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Applications of the Modified Double Sub Equation Method to Nonlinear Partial Differential Equations

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Authors' contributions

This work was carried out in collaboration between all authors. Author SHY designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author HTC managed the analyses of the study and gave author SHY many introductions. All authors read and approved the final manuscript.

Research Article

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ABSTRACT

In this paper, the modified double sub-equation method is proposed to construct complexiton solutions of nonlinear partial differential equations (PDEs). By means of this method, some new complexiton solutions to nonlinear PDEs are obtained, which are non-travelling wave and variable-coefficient function solutions. It is shown that the modified double sub-equation method is effective and straightforward tool to solve nonlinear PDEs.

Keywords: The modified double sub-equation method; nonlinear partial differential equations; complexiton solution.

1. INTRODUCTION

In recent years, both mathematicians and physicists have made many attempts to seek as many and general as possible soliton solutions of nonlinear partial differential equations (PDEs). A number of powerful methods were presented, such as the inverse scattering theory [1], Darboux transformation [2], the sech-function method [3,4], the homogeneous

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balance method [5], the hyperbolic tangent function series method [6], Bäcklund transformations method [7], the Jacobi elliptic function expansion method [8,9], the $\frac{G'}{G}$-expansion method [10] and the multiple exp-function method [11]. One of the most effectively straightforward methods to constructing exact solutions of PDEs is the sub-equation method [12-15]. The complexiton solution, firstly introduced by Ma et al.[16], can be constructed by the multiple Riccati equations rational expansion method [17], which make use of two Riccati equations with the same variable.

Chen [18] has presented the double sub-equation method using two ordinary differential equations with different independent variables. Complexiton solutions combining elementary functions and the Jacobi elliptic functions are obtained by the double sub-equation method [18].

In this paper, the modified double sub-equation method is proposed to construct complexiton solutions of nonlinear partial differential equations (PDEs). We apply this method to the Fisher equation and the Kadomtsev-Petviashvili (KP) equation, and get many new types of complexiton solutions (the non-travelling wave and variable-coefficient function solutions). It makes the modified double sub-equation method more extensively.

2. SUMMARY OF THE MODIFIED DOUBLE SUB-EQUATION METHOD

In the following we would like to establish a modified double sub-equation method with symbolic computation.

Step 1: Given a nonlinear partial differential equations with two variables x and t

$$P(u_t, u_x, u_{tt}, u_{tx}, u_{xx}, \cdots) = 0 \tag{1}$$

Step 2: We assume that the solutions of Eq.(1) are as follows:

$$u(x,t) = A_0 + \frac{A_1\phi(\xi) + A_2\psi(\eta)}{\mu_0 + \mu_1\phi(\xi)\psi(\eta)} \tag{2}$$

where $A_0 = A_0(x,t), A_1 = A_1(x,t), A_2 = A_2(x,t), \xi = \xi(x,t), \eta = \eta(x,t)$ are all functions of x and t, μ_0 and μ_1 are arbitrary nonzero constants. The new functions $\phi(\xi)$ and $\psi(\eta)$ satisfy

$$\phi'(\xi) = e_0 + e_1\phi^2(\xi) \tag{3}$$

where $\phi'(\xi) = \frac{d\phi(\xi)}{d\xi}$ and $\xi = k_1(t)x - c_1(t)$;

$$\psi'(\eta) = h_0 + h_1\psi^2(\eta) \tag{4}$$

where $\psi^{'}(\eta)=\frac{d\psi(\eta)}{d\eta}$ and $\eta=k_2(t)x-c_2(t)$.

Step 3: The solutions of the Riccati equation Eq.(3) [17] are as follows:

$$\phi^{'}(\xi)=e_0+e_1\phi^2(\xi)$$

(I) when $e_0=e_1=1,$

$$\phi(\xi)=\tan(\xi),$$

(II) when $e_0=e_1=-1,$

$$\phi(\xi)=\cot(\xi),$$

(III) when $e_0=1, e_1=-1,$

$$\phi(\xi)=\tanh(\xi),\ \ \phi(\xi)=\cot(\xi),$$

(IV) when $e_0=e_1=\pm\frac{1}{2},$

$$\phi(\xi)=\sec(\xi)\pm\tan(\xi),\ \phi(\xi)=\csc(\xi)\pm\cot(\xi),$$

(V) when $e_0=\frac{1}{2}, e_1=-\frac{1}{2},$

$$\phi(\xi)=\tanh(\xi)\pm i\frac{1}{\cosh(\xi)},\ \phi(\xi)=\coth(\xi)\pm\frac{1}{\sinh(\xi)}$$

