Applied Artificial Intelligence
An International Journal
ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/uaai20

Complex Interval-Valued q-Rung Orthopair 2-Tuple Linguistic Aggregation Operators and Their Application in Multi-Attribute Decision-Making

Shouzhen Zeng, Zeeshan Ali, Tahir Mahmood & Huanhuan Jin

To cite this article: Shouzhen Zeng, Zeeshan Ali, Tahir Mahmood & Huanhuan Jin (2022) Complex Interval-Valued q-Rung Orthopair 2-Tuple Linguistic Aggregation Operators and Their Application in Multi-Attribute Decision-Making, Applied Artificial Intelligence, 36:1, 2033471, DOI: 10.1080/08839514.2022.2033471

To link to this article: https://doi.org/10.1080/08839514.2022.2033471

APPLIED ARTIFICIAL INTELLIGENCE
2022, VOL. 36, NO. 1, e2033471 (1079 pages)
https://doi.org/10.1080/08839514.2022.2033471

Taylor & Francis
Taylor & Francis Group

OPEN ACCESS Check for updates

Complex Interval-Valued q-Rung Orthopair 2-Tuple Linguistic Aggregation Operators and Their Application in Multi-Attribute Decision-Making

Shouzhen Zeng [a], Zeeshan Ali [b], Tahir Mahmood [b], and Huanhuan Jin [c]

[a]School of Business, Ningbo University, Ningbo, Zhejiang, China; [b]Department of Mathematics and Statistics, International Islamic University, Islamabad, Pakistan; [c]Hangzhou College Commerce, Zhejiang Gongshang University, Hangzhou, Zhejiang, China

ABSTRACT

This study aims to study the conception of the complex interval-valued q-rung orthopair 2-tuple linguistic set (CIVQRO2-TLS), which is a new powerful mixture to handle unreliable and vague data in realistic decision concerns. We also explore its fundamental properties as well as important laws. In the occurrence of the above theory, we discover some useful aggregation methods for the CIVQRO2-TLS, including the CIVQRO2-TLWA, CIVQRO2-TLOWA, CIVQRO2-TLHA, CIVQRO2-TLWG, CIVQRO2-TLOWG, and CIVQRO2-TLHG operators. To demonstrate the beneficial features of the invented works, a multi-attribute decision-making (MADM) system is presented and exposed to the supremacy of the presented operators with the help of several examples. In last, we elaborate on the advantages, comparative analysis, and graphical interpretation of the invented approaches.

ARTICLE HISTORY
Received 13 May 2021
Revised 17 December 2021
Accepted 18 January 2022

Introduction

The main theme of MADM is to select the outstanding opportunity in excellence to the limited objects in the presence of the many criteria. In our genuine life, we continuously grapple with distinct sorts of decision-making strategies wherein our focus is to investigate how to make an accurate decision. Roughly, the strategy of decision-making takes impreciseness in the data without determining the vagueness and inconsistent in it. To describe the ambiguity in the data, the main concept of IFS (Atanassov 1986), a revised version of the FS (Zadeh 1965), incorporates the two sorts of terms, called the truth grade (TG) $\mu_{Z_{CQ}}(x)$ and falsity grade (FG) $\eta_{Z_{CQ}}(x)$, then the prominent tool of IFS is of the mathematical shape: $\eta_{Z_{CQ}}(x)$. Thus, IFS has easily evaluated complicated and incorporated data that occurred in genuine life troubles. The IVIFS invented by Atanassov and Gargov (1989) is also of the most important mathematical

CONTACT Huanhuan Jin jinhh06@163.com Hangzhou College Commerce, Zhejiang Gongshang University, Hangzhou 310012, China

structure of the FS, but the resultant values of the IVIFS are in the shape of interval whose both bounds lie within the unit interval. Lots of well-known scholars have done work on IFS and IVIFS in distinct directions (Bouchet et al. 2020; Garg and Kumar 2019, 2020; Kumar and Garg 2018; Zeng, Ali, and Mahmood 2021; Zeng et al. 2022c; Zhang et al. 2021) in recent years. In many dilemmas, the function of IFS can't be working perfectly, for instance, an expert from some enterprises given the pair of data $(0.7, 0.4)$, for TG and FG, it is clear that $(0.7, 0.4)$. After continuing a lot of hardworking to find the solution for the above trouble, Yager (2013) investigated the PFS, by improving the representation range of IFS that: $0 \leq \mu^2_{Z_{CQ}}(x) + \eta^2_{Z_{CQ}}(x) \leq 1$. Later, the IVPFS is discussed by Garg (2017) by considering the interval situation. At present, a lot of individuals employed the conceptions of PFS and IVPFS in disparate specialties and areas (Ejegwa, Onyeke, and Adah 2021; Haktanır and Kahraman 2019; Liang, Darko, and Xu 2018; Peng and Li 2019; Sajjad Ali Khan et al. 2018; Zeng et al. 2022b). Moreover, Yager (2016) deliberated the QROFS with $0 \leq \mu^2_{Z_{CQ}}(x) + \eta^2_{Z_{CQ}}(x) \leq 1$. The parameter q_{SC} used in the tool of QROFS, can help to generalize the representation range of IFS and PFS. Until now, the QROFS and its discrete structures, the IVQROFS (Joshi et al. 2018), have gained a lot of attention from researchers in various problems of chemistry, transportation engineering, biology, sociology, electrical engineering, economics, etc. (Garg and Chen 2020; Hussain, Ali, and Mahmood 2019; Liu and Liu 2018; Liu and Wang 2018).

