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A Study of Thermal Spreading Resistance of Inhomogeneous Silicon Layer in Chip-Multiprocessor

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Authors' contribution

Author YB designed the study, performed the statistical analysis and the literature searches and wrote the protocol. Author DDG managed the analyses of the study and wrote the protocol, and wrote the first draft of the manuscript. All authors read and approved the final manuscript.

Research Article

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ABSTRACT

The phenomenon of thermal Spreading resistance takes places when two rough solids are brought into contact and heat flow is streamed across their asperities. The purpose of the present study is to investigate the heat conduction and thermal spreading resistance of Half-Spaces and semi-infinite Microchannels with Variable conductivity for both heat flux and temperature specified boundary conditions on the contact of silicon and heat sink material in Chip-Multiprocessor. The governing equation is expressed in cylindrical coordinates. A well-known technique (Kirchhoff transformation) is used to linearize the steady state nonlinear heat conduction equation of problem and equations are solved by deriving the analytical solution. Results are presented in contour plots that show the effects of various boundary conditions on the thermal spreading resistance, heat flow rate and temperature distribution.

Keywords: Thermal spreading resistance; heat conduction; kirchhoff transformation; inhomogeneous silicon.

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1. INTRODUCTION

Heat transfer across the interfaces formed by imperfect joints occurs in a wide range of applications, such as micro-electro mechanical systems (MEMS), aerospace, biomedicine, nuclear sciences, heat exchangers and steel industries. Thermal spreading resistance (TSR) is crucial for thermal assessment of contacting bodies in electronic systems. Because of its dependence on the geometric features of solids (asperity slop, surface roughness and etc), kind of deformation, load or apparent contact pressure, type of interstitial fluids and thermal conductivity of solids, it is difficult to develop a precise estimation of TSR. Many studies have been carried out to determine the impact of diverse parameters on the TSRs. General expressions for determining the effect of heat source eccentricity on TSR of finite isotropic and compound rectangular flux channels are presented by Muzychka et al. (2003) [1]. He also reported a review of TSRs in compound and orthotropic systems for both cylindrical and rectangular systems (Muzychka et al., 2004) [2]. The effects of temperature-dependent conductivity, shape and size of contact surfaces and different boundary conditions on the TSR of Silicon have been studied numerically by Rahmani and Shokouhmand (2012) [3]. The effect of heat spreaders in compound systems has been studied by many investigators (Yovanovich et al., 1999; Yovanovich et al., 1998; Yovanovich et al., 1980; Muzychka et al., 2001) [4-7]. Lam et al. (1999) and Ying and Toh, (2000) investigated the effect of orthotropic properties on the TSR [8-9]. Yang et al. (1999) analyzed the TSR of a strip contact spot on a layer of material for the heat-flux specified boundary condition on the contact zone [10]. Many researches on TSR have been done showing the significance of this issue. This work plans to develop analytical solution for determination of TSR with inhomogeneous thermal conductivity. TSR can be analyzed as two problems which two boundary conditions will be considered for the contact area, isothermal contact area and heat flux-specified condition. The solution for the inhomogeneous silicon will be obtained by means of Kirchhoff Transformation and Separation values methods.

Nomenclature

A	=	cross section area, m^2
b	=	larger radius of Flux Channel, m
c	=	radius of contact surface, m
cte	=	constant value
D	=	Area-averaged temperature defined by Eq. 20, K
$J_0(x)$	=	Bessel function of the first kind of order zero
$J_1(x)$	=	Bessel function of the first kind of order one
k	=	thermal conductivity, W/mK
Q	=	heat flow rate, W
q"	=	heat flux, W/m^2
R	=	thermal resistance, K/W
r	=	radial coordinate, m
t	=	arbitrary variable
T	=	temperature, K
z	=	length coordinate, m

Greek Letter Symbols

θ	=	variable temperature substitution, K
ε	=	flux channel relative radius, *c/b*
λ	=	dummy variable

φ = function of contact/total radius ratio *(*where the flux channel relative radius is equal to c/b*)*
∇^2 = Laplacian operator

Subscripts

n = normal component counter
r = reference variable
c = contact surface
∞ = sink properties
s = spreading
Sink = Heat Sink
k=cte = Constant value of thermal conductivity

2. THERMAL SPREADING RESISTANCE (TSR)

Fig. 1 simply shows the schematic configuration of chip-multiprocessor in which Silicon layer is in contact with heat sink material which can be Aluminum, Copper, Diamond and etc. In order to simulate this contact, many domains are suggested and employed; two common geometries which are used in this study are shown in Figs. 2 and 3. Spreading resistance occurs when thermal energy is transformed from one solid to another one by conduction through a small contact area. In a heat sink, this means that heat does not distribute uniformly through the heat sink base.