(VI) when $e_0=0, e_1=1,$

$$\phi(\xi)=-\frac{1}{e_1\xi+c}.$$

Step 4: Substituting Eq.(2) along with Eq.(3) and Eq.(4) into Eq.(1) yields a system of equations w.r.t $\phi^i\psi^j$ $(i=0,1,2,\cdots, j=0,1,2,\cdots)$, setting the coefficients of $\phi^i\psi^j$ in the obtained system of equations to be zero, we obtain a set of over-determined PDEs (or ODEs) with respect to $A_0, A_1, A_2, k_1(t), c_1(t), k_2(t), c_2(t), \mu_0, \mu_1$.

Step 5: Solving the over-determined PDEs (or ODEs) (e.g. Maple), we would gain the explicit expressions for $A_0, A_1, A_2, k_1(t), c_1(t), k_2(t), c_2(t), \mu_0, \mu_1$.

Step 6: By using the results obtained in the above steps and the various solutions of Eq.(3,4), we can derive many solutions for Eq.(1).

Remark 1: The the method proposed here is to take full advantage of two solvable ordinary differential equations with different independent variables. The success and appeal of the method is the fact that the new complexiton solutions obtained are the non-travelling wave and variable-coefficient function solutions.

Remark 2: In order to simplify the calculation and solve the PDEs derived in Step 4, we usually choose special forms of A_0, A_1, A_2 as we do in Section 3.

3. APPLICATION

3.1 Example 1

The Fisher's equation is the simplest nonlinear reaction diffusion equation,

$$u_t - \alpha u_{xx} - \beta u(1-u) = 0 \tag{5}$$

where α and β are greater than zero, α is the diffusion coefficient, β is the reaction coefficient. According to the method, we assume that the solutions of Eq.(5) are as follows:

$$u(x,t) = A_0 + \frac{A_1\phi(\xi) + A_2\psi(\eta)}{\mu_0 + \mu_1\phi(\xi)\psi(\eta)} \tag{6}$$

Where $A_0 = A_0(t), A_1 = A_1(t), A_2 = A_2(t),\ \xi = k_1(t)x - c_1(t)$, $\eta = k_2(t)x - c_2(t)$, μ_0 and μ_1 are arbitrary nonzero constants.

Substituting Eq.(6) along with Eq.(3) and Eq.(4) into Eq.(5) yields a system of equations w.r.t $\phi^i\psi^j\ (i = 0,1,2,3,\ j = 0,1,2,3)$, setting the coefficients of $\phi^i\psi^j$ in the obtained system of equations to be zero,we obtain a set of over-determined PDEs (or ODEs) with respect to $A_0, A_1, A_2, k_1(t), c_1(t), k_2(t), c_2(t), \mu_0, \mu_1$. Solving the over-determined PDEs (or ODEs) by use of Maple, we can obtain the following result.

Case 1: $\mu_0 = \mu_0,\ \mu_1 = -\dfrac{h_1\mu_0 e_1}{\Delta_1},\ c_1(t) = c_1(t),\ k_1(t) = k_1(t),\ k_2 = -\dfrac{\Delta_1 k_1(t)}{h_0 h_1},$

$$A_0(t) = \frac{\Delta_2}{\Delta 3},\ A_1(t) = \frac{4C_1 e_1\beta^2{\mu_0}^2}{\Delta_3 e^{\beta t}},\ A_2(t) = \frac{4C_1\beta^2{\mu_0}^2 e_0 h_1}{\Delta_1\Delta_3 e^{\beta t}},$$

$$c_2(t) = -\frac{\Delta_1\beta t c_1(t)}{h_0 h_1 \ln e^{\beta t}} + \frac{\arctan(\dfrac{\Delta_4}{4\beta\mu_0\sqrt{e_0 e_1}})\Delta_1\beta t}{\sqrt{e_0 e_1}\, h_0 h_1 \ln e^{\beta t}} - \frac{C_3\beta t}{h_0 h_1 \ln e^{\beta t}} + C_3,$$