To generalize the unit disc of FSs to unit interval situation, a lot of researchers have worked new extensions and suggested several new fuzzy tools by generalizing the unit disc in the range of FSs. Invented by Alkouri and Salleh (2012), the CIFS, a modified structure of the CFS (Ramot et al. 2002), depends on two terms in the shape of TG $\mu_{Z_{CQ}} = \mu_{Z_{RP}} e^{i2\pi(\mu_{Z_{IP}})}$ and FG $\mu_{Z_{CQ}} = \mu_{Z_{RP}} e^{i2\pi(\mu_{Z_{IP}})}$, then the prominent tool of CIFS is of the shape: $0 \leq \mu_{Z_{CQ}}(x) + \eta_{Z_{CQ}}(x) \leq 1$ and $0 \leq \mu_{Z_{CQ}}(x) + \eta_{Z_{CQ}}(x) \leq 1$. Some important and beneficial implementations of CIFS and CIVIFS are then studied in diverse disciplines (Akram and Naz 2019; Garg and Rani 2019; Rani and Garg 2017). Moreover, Ullah et al. (2020b) expanded the conception of CIFS to invent the CPFS with $0 \leq \mu^2_{Z_{RP}}(x) + \eta^2_{Z_{RP}}(x) \leq 1$ and $0 \leq \mu^2_{Z_{RP}}(x) + \eta^2_{Z_{RP}}(x) \leq 1$. In many dilemmas, the tool of CPFS can't be working perfectly, for instance, an expert from some enterprises given the pair of data $(0.9e^{i2\pi(0.8)}, 0.8e^{i2\pi(0.7)})$, for TG and FG, it is clear that $0.9^2 + 0.8^2 = 0.81 + 0.64 = 1.45 > 1$, and $0.9^2 + 0.8^2 = 0.81 + 0.64 = 1.45 > 1$. To settle this Liu, Ali, and

Mahmood (2019, 2020) diagnosed the novel and well-known theory of CQROFS with a new strategy $0 \leq \mu^{q_{SC}}_{Z_{RP}}(x) + \eta^{q_{SC}}_{Z_{RP}}(x) \leq 1$ and $0 \leq \mu^{q_{SC}}_{Z_{RP}}(x) + \eta^{q_{SC}}_{Z_{RP}}(x) \leq 1$. Furthermore, Garg, Ali, and Mahmood (2020b) developed the CIVQROFS on the basis of the presence of the above-invented works, whose construct composes the grade of truth and falsity, with the real part and imaginary part in the form of a subset of the unit interval. So far, several researchers have contributed the development of CIVQROFS in the region of diverse disciplines (Ali and Mahmood 2020b; Garg et al. 2020a).

To depict the ambiguity and awkward data in genuine life procedures, the 2-tuple linguistic (2-TL) is introduced by Herrera and Martinez (2001). 2-TL theory plays a beneficial role in the environment of fuzzy sets theory and due to its beneficial worth, a lot of individuals made contributions in the development of 2-TL theory. For instance, Herrera and Martinez (2000) combined the FS with 2-TL, introduced the conception of fuzzy 2-TLS. Beg and Rashid (2016), Liu and Chen (2018) put forth the intuitionistic 2-TL set by taking the advantages of IFS. Faizi, Rashid, and Zafar (2018), (2020) invented some useful aggregation operators for intuitionistic 2-TLS. Wei et al. (2017) elaborated the Pythagorean 2-TLS and explored its application. He et al. (2019) exposed the Taxonomy method for Pythagorean 2-TLS. Ju et al. (2020) diagnosed the q-rung orthopair 2-TLS. Finally, the interval-valued q-rung orthopair 2-TLS deliberated explored by Wang, Garg, and Li (2019).

Our perspective behind the introduction to CIVQRO2-TLS is given below:

(1) CIVQROFS is a perfect blend of QROFS and CPFS. It assigns interval grades to the elements, which posses parametric characterization. However, it isn't utilized to portray qualitative expression with 2-TLS, which is more convenient for decision makers to express their subjective preferences during decision (Herrera and Martinez 2000). In order to combine the both properties of the above discussed models and to handle the existing drawbacks, we shall put forward the CIVQRO2-TLS.

(2) The 2-TLS has been explored so far in kinds of extensions of fuzzy set theory. However, it is no discussion over the degrees of interval-valued truth grade and the interval-valued grade of falsity in the literature. This motivated us to initiate the mathematical mixture of CIVQRO2 and 2-TLS.

Keeping all these facts in our mind, the main contributions of this study to the current literature deliberated in the following way:

(1) To present the conception of the CIVQRO2-TLS and its fundamental laws.
(2) To discover the conceptions of CIVQRO2-TLWA, CIVQRO2-TLOWA, CIVQRO2-TLHA, CIVQRO2-TLWG, CIVQRO2-TLOWG, CIVQRO2-TLHG operators and diagnose their important properties.
(3) A MADM system is developed based on the proposed operators, and its application is carried out in the CIVQRO2-TL situation with the help of several examples.
(4) To elaborate on the advantages, comparative analysis, and graphical interpretation of the invented approaches are presented.