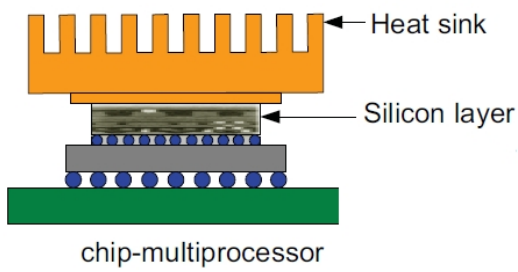

Fig. 1. Chip-Multiprocessor Configuration

In order to determine the TSR, two schematic geometries are commonly used:

i) Heat source on a half-space in which microcontacts are assumed to be located far from each other, the top surface (z=0) of the half-space outside the contact area is assumed to be adiabatic and contact surface has isothermal or isoflux boundary conditions. The sink area is much larger than the contact area, thus, it can be assumed isothermal ($T_{sink}=T_\infty$). The TSR of a circular microcontact of radius C is defined as (see Fig. 2) (Yovanovich and Marotta, 2003) [11]:

$$TSR=\frac{\overline{T}_c-\overline{T}_{Sink}}{Q}=\begin{cases}\dfrac{8}{4KC\pi^2}, isoflux, k=cte\\ \dfrac{1}{4KC}, isothermal, k=cte\end{cases} \quad (1)$$

Where the area-averaged temperature (AAT) is given by:

$$\overline{T} = \frac{1}{A}\iint_A T dA \tag{2}$$

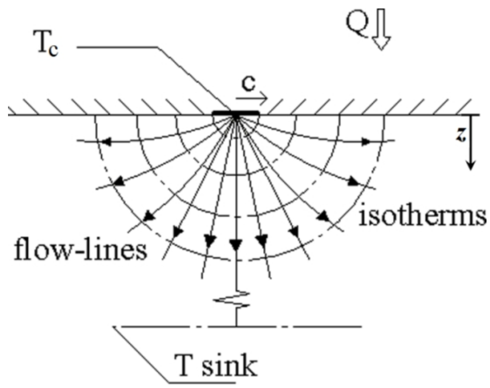

Fig. 2. Half-Space

ii) A circular contact area (A_c) is in contact with semi-infinite circular flux channel with adiabatic edges. Cooper et al. (1969) suggested a precise relation for determining the circular flux channel TSR (see Fig. 3) [12]:

$$TSR = \frac{\overline{T}_c - \overline{T}_{z=0}}{Q} = \frac{\varphi(\varepsilon)}{4KC},\ k = cte \tag{3}$$

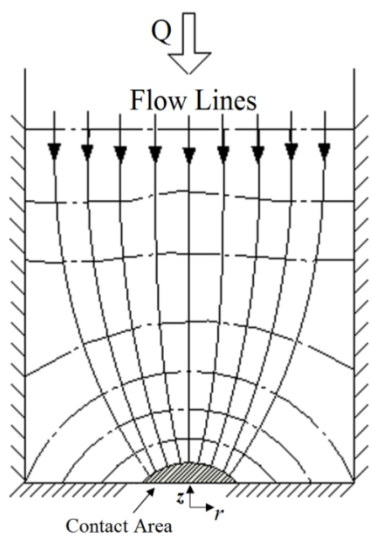

Fig. 3. Flux Channel

Where φ (ε) is dimensionless TSR factor (ε = c/b) that was calculated in many studies (Cooper et al., 1969; Gibson, 1976; Negus and Yovanovich, 1984; Mikic and Rohsenow, 1966) [12-15].