Case 2: $\mu_0 = \mu_0,\ \mu_1 = \dfrac{h_1\mu_0 e_1}{\Delta_1},\ c_1(t) = c_1(t),\ k_1(t) = k_1(t),\ k_2 = \dfrac{\Delta_1 k_1(t)}{h_0 h_1},$

$$A_0(t) = \frac{\Delta_2}{\Delta 3},\ A_1(t) = \frac{4C_1 e_1\beta^2\mu_0^{\ 2}}{\Delta_3 e^{\beta t}},\ A_2(t) = -\frac{4C_1\beta^2\mu_0^{\ 2}e_0 h_1}{\Delta_1\Delta_3 e^{\beta t}},$$

$$c_2(t) = \frac{\Delta_1\beta t c_1(t)}{h_0 h_1 \ln e^{\beta t}} - \frac{\arctan(\frac{\Delta_4}{4\beta\mu_0\sqrt{e_0 e_1}})\Delta_1\beta t}{\sqrt{e_0 e_1}h_0 h_1 \ln e^{\beta t}} + \frac{C_3\beta t}{h_0 h_1 \ln e^{\beta t}} + C_3,$$

Where $\Delta_1 = \sqrt{e_0 h_0 e_1 h_1}$, $\Delta_3 = 16C_1^{\ 2}e_1\beta^4 e_0\mu_0^{\ 2} + (e^{-\beta t})^2 + 2e^{-\beta t}C_2\beta + C_2^{\ 2}\beta^2$,

$\Delta_2 = 16C_1^{\ 2}h_1\beta^4 h_0\mu_0^{\ 2} + \beta e^{-\beta t}C_2 + C_2^{\ 2}\beta^2$, $\Delta_4 = 16C_1^{\ 2}e_1\beta^3 e_0\mu_0^{\ 2}e^{\beta t} + \beta e^{\beta t}C_2^{\ 2} + C_2$

C_1, C_2, C_3 and μ_0 are arbitrary constants, $k_1(t), c_1(t)$ are arbitrary functions of t.

According to case 2, when $C_1 = C_2 = 1, C_3 = 0,\ e_0 = e_1 = 1, h_0 = 1, h_1 = -1$, we can get combining $\tan$ and $\tanh, \coth$ function complexiton solutions:

$$u_1 = \frac{-16\beta^4\mu_0^{\ 2} + \beta e^{-\beta t} + \beta^2}{16\beta^4\mu_0^{\ 2} + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2} + \frac{A_1(t)\tan(\xi) + A_2(t)\tanh(\eta)}{\mu_0 \pm i\mu_0 \tan(\xi)\tanh(\eta)},$$

$$u_2 = \frac{-16\beta^4\mu_0^{\ 2} + \beta e^{-\beta t} + \beta^2}{16\beta^4\mu_0^{\ 2} + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2} + \frac{A_1(t)\tan(\xi) + A_2(t)\coth(\eta)}{\mu_0 \pm i\mu_0 \tan(\xi)\coth(\eta)},$$

Where $\xi = k_1(t)x - c_1(t)$, $\eta = k_2(t)x - c_2(t)$, $\mu_1 = \pm i\mu_0$,

$$A_1(t) = \frac{4\beta^2\mu_0^{\ 2}}{(16\beta^4\mu_0^{\ 2} + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2)e^{\beta t}},$$

$$A_2(t) = \frac{\pm 4i\beta^2\mu_0^{\ 2}}{(16\beta^4\mu_0^{\ 2} + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2)e^{\beta t}},$$

$$c_2(t) = \frac{\pm\beta t c_1(t)}{\ln e^{\beta t}} \pm i\frac{\arctan(\frac{16\beta^3\mu_0^{\ 2}e^{\beta t} + \beta e^{\beta t} + 1}{4\beta\mu_0})\beta t}{\ln e^{\beta t}},\ k_2(t) = \mp i k_1(t).$$

When $C_1 = C_2 = 1, C_3 = 0,\ e_0 = e_1 = -\frac{1}{2}, h_0 = 1, h_1 = -1$, we can get combining $\sec - \tan, \csc - \cot$ and $\tanh, \coth$ function complexiton solutions:

$$u_3 = \frac{-16\beta^4 {\mu_0}^2 + \beta e^{-\beta t} + \beta^2}{16\beta^4 {\mu_0}^2 + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2} + \frac{A_1(t)(\sec(\xi) - \tan(\xi)) + A_2(t)\tanh(\eta)}{\mu_0 \pm i\mu_0 (\sec(\xi) - \tan(\xi))\tanh(\eta)},$$