The rest contents of this manuscript are deliberated in the following manner: Section 2 reviews some related fundamental theories of CQROFS, CIVQROFS, and 2-TLS. Section 3 discusses the conception of the CIVQRO2-TLS and its important operational laws. In section 4, we discover the thought of CIVQRO2-TLWA, CIVQRO2-TLOWA, CIVQRO2-TLHA, CIVQRO2-TLWG, CIVQRO2-TLOWG, CIVQRO2-TLHG and diagnosed their fundamental properties. In section 5, we demonstrate the beneficial features of the invented works, a MADM system is then presented, and its supremacy of the diagnosed operators is discussed with the help of several examples. In last, we elaborate on the advantages, comparative analysis, and graphical interpretation of the invented approaches. The conclusion of this scenario is described in section 6.

Preliminaries

Firstly, some mathematical terms used in the current studies are deliberated in Table 1.

Using the above terminologies, we reviews several basic theories.

Table 1. Mathematical terms and their meanings.

Symbols	Meanings	Symbols	Meanings
$q_{SC}, \Upsilon_{SC} \geq 1$	Positive integers	X_{UNI}	Universal set
$\mu_{Z_{CQ}}(x)$	Truth grade	$\eta_{Z_{CQ}}(x)$	Falsity grade
$\mu_{Z_{RP}} e^{i2\pi(\mu_{Z_{IP}})}$	Complex-valued truth grade	$\eta_{Z_{RP}} e^{i2\pi(\eta_{Z_{IP}})}$	Complex-valued falsity grade
$\mu_{Z_{RP}}$	A real part of truth grade	$\mu_{Z_{RP}}$	A real part of falsity grade
$\mu_{Z_{IP}}$	The imaginary part of truth grade	$\eta_{Z_{IP}}$	The imaginary part of falsity grade
Z_{CQ}	Complex q-rung orthopair fuzzy sets	Γ_{SF}	Score function
Ψ_{AF}	Accuracy function	Z_{CIVT}	Complex interval-valued q-rung orthopair fuzzy 2-tuple linguistic sets
$\check{s}_{\check{S}_{LT}}$	Linguistic term	$\left(\check{s}_{\check{S}_{LT}}, \alpha_{SC}\right)$	2-tuple linguistic set
x	Element of the universal set	ϖ_{W-j}	Weight vector

Definition 1 (Liu, Ali, and Mahmood 2019). In the consideration of the universal set X_{UNI}, a CQROFS X_{UNI} is deliberated by:

$$Z_{CQ} = \left\{ \left(\mu_{Z_{CQ}}(x), \eta_{Z_{CQ}}(x) \right) : x \in X_{UNI} \right\} \tag{1}$$

The complex mathematical form of TG and TG is stated: $\mu_{Z_{CQ}} = \mu_{Z_{RP}} e^{i2\pi\left(\mu_{Z_{IP}}\right)}$ and $\mu_{Z_{CQ}} = \mu_{Z_{RP}} e^{i2\pi\left(\mu_{Z_{IP}}\right)}$, with $0 \leq \mu_{Z_{RP}}^{\|\mathbb{SC}}(x) + \eta_{Z_{RP}}^{\|\mathbb{SC}}(x) \leq 1$ and $0 \leq \mu_{Z_{RP}}^{q_{SC}}(x) + \eta_{Z_{RP}}^{q_{SC}}(x) \leq 1$. Additionally, the mathematical shape of refusal grade is stated: $\pi_{Z_{CQ}} = \left(1 - \left(\mu_{Z_{RP}}^{q_{SC}} + \eta_{Z_{RP}}^{q_{SC}}\right)\right)^{\frac{1}{q_{SC}}} e^{i2\pi\left(1-\left(\mu_{Z_{IP}}^{q_{SC}} + \eta_{Z_{IP}}^{q_{SC}}\right)\right)^{\frac{1}{q_{SC}}}}$.

The mathematical term of CQROFNs is of the shape: $Z_{CQ} = \left(\mu_{Z_{RP}} e^{i2\pi\left(\mu_{Z_{IP}}\right)}, \eta_{Z_{RP}} e^{i2\pi\left(\eta_{Z_{IP}}\right)}\right)$.

Definition 2 (Garg, Ali, and Mahmood 2020b). In the consideration of the universal set X_{UNI}, a CIVQROFS $_{CIVQ}$ is deliberated by:

$$Z_{CIVQ} = \left\{ \left(\mu_{Z_{CIVQ}}(x), \eta_{Z_{CIVQ}}(x) \right) : x \in X_{UNI} \right\} \tag{2}$$

The complex mathematical form of interval-valued TG and interval-valued TG is stated: $\mu_{Z_{CIVQ}} = \left[\mu_{Z_{RP}}^{-}, \mu_{Z_{RP}}^{+}\right] e^{i2\pi\left[\mu_{Z_{IP}}^{-}, \mu_{Z_{IP}}^{+}\right]}$ and $\mu_{Z_{CIVQ}} = \left[\mu_{Z_{RP}}^{-}, \mu_{Z_{RP}}^{+}\right] e^{i2\pi\left[\mu_{Z_{IP}}^{-}, \mu_{Z_{IP}}^{+}\right]}$, with $0 \leq \mu^{+q_{SC}}_{Z_{RP}} + \eta^{+q_{SC}}_{Z_{RP}} \leq 1$ and $0 \leq \mu^{+q_{SC}}_{Z_{RP}} + \eta^{+q_{SC}}_{Z_{RP}} \leq 1$. Additionally, the mathematical shape of refusal grade is stated:

$$\pi_{Z_{CIVQ}} = \left[\left(1 - \left(\mu^{-q_{SC}}_{Z_{RP}} + \eta^{-q_{SC}}_{Z_{RP}}\right)\right)^{\frac{1}{q_{SC}}}, \left(1 - \left(\mu^{+q_{SC}}_{Z_{RP}} + \eta^{+q_{SC}}_{Z_{RP}}\right)\right)^{\frac{1}{q_{SC}}}\right] e^{i2\pi\left[\left(1-\left(\mu^{-q_{SC}}_{Z_{IP}} + \eta^{-q_{SC}}_{Z_{IP}}\right)\right)^{\frac{1}{q_{SC}}}, \left(1-\left(\mu^{+q_{SC}}_{Z_{IP}} + \eta^{+q_{SC}}_{Z_{IP}}\right)\right)^{\frac{1}{q_{SC}}}\right]}$$