3. KIRCHHOFF TRANSFORMATION

Assessment of the TSR needs the solution of Laplace's equation. Laplace's equation for inhomogeneous silicon has the form:

$$\nabla.(k\nabla T) = 0 \tag{4}$$

This nonlinear equation cannot be solved easily by analytical methods, but there is a simple method named Kirchhoff transformation for transforming the expression of the non-linear diffusive process into the linear equation. Introducing a new variable:

$$\theta = \frac{1}{k_r}\int_{T_r}^{T} k(T)dT \tag{5}$$

Where the reference conductivity is a function of temperature k_r=k(T_r). The aforementioned nonlinear heat conduction equation Eq. (4) becomes a linear Laplace's equation

$$k_r \nabla^2 \theta = 0 \tag{6}$$

Once θ is obtained, the transformation can be used to convert back to temperature. The temperature dependence of k (T) of the Silicon can be modeled approximately by following relation (Glassbrenner and Slack, 1964) [16].

$$k(T) = k_r \times \exp(1 - \tfrac{T}{T_r}) \tag{7}$$

Where k_r=148 W/mK is the reference value for the thermal conductivity of Silicon at the reference value of temperature $T_{r=}$300 K. If the reference temperature T_r of the Kirchhoff transformation is assumed to be the temperature at the infinity distance in z and r directions (T_∞, K_∞) and the contact surface temperature (T_c, K_c) for the Half-Space and the Flux Channel, respectively, the governing equations and boundary conditions will be transformed using Kirchhoff transformation, as shown below:

$$\text{Half-Space:}\begin{cases} k_\infty \nabla^2\theta = 0 \\ z = 0 \to \begin{cases} 0 \le r \le c \begin{cases} \theta = \theta_c \\ or \\ -k_\infty \dfrac{\partial\theta}{\partial z} = \dfrac{Q}{\pi c^2} = q'' \end{cases} \\ r > c, \dfrac{\partial\theta}{\partial z} = 0 \end{cases} \\ z \ne 0 \to \begin{cases} r \to \infty, \theta = \theta_\infty = 0 \\ z \to \infty, \theta = \theta_\infty = 0 \end{cases} \end{cases} \tag{8}$$

and

$$\text{Flux Channel:}\begin{cases} k_c \nabla^2\theta = 0 \\ z = 0 \to \begin{cases} 0 \le r \le c, \theta = \theta_c = 0 \\ r \rangle c, \to \dfrac{\partial\theta}{\partial z} = 0 \end{cases} \\ z \ne 0 \to \begin{cases} k_c \dfrac{\partial\theta}{\partial z} = 0, r \to b \\ k_c \dfrac{\partial\theta}{\partial z} = \dfrac{Q}{\pi b^2} = q'', z \to \infty \end{cases} \end{cases} \tag{9}$$

4. TSR OF INHOMOGENOUS SILICON

4.1 Half Space

4.1.1 Isothermal contact surface

The general solution of temperature distribution throughout the Half-Space can be obtained by means of separation of variables method as follows (Yovanovich and Marotta, 2003) [11]:

$$\theta = \frac{2}{\pi}\theta_c \sin^{-1}\frac{2c}{\sqrt{(r-c)^2+z^2}+\sqrt{(r+c)^2+z^2}} \tag{10}$$

The temperature distribution in the original system T(r,z) can be obtained using Kirchhoff transformation and the temperature-dependent conductivity relation (7) as

$$T(r,z) = T_\infty(1 - \ln(\frac{2}{\pi}(1 - \exp(1 - \frac{T_c}{T_\infty}))\sin^{-1}\frac{2c}{\sqrt{(r-c)^2 + z^2} + \sqrt{(r+c)^2 + z^2}}) \qquad (11)$$

The heat flow rate (HFR) through the Half-Space becomes as follows:

$$Q = 4k_\infty T_\infty c\left(1 - \exp(1 - \frac{T_c}{T_\infty})\right) \qquad (12)$$

The final result for the TSR becomes as follows:

$$TSR = \frac{(T_c - T_\infty)}{4k_\infty T_\infty c(1 - \exp(1 - \frac{T_c}{T_\infty}))} \qquad (13)$$