$$u_4 = \frac{-16\beta^4 {\mu_0}^2 + \beta e^{-\beta t} + \beta^2}{16\beta^4 {\mu_0}^2 + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2} + \frac{A_1(t)(\sec(\xi) - \tan(\xi)) + A_2(t)\coth(\eta)}{\mu_0 \pm i\mu_0 (\sec(\xi) - \tan(\xi))\coth(\eta)},$$

$$u_5 = \frac{-16\beta^4 {\mu_0}^2 + \beta e^{-\beta t} + \beta^2}{16\beta^4 {\mu_0}^2 + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2} + \frac{A_1(t)(\csc(\xi) - \cot(\xi)) + A_2(t)\tanh(\eta)}{\mu_0 \pm i\mu_0 (\csc(\xi) - \cot(\xi))\tanh(\eta)},$$

$$u_6 = \frac{-16\beta^4 {\mu_0}^2 + \beta e^{-\beta t} + \beta^2}{16\beta^4 {\mu_0}^2 + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2} + \frac{A_1(t)(\csc(\xi) - \cot(\xi)) + A_2(t)\coth(\eta)}{\mu_0 \pm i\mu_0 (\csc(\xi) - \cot(\xi))\cot(\eta)},$$

where $\xi = k_1(t)x - c_1(t)$, $\eta = k_2(t)x - c_2(t)$, $\mu_1 = \pm i\mu_0$,

$$A_1(t) = \frac{-2\beta^2 {\mu_0}^2}{(4\beta^4 {\mu_0}^2 + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2)e^{\beta t}},$$

$$A_2(t) = \frac{\pm 2i\beta^2 {\mu_0}^2}{(4\beta^4 {\mu_0}^2 + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2)e^{\beta t}},$$

$$c_2(t) = \frac{\pm \beta t c_1(t)}{2\ln e^{\beta t}} \pm i \frac{2\arctan\left(\frac{16\beta^3 {\mu_0}^2 e^{\beta t} + \beta e^{\beta t} + 1}{2\beta\mu_0}\right)\beta t}{\ln e^{\beta t}}, \quad k_2(t) = \mp \frac{i}{2} k_1(t).$$

When $C_1 = C_2 = 1, C_3 = 0,\ e_0 = e_1 = -1, h_0 = \frac{1}{2}, h_1 = -\frac{1}{2}$, we can get combining $\cot$ and $\tanh, \coth, \sinh, \cosh$ function complexiton solutions:

$$u_7 = \frac{-4\beta^4 {\mu_0}^2 + \beta e^{-\beta t} + \beta^2}{16\beta^4 {\mu_0}^2 + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2} + \frac{A_1(t)\cot(\xi) + A_2(t)\left(\tanh(\eta) \pm i\frac{1}{\cosh(\eta)}\right)}{\mu_0 \pm i\mu_0 \cot(\xi)\left(\tanh(\eta) \pm i\frac{1}{\cosh(\eta)}\right)},$$

$$u_8 = \frac{-4\beta^4 {\mu_0}^2 + \beta e^{-\beta t} + \beta^2}{16\beta^4 {\mu_0}^2 + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2} + \frac{A_1(t)\cot(\xi) + A_2(t)(\coth(\eta) \pm i\frac{1}{\sinh(\eta)})}{\mu_0 \pm i\mu_0 \cot(\xi)(\coth(\eta) \pm i\frac{1}{\sinh(\eta)})},$$

where $\xi = k_1(t)x - c_1(t)$, $\eta = k_2(t)x - c_2(t)$, $\mu_1 = \pm i\mu_0$,

$$A_1(t) = \frac{-4\beta^2 {\mu_0}^2}{(16\beta^4 {\mu_0}^2 + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2)e^{\beta t}},$$

$$A_2(t) = \frac{\pm 4i\beta^2 {\mu_0}^2}{(16\beta^4 {\mu_0}^2 + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2)e^{\beta t}},$$

$$c_2(t) = \frac{\pm 2\beta t c_1(t)}{\ln e^{\beta t}} \pm i\frac{\arctan(\frac{16\beta^3 {\mu_0}^2 e^{\beta t} + \beta e^{\beta t} + 1}{4\beta\mu_0})\beta t}{\ln e^{\beta t}},\ k_2(t) = \mp 2ik_1(t).$$