Moreover, the mathematical term of CIVQROFNs is of the shape: $Z_{CIVQ} = \left(\left[\mu_{Z_{RP}}^{-}, \mu_{Z_{RP}}^{+}\right] e^{i2\pi\left[\mu_{Z_{IP}}^{-}, \mu_{Z_{IP}}^{+}\right]}, \left[\eta_{Z_{RP}}^{-}, \eta_{Z_{RP}}^{+}\right] e^{i2\pi\left[\eta_{Z_{IP}}^{-}, \eta_{Z_{IP}}^{+}\right]}\right)$. Moreover, for any two CIVQROFNs

$Z_{CIVQ} = \left(\left[\mu_{Z_{RP}}^{-}, \mu_{Z_{RP}}^{+}\right] e^{i2\pi\left[\mu_{Z_{IP}}^{-}, \mu_{Z_{IP}}^{+}\right]}, \left[\eta_{Z_{RP}}^{-}, \eta_{Z_{RP}}^{+}\right] e^{i2\pi\left[\eta_{Z_{IP}}^{-}, \eta_{Z_{IP}}^{+}\right]}\right)$ and

$Z_{CIVQ-2} = \left(\left(\left[\mu_{Z_{RP-2}}^{-}, \mu_{Z_{RP-2}}^{+}\right] e^{i2\pi\left[\mu_{Z_{IP-2}}^{-}, \mu_{Z_{IP-2}}^{+}\right]}, \left[\eta_{Z_{RP-2}}^{-}, \eta_{Z_{RP-2}}^{+}\right] e^{i2\pi\left[\eta_{Z_{IP-2}}^{-}, \eta_{Z_{IP-2}}^{+}\right]}\right)\right)$

$$Z_{CIVT-1} \oplus_{CIVT} Z_{CIVT-2} = \left(\begin{array}{c} \left[\begin{array}{l} \left(\mu^{-q_{SC}}_{Z_{RP-1}} + \mu^{-q_{SC}}_{Z_{RP-2}} - \mu^{-q_{SC}}_{Z_{RP-1}} \mu^{-q_{SC}}_{Z_{RP-2}} \right)^{\frac{1}{q_{SC}}}, \\ \left(\mu^{+q_{SC}}_{Z_{RP-1}} + \mu^{+q_{SC}}_{Z_{RP-2}} - \mu^{+q_{SC}}_{Z_{RP-1}} \mu^{+q_{SC}}_{Z_{RP-2}} \right)^{\frac{1}{q_{SC}}} \end{array} \right] e^{i2\pi \left[\begin{array}{l} \left(\mu^{-q_{SC}}_{Z_{IP-1}} + \mu^{-q_{SC}}_{Z_{IP-2}} - \mu^{-q_{SC}}_{Z_{IP-1}} \mu^{-q_{SC}}_{Z_{IP-2}} \right)^{\frac{1}{q_{SC}}}, \\ \left(\mu^{+q_{SC}}_{Z_{IP-1}} + \mu^{+q_{SC}}_{Z_{IP-2}} - \mu^{+q_{SC}}_{Z_{IP-1}} \mu^{+q_{SC}}_{Z_{IP-2}} \right)^{\frac{1}{q_{SC}}} \end{array} \right]}, \\ \left[\eta^{-}_{Z_{RP-1}} \eta^{-}_{Z_{RP-2}}, \eta^{+}_{Z_{RP-1}} \eta^{+}_{Z_{RP-2}} \right] e^{i2\pi \left[\eta^{-}_{Z_{IP-1}} \eta^{-}_{Z_{IP-2}}, \eta^{+}_{Z_{IP-1}} \eta^{+}_{Z_{IP-2}} \right]} \end{array} \right) \tag{3}$$

$$Z_{CIVT-1} \otimes_{CIVT} Z_{CIVT-2} = \left(\begin{array}{c} \left[\mu^{-}_{Z_{RP-1}} \mu^{-}_{Z_{RP-2}}, \mu^{+}_{Z_{RP-1}} \mu^{+}_{Z_{RP-2}} \right] e^{i2\pi \left[\mu^{-}_{Z_{IP-1}} \mu^{-}_{Z_{IP-2}}, \mu^{+}_{Z_{IP-1}} \mu^{+}_{Z_{IP-2}} \right]}, \\ \left[\begin{array}{l} \left(\eta^{-q_{SC}}_{Z_{RP-1}} + \eta^{-q_{SC}}_{Z_{RP-2}} - \eta^{-q_{SC}}_{Z_{RP-1}} \eta^{-q_{SC}}_{Z_{RP-2}} \right)^{\frac{1}{q_{SC}}}, \\ \left(\eta^{+q_{SC}}_{Z_{RP-1}} + \eta^{+q_{SC}}_{Z_{RP-2}} - \eta^{+q_{SC}}_{Z_{RP-1}} \eta^{+q_{SC}}_{Z_{RP-2}} \right)^{\frac{1}{q_{SC}}} \end{array} \right] e^{i2\pi \left[\begin{array}{l} \left(\eta^{-q_{SC}}_{Z_{IP-1}} + \eta^{-q_{SC}}_{Z_{IP-2}} - \eta^{-q_{SC}}_{Z_{IP-1}} \eta^{-q_{SC}}_{Z_{IP-2}} \right)^{\frac{1}{q_{SC}}}, \\ \left(\eta^{+q_{SC}}_{Z_{IP-1}} + \eta^{+q_{SC}}_{Z_{IP-2}} - \eta^{+q_{SC}}_{Z_{IP-1}} \eta^{+q_{SC}}_{Z_{IP-2}} \right)^{\frac{1}{q_{SC}}} \end{array} \right]} \end{array} \right) \tag{4}$$

