Espejo M. On linear regression and ratio-product estimation of a finite population mean. The Statistician. 2003;52(1):59-67.

[5] Evans WD. On the variance of estimates of the standard deviation and variance. Journal of the American Statistical Association. 1951;46:220-224.

[6] Hansen MH, Hurwitz WN, Madow WG. Sample survey methods and theory. Wiley, New York; 1953.

[7] Isaki CT. Variance estimation using auxiliary information. Journal of the American Statistical Association. 1983;78:117–123.

[8] Das AK, Tripathi TP. Use of auxiliary information in estimating the finite population variance. Sankhya. 1978;40:139-148.

[9] Gupta S, Shabbir J. Variance estimation in simple random sampling using auxiliary information. Hacettepe Journal of Mathematics and Statistics. 2008;37:57-67.

[10] Tailor R, Sharma B. Modified estimators of population variance in presence of auxiliary information. Statistics in Transition-New Series. 2012;13(1):37-46.

[11] Audu A, Adewara AA, Singh RVK. Class of ratio estimators with known functions of auxiliary variable for estimating finite population variance. Asian Journal of Mathematics and Computer Research. 2016;12(1):63-70.

[12] Subramani J. Generalized modified ratio type estimator for estimation of population variance. Sri-Lankan J. Appl. Statist. 2015;16(1):69-90.

[13] Bhat MA, Maqbool S, Mir SA. Variance estimation using linear combination of non-conventional measures, quartiles average and deciles mean. Internat. Res. J. Agric. Eco. and Stat. 2017;8(2):431-434.

[14] Murthy MN. Sampling theory and methods. Statistical Publishing Society, Calcutta. India; 1967.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sdiarticle4.com/review-history/58031