4.1.2 Isoflux contact surface

The general solution of temperature distribution throughout the Half-Space can be obtained by means of separation of variables method as follows (Yovanovich and Marotta, 2003) [11]:

$$\theta = \frac{q''c}{k}\int_0^\infty e^{-\lambda_n z} J_0(\lambda_n r) J_1(\lambda_n c)\frac{d\lambda}{\lambda} \qquad (14)$$

The temperature distribution in the original system T(r,z) can be obtained using Kirchhoff transformation and the temperature-dependent conductivity relation (7) as follows:

$$T(r,z) = T_\infty(1 - \ln(1 - \frac{Q}{k_\infty \pi c T_\infty}\int_0^\infty e^{-\lambda_n z} J_0(\lambda_n r) J_1(\lambda_n c)\frac{d\lambda}{\lambda_n}) \qquad (15)$$

The AAT of contact surface is given as follows (Yovanovich and Marotta, 2003) [11]:

$$\bar{\theta}_c = (\frac{q''c}{k_\infty}) \qquad (16)$$

Also:

$$\bar{T}_c = T_\infty(1 - \ln(1 - \frac{8Q}{3\pi^2 k_\infty c T_\infty})) \qquad (17)$$

Finally, the TSR becomes:

$$TSR = \frac{-T_\infty \ln(1 - \frac{8Q}{3\pi^2 k_\infty c T_\infty})}{Q} \qquad (18)$$

4.2 Flux Channel

4.2.1 Isothermal contact surface

The general solution of temperature distribution throughout the Flux channel can be obtained by means of separation of variables method as follows (Yovanovich and Marotta, 2003) [11]:

$$\theta(r,z) = -\frac{Q}{k\pi b^2} z + \frac{Q}{k\pi b^2 c}\sum_{n=1}^{\infty}\frac{e^{-\lambda_n z} J_0(\lambda_n r)\sin(\lambda_n c)}{\lambda_n^2 J_0^2(\lambda_n b)} + D \tag{19}$$

Where D is equal to the AAT on Z=0 plane.

$$\overline{\theta}_{z=0} = \frac{1}{\pi b^2}\int_0^b \theta(r,0) 2\pi r dr = D \tag{20}$$

The temperature distribution in the original system T(r,z) can be obtained using Kirchhoff transformation and the temperature-dependent conductivity relation (7) as follows:

$$T(r,z) = T_c(1-\ln(1+\frac{Q}{k\pi T_c b^2} z - \frac{Q}{k\pi T_c b^2 c}\sum_{n=1}^{\infty}\frac{e^{-\lambda_n z} J_0(\lambda_n r)\sin(\lambda_n c)}{\lambda_n^2 J_0^2(\lambda_n b)} + D)) \tag{21}$$

The TSR becomes as follows:

$$TSR = \frac{T_c\{\ln(1-\frac{D}{T_c}) - \ln(1-\frac{D+\frac{Q}{4k_c c}\varphi(\varepsilon)}{T_c})\}}{Q} \tag{22}$$

In this equation, increasing the total HFR from the contact surface to the flux channel, Q, causes an increase in the AAT of contact surface with θ(r,z) distribution, *D*.
The TSR ratio TSR/TSR $_{(k=cte)}$ is:

$$\frac{TSR}{TSR_{(k=cte)}} = \frac{\frac{T_c\{\ln(1-\frac{D}{T_c}) - \ln(1-\frac{D+\frac{Q}{4k_c c}\varphi(\varepsilon)}{T_c})\}}{Q}}{\frac{1}{4k_c c}\varphi(\varepsilon)} > 1 \tag{23}$$

This relation indicates that when the thermal conductivity is assumed to be variable, the amount of TSR related to this assumption is more than the TSR with constant conductivity.

5. DISCUSSION

Thermal spreading resistance of silicon (Si) as a most widely used semiconductor in microelectronics industry was expressed by analytical method. To demonstrate the importance of the present work, some analytical graphs from aforementioned mathematical results are presented. Two example of Half-Space with two different boundary conditions (isoflux and isothermal contact surface) are presented. First, an example is given which shows the effects of temperature and size of contact surface on the HFR and TSR. Second one shows the effects of HFR and size of contact surface on the AAT and TSR.