$$\Upsilon_{SC} Z_{CIVT-1} = \left(\begin{array}{c} \left[\begin{array}{l} \left(1 - \left(1 - \mu^{-}_{Z_{RP-1}}{}^{q_{SC}} \right)^{\Upsilon_{SC}} \right)^{\frac{1}{q_{SC}}}, \\ \left(1 - \left(1 - \mu^{+}_{Z_{RP-1}}{}^{q_{SC}} \right)^{\Upsilon_{SC}} \right)^{\frac{1}{q_{SC}}} \end{array} \right] e^{i2\pi \left[\begin{array}{l} \left(1 - \left(1 - \mu^{-}_{Z_{IP-1}}{}^{q_{SC}} \right)^{\Upsilon_{SC}} \right)^{\frac{1}{q_{SC}}}, \\ \left(1 - \left(1 - \mu^{+}_{Z_{IP-1}}{}^{q_{SC}} \right)^{\Upsilon_{SC}} \right)^{\frac{1}{q_{SC}}} \end{array} \right]}, \\ \left[\eta^{-\Upsilon_{SC}}_{Z_{RP-1}}, \eta^{+\Upsilon_{SC}}_{Z_{RP-1}} \right] e^{i2\pi \left[\eta^{-\Upsilon_{SC}}_{Z_{IP-1}}, \eta^{+\Upsilon_{SC}}_{Z_{IP-1}} \right]} \end{array} \right) \tag{5}$$

$$Z^{\Upsilon_{SC}}_{CIVT-1} = \left(\begin{array}{c} \left[\mu^{-\Upsilon_{SC}}_{Z_{RP-1}}, \mu^{+\Upsilon_{SC}}_{Z_{RP-1}} \right] e^{i2\pi \left[\mu^{-\Upsilon_{SC}}_{Z_{IP-1}}, \mu^{+\Upsilon_{SC}}_{Z_{IP-1}} \right]}, \\ \left[\begin{array}{l} \left(1 - \left(1 - \eta^{-}_{Z_{RP-1}}{}^{q_{SC}} \right)^{\Upsilon_{SC}} \right)^{\frac{1}{q_{SC}}}, \\ \left(1 - \left(1 - \eta^{+}_{Z_{RP-1}}{}^{q_{SC}} \right)^{\Upsilon_{SC}} \right)^{\frac{1}{q_{SC}}} \end{array} \right] e^{i2\pi \left[\begin{array}{l} \left(1 - \left(1 - \eta^{-}_{Z_{IP-1}}{}^{q_{SC}} \right)^{\Upsilon_{SC}} \right)^{\frac{1}{q_{SC}}}, \\ \left(1 - \left(1 - \eta^{+}_{Z_{IP-1}}{}^{q_{SC}} \right)^{\Upsilon_{SC}} \right)^{\frac{1}{q_{SC}}} \end{array} \right]} \end{array} \right) \tag{6}$$

The score function (SF) and accuracy function (AF) is stated by (Garg, Ali, and Mahmood 2020b):

$$\Gamma_{SF}(\mathrm{Z}_{CIVQ-1}) = \frac{1}{4}\begin{pmatrix} \left(\mu^{+q_{SC}}_{\mathrm{Z}_{RP-1}} + \mu^{+q_{SC}}_{\mathrm{Z}_{IP-1}} - \eta^{+q_{SC}}_{\mathrm{Z}_{RP-1}} - \eta^{+q_{SC}}_{\mathrm{Z}_{IP-1}}\right) - \\ \left(\mu^{-q_{SC}}_{\mathrm{Z}_{RP-1}} + \mu^{-q_{SC}}_{\mathrm{Z}_{IP-1}} - \eta^{-q_{SC}}_{\mathrm{Z}_{RP-1}} - \eta^{-q_{SC}}_{\mathrm{Z}_{IP-1}}\right) \end{pmatrix} \tag{7}$$

$$\Psi_{SF}(\mathrm{Z}_{CIVQ-1}) = \frac{1}{4}\begin{pmatrix} \left(\mu^{+q_{SC}}_{\mathrm{Z}_{RP-1}} + \mu^{+q_{SC}}_{\mathrm{Z}_{IP-1}} + \eta^{+q_{SC}}_{\mathrm{Z}_{RP-1}} + \eta^{+q_{SC}}_{\mathrm{Z}_{IP-1}}\right) - \\ \left(\mu^{-q_{SC}}_{\mathrm{Z}_{RP-1}} + \mu^{-q_{SC}}_{\mathrm{Z}_{IP-1}} + \eta^{-q_{SC}}_{\mathrm{Z}_{RP-1}} + \eta^{-q_{SC}}_{\mathrm{Z}_{IP-1}}\right) \end{pmatrix} \tag{8}$$