When $C_1 = C_2 = 1, C_3 = 0,\ e_0 = e_1 = 1, h_0 = h_1 = -1$, we can get combining tan and cot function complexiton solutions:

$$u_9 = \frac{16\beta^4 {\mu_0}^2 + \beta e^{-\beta t} + \beta^2}{16\beta^4 {\mu_0}^2 + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2} + \frac{A_1(t)\tan(\xi) + A_2(t)\cot(\eta)}{\mu_0 + \mu_0 \tan(\xi)\cot(\eta)},$$

where $\xi = k_1(t)x - c_1(t)$, $\eta = k_2(t)x - c_2(t)$,

$$A_1(t) = \frac{4\beta^2 {\mu_0}^2}{(16\beta^4 {\mu_0}^2 + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2)e^{\beta t}},$$

$$A_2(t) = \frac{\pm 4\beta^2 {\mu_0}^2}{(16\beta^4 {\mu_0}^2 + e^{-2\beta t} + 2\beta e^{-\beta t} + \beta^2)e^{\beta t}},$$

$$c_2(t) = \frac{\beta t c_1(t)}{\ln e^{\beta t}} - \frac{\arctan(\frac{16\beta^3 {\mu_0}^2 e^{\beta t} + \beta e^{\beta t} + 1}{4\beta\mu_0})\beta t}{\ln e^{\beta t}},\ k_2(t) = \mp 2ik_1(t).$$

Similarly, we can write down the other complexiton solutions of equation eq.(5) which are omitted for convenience.

3.2 Example 2

In this section, we consider the Kadomtsev-Petviashvili(KP) equation

$$v_{tx} + \alpha(v_x^2 + vv_{xx}) + \gamma v_{xxxx} + \varepsilon v_{yy} = 0, \tag{7}$$

where α, γ and ε are arbitrary nonzero constants. According to the method, we assume that the solutions of Eq.(5) are as follows:

$$v(x, y, t) = B_0 + \frac{B_1\phi(\xi) + B_2\psi(\eta)}{\mu_0 + \mu_1\phi(\xi)\psi(\eta)} \tag{8}$$

where $B_0 = B_0(t), B_1 = B_1(t), B_2 = B_2(t),\ \xi = k_1(t)x - c_1(t) + w_1(t)y$,

$\eta = k_2(t)x - c_2(t) + w_2(t)y$, μ_0 and μ_1 are arbitrary nonzero constants.

Substituting Eq.(8) along with Eq.(3) and Eq.(4) into Eq.(7) and using Maple yields a system of equations w.r.t $\phi^i\psi^j\ (0 \le i, j \le 5)$, setting the coefficients of $\phi^i\psi^j$ in the obtained system of equations to be zero, we obtain a set of over-determined PDEs (or ODEs) with respect to $B_0, B_1, B_2, k_1(t), c_1(t), w_1(t), k_2(t), c_2(t), w_2(t), \mu_0, \mu_1$. Solving the over-determined PDEs by use of Maple, we can obtain the following result.

Case 1 $\mu_0 = \mu_0, B_0(t) = B_0(t), B_1(t) = B_1(t), c_1(t) = c_1(t), c_2(t) = c_2(t), k_1(t) = k_1(t),$

$$\mu_1 = \frac{\sqrt{e_0h_0h_1e_1}\mu_0}{e_0h_0},\ B_2(t) = -\frac{\sqrt{e_0h_0h_1e_1}B_1(t)}{h_0e_1},\ k_2(t) = \frac{\sqrt{e_0h_0h_1e_1}k_1(t)}{h_1h_0},\ w_1(t) = w_1(t),$$

$$w_2(t) = \frac{\sqrt{e_0h_0h_1e_1}w_1(t)}{h_1h_0},$$

Where μ_0 is arbitrary nonzero constant, $B_0(t) = B_0(t), B_1(t) = B_1(t), c_1(t) = c_1(t),$

$c_2(t) = c_2(t), k_1(t) = k_1(t), w_1 = w_1(t)$ are arbitrary functions of t.

Case 2 $\mu_0 = \mu_0, B_0(t) = B_0(t), B_1(t) = B_1(t), c_1(t) = c_1(t), c_2(t) = c_2(t), k_1(t) = k_1(t),$

$$\mu_1 = -\frac{\sqrt{e_0h_0h_1e_1}\mu_0}{e_0h_0},\ B_2(t) = \frac{\sqrt{e_0h_0h_1e_1}B_1(t)}{h_0e_1},\ k_2(t) = -\frac{\sqrt{e_0h_0h_1e_1}k_1(t)}{h_1h_0},\ w_1(t) = w_1(t),$$

$$w_2(t) = -\frac{\sqrt{e_0 h_0 h_1 e_1}\, w_1(t)}{h_1 h_0},$$

Where μ_0 is arbitrary nonzero constant, $B_0(t) = B_0(t), B_1(t) = B_1(t), c_1(t) = c_1(t),$

$c_2(t) = c_2(t), k_1(t) = k_1(t), w_1 = w_1(t)$ are arbitrary functions of t.