Variation of ratio $TSR/TSR_{(k=\ cte)}$ in the heat flow range of rate from 1 W to 5 W and in the temperature range of 300 to 350 K for the isoflux and isothermal contact surface boundary conditions, are shown in Figs. 4 and 5, respectively.

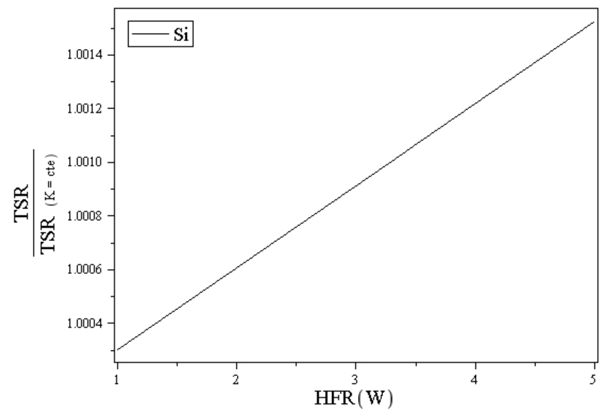

Fig. 4 Variation of $TSR/TSR_{(k=cte)}$, isoflux contact area

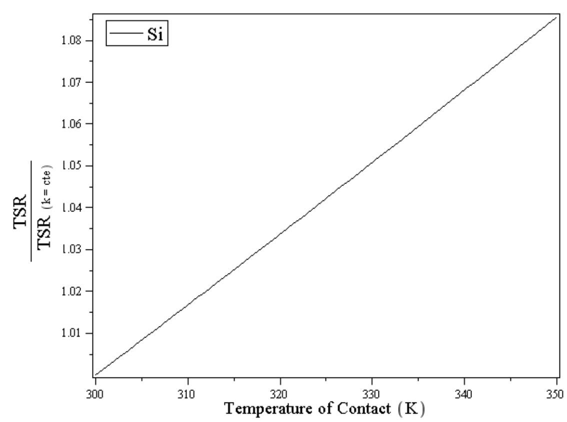

Fig. 5 Variation of $TSR/TSR_{(k=cte)}$, isothermal contact area

Increasing linear changes can be seen in both graphs that could be ascribed to the opposite relationship between temperature value and thermal conductivity of silicon within this temperature range (see Eq. 7). As the difference between the contact surface temperature and the sink temperature becomes more evident, the difference between the TSR $_{(k=cte)}$ and the TSR becomes more significant, as can be observed in following figures.

Additionally, the symbolic mathematics program Maple 13 (2009) is employed and some contour plots from mathematical results given previously are presented in this work [17].

For illustration purposes we consider the following two surface boundary conditions, figs. 6, and 7 are given for the Isothermal contact surface boundary condition and Figs. 8, 9 and 10 are given for isoflux contact.

Isothermal contact surface boundary condition: In this boundary condition, HFR and TSR are the most important parameter in the thermal evaluation of system. Fig. 6 shows the changes in the HFR when the contact radius and temperature are increased, the values of HFR in some pints are given (24, 50…251).

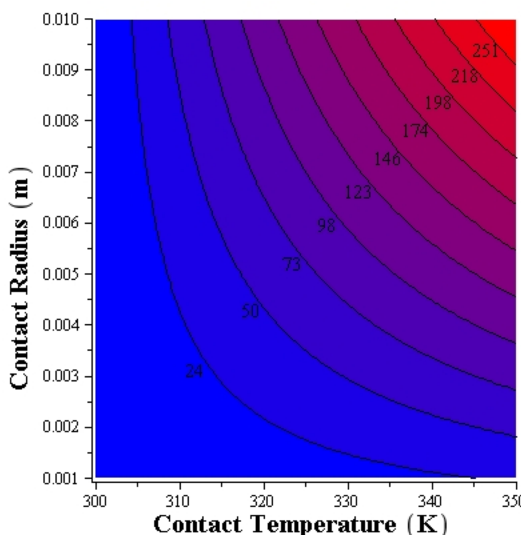

Fig. 6. 2D contour plot of HFR, isothermal contact area

As can be seen in Fig. 6 the values of HFR increase with increase of contact radius. Also, HFR increases slightly as contact temperature increases, while the contact size is held constant. As shown in Fig. 7, an increase in contact radius results in an increase in the TSR, while, TSR decreases with increasing the Temperature of contact surface (T_c). These results can be drawn from Eqs. 12 and 13.