For CIVQROFNs Z_{CIVQ-1} and Z_{CIVQ-1}, two techniques for comparison are dominated as (Garg, Ali, and Mahmood 2020b);

(1) If $\Gamma_{SF}(\mathrm{Z}_{CIVQ-1}) > \Gamma_{SF}(\mathrm{Z}_{CIVQ-2})$, then $\Gamma_{SF}(\mathrm{Z}_{CIVQ-1}) > \Gamma_{SF}(\mathrm{Z}_{CIVQ-2})$,
- If $\mathrm{Z}_{CIVQ-1} = \mathrm{Z}_{CIVQ-2}$, then
- If $\mathrm{Z}_{CIVQ-1} = \mathrm{Z}_{CIVQ-2}$, then $\mathrm{Z}_{CIVQ-1} > \mathrm{Z}_{CIVQ-2}$;

CIVQROF2-TLS

This study aims to present the conception of the CIVQRO2-TLS and its fundamental laws.

Definition 3: In the consideration of the universal set X_{UNI}, a CIVQRO2-TLS X_{UNI} is deliberated by:

$$\mathrm{Z}_{CIVT} = \left\{\left(\left(\check{s}_{\check{S}_{LT}}, \alpha_{SC}\right), \left(\mu_{\mathrm{Z}_{CIVQ}}(x), \eta_{\mathrm{Z}_{CIVQ}}(x)\right)\right) : x \in X_{UNI}\right\} \tag{9}$$

The complex mathematical form of TG and TG is stated: $\mu_{\mathrm{Z}_{CIVQ}} = \left[\mu^{-}_{\mathrm{Z}_{RP}}, \mu^{+}_{\mathrm{Z}_{RP}}\right] e^{i2\pi\left[\mu^{-}_{\mathrm{Z}_{IP}}, \mu^{+}_{\mathrm{Z}_{IP}}\right]}$ and $\eta_{\mathrm{Z}_{CQ}} = \left[\eta^{-}_{\mathrm{Z}_{RP}}, \eta^{+}_{\mathrm{Z}_{RP}}\right] e^{i2\pi\left[\eta^{-}_{\mathrm{Z}_{IP}}, \eta^{+}_{\mathrm{Z}_{IP}}\right]}$, with $0 \leq \mu^{+q_{SC}}_{\mathrm{Z}_{RP}} + \eta^{+q_{SC}}_{\mathrm{Z}_{RP}} \leq 1$ and $0 \leq \mu^{+q_{SC}}_{\mathrm{Z}_{RP}} + \eta^{+q_{SC}}_{\mathrm{Z}_{RP}} \leq 1$, and $\left(\check{s}_{\check{S}_{LT}}, \alpha_{SC}\right)$ is 2-TLS. Additionally, the mathematical shape of refusal grade is stated: $\pi_{\mathrm{Z}_{CIVQ}} = \begin{bmatrix} \left(1 - \left(\mu^{-q_{SC}}_{\mathrm{Z}_{RP}} + \eta^{-q_{SC}}_{\mathrm{Z}_{RP}}\right)\right)^{\frac{1}{q_{SC}}}, \\ \left(1 - \left(\mu^{+q_{SC}}_{\mathrm{Z}_{RP}} + \eta^{+q_{SC}}_{\mathrm{Z}_{RP}}\right)\right)^{\frac{1}{q_{SC}}} \end{bmatrix} e^{i2\pi\begin{bmatrix} \left(1-\left(\mu^{-q_{SC}}_{\mathrm{Z}_{IP}} + \eta^{-q_{SC}}_{\mathrm{Z}_{IP}}\right)\right)^{\frac{1}{q_{SC}}}, \\ \left(1-\left(\mu^{+q_{SC}}_{\mathrm{Z}_{IP}} + \eta^{+q_{SC}}_{\mathrm{Z}_{IP}}\right)\right)^{\frac{1}{q_{SC}}} \end{bmatrix}}$. The mathematical term of CIVQRO2-TLN is of the shape: $\mathrm{Z}_{CIVT} = \left(\left(\check{s}_{\check{S}_{LT}}, \alpha_{SC}\right), \begin{pmatrix} \left[\mu^{-}_{\mathrm{Z}_{RP}}, \mu^{+}_{\mathrm{Z}_{RP}}\right] e^{i2\pi\left[\mu^{-}_{\mathrm{Z}_{IP}}, \mu^{+}_{\mathrm{Z}_{IP}}\right]}, \\ \left[\eta^{-}_{\mathrm{Z}_{RP}}, \eta^{+}_{\mathrm{Z}_{RP}}\right] e^{i2\pi\left[\eta^{-}_{\mathrm{Z}_{IP}}, \eta^{+}_{\mathrm{Z}_{IP}}\right]} \end{pmatrix}\right)$. As well, assume

CIVQRO2-TLNs $Z_{CIVT} = \left(\left(\check{s}_{\check{S}_{LT}}, \alpha_{SC}\right), \left(\begin{array}{c} \left[\mu^{-}_{Z_{RP}}, \mu^{+}_{Z_{RP}}\right] e^{i2\pi\left[\mu^{-}_{Z_{IP}}, \mu^{+}_{Z_{IP}}\right]}, \\ \left[\eta^{-}_{Z_{RP}}, \eta^{+}_{Z_{RP}}\right] e^{i2\pi\left[\eta^{-}_{Z_{IP}}, \eta^{+}_{Z_{IP}}\right]} \end{array} \right) \right)$ and