When $e_0 = e_1 = 1, h_0 = 1, h_1 = -1$, we can get combining $\tan$ and $\tanh, \coth$ function complexiton solutions:

$$v_1 = B_0(t) + \frac{B_1(t)\tan(\xi) \mp iB_1(t)\tanh(\eta)}{\mu_0 \pm i\mu_0 \tan(\xi)\tanh(\eta)},$$

$$v_2 = B_0(t) + \frac{B_1(t)\tan(\xi) \mp iB_1(t)\coth(\eta)}{\mu_0 \pm i\mu_0 \tan(\xi)\coth(\eta)},$$

Where $\xi = k_1(t)x - c_1(t) + w_1(t)y$, $\eta = \mp ik_1(t)x - c_2(t) \mp iw_1(t)y$.

When $e_0 = e_1 = \frac{1}{2}, h_0 = 1, h_1 = -1$, we can get combining the new complexiton solutions:

$$v_3 = B_0(t) + \frac{B_1(t)(\sec(\xi) + \tan(\xi)) \mp iB_1(t)\tanh(\eta)}{\mu_0 \pm i\mu_0 (\sec(\xi) + \tan(\xi))\tanh(\eta)},$$

$$v_4 = B_0(t) + \frac{B_1(t)(\sec(\xi) + \tan(\xi)) \mp iB_1(t)\coth(\eta)}{\mu_0 \pm i\mu_0 (\sec(\xi) + \tan(\xi))\coth(\eta)},$$

$$v_5 = B_0(t) + \frac{B_1(t)(\csc(\xi) + \cot(\xi)) \mp iB_1(t)\tanh(\eta)}{\mu_0 \pm i\mu_0 (\csc(\xi) + \cot(\xi))\tanh(\eta)},$$

$$v_6 = B_0(t) + \frac{B_1(t)(\csc(\xi) + \cot(\xi)) \mp iB_1(t)\coth(\eta)}{\mu_0 \pm i\mu_0 (\csc(\xi) + \cot(\xi))\coth(\eta)},$$

Where $\xi = k_1(t)x - c_1(t) + w_1(t)y$, $\eta = \mp \frac{i}{2} k_1(t)x - c_2(t) \mp \frac{i}{2} w_1(t)y$.

When $e_0 = e_1 = -1, h_0 = \frac{1}{2}, h_1 = -\frac{1}{2}$, we can get combining the new complexiton solutions:

$$v_7 = B_0(t) + \frac{B_1(t)\cot(\xi) \mp iB_1(t)(\tanh(\eta) \pm i\frac{1}{\cosh(\eta)})}{\mu_0 \pm i\mu_0\cot(\xi)(\tanh(\eta) \pm i\frac{1}{\cosh(\eta)})},$$

$$v_8 = B_0(t) + \frac{B_1(t)\cot(\xi) \mp iB_1(t)(\coth(\eta) \pm i\frac{1}{\sinh(\eta)})}{\mu_0 \pm i\mu_0\cot(\xi)(\coth(\eta) \pm i\frac{1}{\sinh(\eta)})},$$

Where $\xi = k_1(t)x - c_1(t) + w_1(t)y$, $\eta = \mp 2ik_1(t)x - c_2(t) \mp 2iw_1(t)y$.

Similarly, we can write down the other complexiton solutions of eq.(7) which are omitted for convenience.

4. CONCLUSION AND DISCUSSION

we present the modified double sub-equation method and use it to solve the fisher and Kadomtsev-Petviashvili(KP)equations. We obtain not only complexiton solutions but also non-travelling wave and variable-coefficient function solutions. Complexiton solutions obtained in this paper cannot be found in other references. It is clear that this method is different from other methods, very straightforward and effective to get non-travelling wave and variable-coefficient function solutions. The method can also be applied to other nonlinear differential equations in mathematical physics. We are investigating new ansatz and new auxiliary ordinary differential equations to construct more types of exact solutions.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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