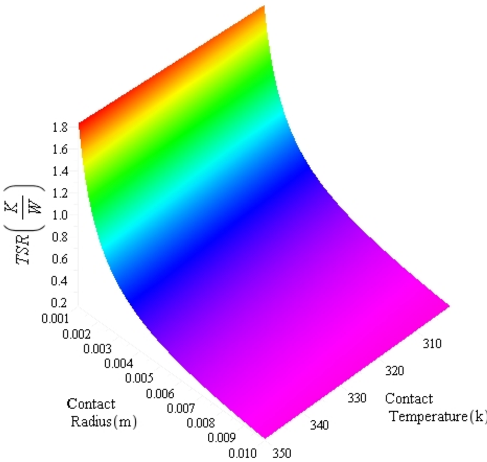

Fig. 7. 3D contour plot of TSR, isothermal contact area

The Isoflux contact surface boundary condition: It can be observed in Figs. 8, 9 and Eq. 17 that the AAT of contact surface increases linearly by increasing the HFR and decreases logarithmically with contact radius increase. Also, the TSR increases linearly by increasing the HFR and decreases logarithmically with contact radius increase as illustrated in Fig. 10 and Eq. 18.

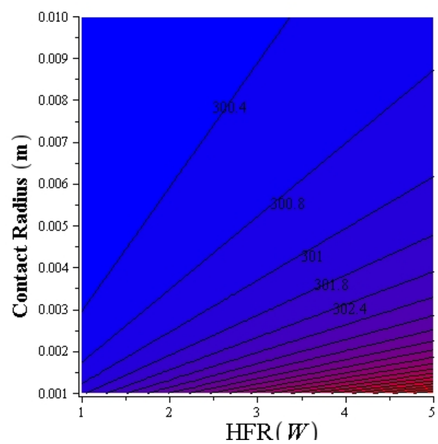

Fig. 8. 2D contour plot of AAT of contact surface, isoflux contact area

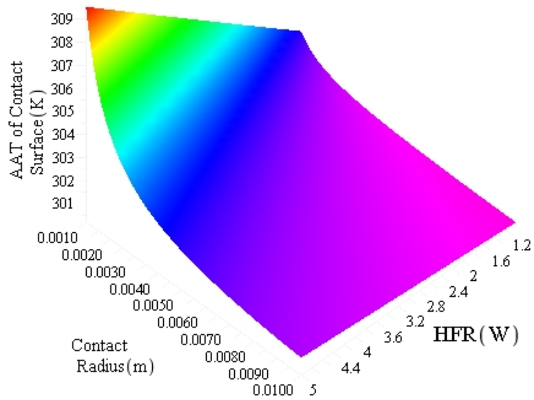

Fig. 9. 3D contour plot of contact surface AAT, isoflux contact area

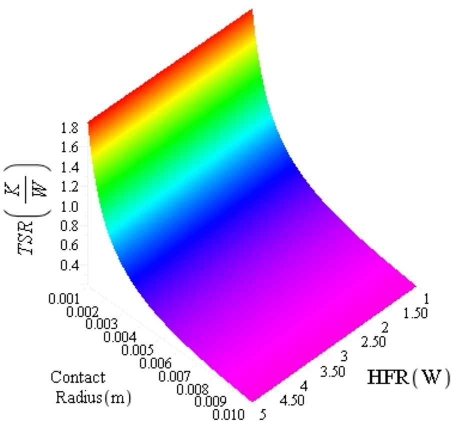

Fig. 10. 3D contour plot of TSR

6. CONCLUSION

A simple direct method to estimate the TSR of inhomogeneous silicon is proposed. The proposed model is based on the Kirchhoff transformation, which transforms the nonlinear heat Conduction equation into the Laplace equation whose analytic solution can be obtained easily. The effect of contact surface radius and different boundary conditions on the Heat flux ratio and thermal spreading resistance were assessed. Simple analytical expressions for determining the TSR of silicon were derived and results were presented in some contour plots. The effects of various boundary conditions on the thermal spreading resistance, heat flow rate and temperature distribution were investigated.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

.

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