$Z_{CIVT-2} = \left(\begin{array}{c} \left(\check{s}_{\check{S}_{LT-2}}, \alpha_{SC-2}\right), \\ \left(\begin{array}{c} \left[\mu^{-}_{Z_{RP-2}}, \mu^{+}_{Z_{RP-2}}\right] e^{i2\pi\left[\mu^{-}_{Z_{IP-2}}, \mu^{+}_{Z_{IP-2}}\right]}, \\ \left[\eta^{-}_{Z_{RP-2}}, \eta^{+}_{Z_{RP-2}}\right] e^{i2\pi\left[\eta^{-}_{Z_{IP-2}}, \eta^{+}_{Z_{IP-2}}\right]} \end{array} \right) \end{array} \right)$, then:

$$Z_{CIVT-1} \oplus_{CIVT} Z_{CIVT-2} = \left(\begin{array}{c} \Delta_{LT}\left(\Delta^{-1}_{LT}\left(\check{s}_{\check{S}_{LT-1}}, \alpha_{SC-1}\right) + \Delta^{-1}_{LT}\left(\check{s}_{\check{S}_{LT-2}}, \alpha_{SC-2}\right) - \frac{\Delta^{-1}_{LT}\left(\check{s}_{\check{S}_{LT-1}}, \alpha_{SC-1}\right)\Delta^{-1}_{LT}\left(\check{s}_{\check{S}_{LT-2}}, \alpha_{SC-2}\right)}{t} \right), \\ \left(\begin{array}{c} \left[\begin{array}{c} \left(\mu^{-q_{SC}}_{Z_{RP-1}} + \mu^{-q_{SC}}_{Z_{RP-2}} - \mu^{-q_{SC}}_{Z_{RP-1}}\mu^{-q_{SC}}_{Z_{RP-2}}\right)^{\frac{1}{q_{SC}}}, \\ \left(\mu^{+q_{SC}}_{Z_{RP-1}} + \mu^{+q_{SC}}_{Z_{RP-2}} - \mu^{+q_{SC}}_{Z_{RP-1}}\mu^{+q_{SC}}_{Z_{RP-2}}\right)^{\frac{1}{q_{SC}}} \end{array} \right] e^{i2\pi \left[\begin{array}{c} \left(\mu^{-q_{SC}}_{Z_{IP-1}} + \mu^{-q_{SC}}_{Z_{IP-2}} - \mu^{-q_{SC}}_{Z_{IP-1}}\mu^{-q_{SC}}_{Z_{IP-2}}\right)^{\frac{1}{q_{SC}}}, \\ \left(\mu^{+q_{SC}}_{Z_{IP-1}} + \mu^{+q_{SC}}_{Z_{IP-2}} - \mu^{+q_{SC}}_{Z_{IP-1}}\mu^{+q_{SC}}_{Z_{IP-2}}\right)^{\frac{1}{q_{SC}}} \end{array} \right]}, \\ \left[\eta^{-}_{Z_{RP-1}}\eta^{-}_{Z_{RP-2}}, \eta^{+}_{Z_{RP-1}}\eta^{+}_{Z_{RP-2}}\right] e^{i2\pi\left[\eta^{-}_{Z_{IP-1}}\eta^{-}_{Z_{IP-2}}, \eta^{+}_{Z_{IP-1}}\eta^{+}_{Z_{IP-2}}\right]} \end{array} \right) \end{array} \right) \tag{10}$$

$$Z_{CIVT-1} \otimes_{CIVT} Z_{CIVT-2} = \left(\begin{array}{c} \Delta_{LT}\left(\frac{\Delta^{-1}_{LT}\left(\check{s}_{\check{S}_{LT-1}}, \alpha_{SC-1}\right)\Delta^{-1}_{LT}\left(\check{s}_{\check{S}_{LT-2}}, \alpha_{SC-2}\right)}{t} \right), \\ \left(\begin{array}{c} \left[\mu^{-}_{Z_{RP-1}}\mu^{-}_{Z_{RP-2}}, \mu^{+}_{Z_{RP-1}}\mu^{+}_{Z_{RP-2}}\right] e^{i2\pi\left[\mu^{-}_{Z_{IP-1}}\mu^{-}_{Z_{IP-2}}, \mu^{+}_{Z_{IP-1}}\mu^{+}_{Z_{IP-2}}\right]}, \\ \left[\begin{array}{c} \left(\eta^{-q_{SC}}_{Z_{RP-1}} + \eta^{-q_{SC}}_{Z_{RP-2}} - \eta^{-q_{SC}}_{Z_{RP-1}}\eta^{-q_{SC}}_{Z_{RP-2}}\right)^{\frac{1}{q_{SC}}}, \\ \left(\eta^{+q_{SC}}_{Z_{RP-1}} + \eta^{+q_{SC}}_{Z_{RP-2}} - \eta^{+q_{SC}}_{Z_{RP-1}}\eta^{+q_{SC}}_{Z_{RP-2}}\right)^{\frac{1}{q_{SC}}} \end{array} \right] e^{i2\pi \left[\begin{array}{c} \left(\eta^{-q_{SC}}_{Z_{IP-1}} + \eta^{-q_{SC}}_{Z_{IP-2}} - \eta^{-q_{SC}}_{Z_{IP-1}}\eta^{-q_{SC}}_{Z_{IP-2}}\right)^{\frac{1}{q_{SC}}}, \\ \left(\eta^{+q_{SC}}_{Z_{IP-1}} + \eta^{+q_{SC}}_{Z_{IP-2}} - \eta^{+q_{SC}}_{Z_{IP-1}}\eta^{+q_{SC}}_{Z_{IP-2}}\right)^{\frac{1}{q_{SC}}} \end{array} \right]} \end{array} \right) \end{array} \right) \tag{11}$$

$$\Upsilon_{SC} Z_{CIVT-1} = \left(\begin{array}{c} \Delta_{LT}\left(t\left(1-\left(1-\frac{\Delta_{LT}^{-1}\left(\check{s}_{\check{S}_{LT-1}}, \alpha_{SC-1}\right)}{t} \right)^{\Upsilon_{SC}} \right) \right), \\ \left(\begin{array}{c} \left[\begin{array}{c} \left(1-\left(1-\mu_{Z_{RP-1}}^{-}{}^{q_{SC}}\right)^{\Upsilon_{SC}}\right)^{\frac{1}{q_{SC}}}, \\ \left(1-\left(1-\mu_{Z_{RP-1}}^{+}{}^{q_{SC}}\right)^{\Upsilon_{SC}}\right)^{\frac{1}{q_{SC}}} \end{array} \right] e^{i2\pi \left[\begin{array}{c} \left(1-\left(1-\mu_{Z_{IP-1}}^{-}{}^{q_{SC}}\right)^{\Upsilon_{SC}}\right)^{\frac{1}{q_{SC}}}, \\ \left(1-\left(1-\mu_{Z_{IP-1}}^{+}{}^{q_{SC}}\right)^{\Upsilon_{SC}}\right)^{\frac{1}{q_{SC}}} \end{array} \right]}, \\ \left[\eta_{Z_{RP-1}}^{-\Upsilon_{SC}}, \eta_{Z_{RP-1}}^{+\Upsilon_{SC}}\right] e^{i2\pi \left[\eta_{Z_{IP-1}}^{-\Upsilon_{SC}}, \eta_{Z_{IP-1}}^{+\Upsilon_{SC}}\right]} \end{array} \right) \end{array} \right) \tag{12}$$

$$Z_{CIVT-1}^{\Upsilon_{SC}} = \left(\begin{array}{c} \Delta_{LT}\left(\left(\Delta_{LT}^{-1}\left(\check{s}_{\check{S}_{LT-1}}, \alpha_{SC-1}\right)\right)^{\Upsilon_{SC}} t^{1-\Upsilon_{SC}} \right), \\ \left(\begin{array}{c} \left[\mu_{Z_{RP-1}}^{-\Upsilon_{SC}}, \mu_{Z_{RP-1}}^{+\Upsilon_{SC}}\right] e^{i2\pi \left[\mu_{Z_{IP-1}}^{-\Upsilon_{SC}}, \mu_{Z_{IP-1}}^{+\Upsilon_{SC}}\right]}, \\ \left[\begin{array}{c} \left(1-\left(1-\eta_{Z_{RP-1}}^{-}{}^{q_{SC}}\right)^{\Upsilon_{SC}}\right)^{\frac{1}{q_{SC}}}, \\ \left(1-\left(1-\eta_{Z_{RP-1}}^{+}{}^{q_{SC}}\right)^{\Upsilon_{SC}}\right)^{\frac{1}{q_{SC}}} \end{array} \right] e^{i2\pi \left[\begin{array}{c} \left(1-\left(1-\eta_{Z_{IP-1}}^{-}{}^{q_{SC}}\right)^{\Upsilon_{SC}}\right)^{\frac{1}{q_{SC}}}, \\ \left(1-\left(1-\eta_{Z_{IP-1}}^{+}{}^{q_{SC}}\right)^{\Upsilon_{SC}}\right)^{\frac{1}{q_{SC}}} \end{array} \right]} \end{array} \right) \end{array} \right) \tag{13}$$

Moreover, the SF and AF for CIVQRO2-TLS are defined as:

$$\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{\frac{1}{4}\left((\check{S}_{LT-1}+\alpha_{SC-1}) \times \left(\begin{array}{c} \left(\mu_{Z_{RP-1}}^{+q_{SC}} + \mu_{Z_{IP-1}}^{+q_{SC}} - \eta_{Z_{RP-1}}^{+q_{SC}} - \eta_{Z_{IP-1}}^{+q_{SC}}\right) - \\ \left(\mu_{Z_{RP-1}}^{-q_{SC}} + \mu_{Z_{IP-1}}^{-q_{SC}} - \eta_{Z_{RP-1}}^{-q_{SC}} - \eta_{Z_{IP-1}}^{-q_{SC}}\right) \end{array} \right) \right)} \tag{14}$$

$$\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{\frac{1}{4}\left((\check{S}_{LT-1}+\alpha_{SC-1}) \times \left(\begin{array}{c} \left(\mu_{Z_{RP-1}}^{+q_{SC}} + \mu_{Z_{IP-1}}^{+q_{SC}} - \eta_{Z_{RP-1}}^{+q_{SC}} - \eta_{Z_{IP-1}}^{+q_{SC}}\right) - \\ \left(\mu_{Z_{RP-1}}^{-q_{SC}} + \mu_{Z_{IP-1}}^{-q_{SC}} - \eta_{Z_{RP-1}}^{-q_{SC}} - \eta_{Z_{IP-1}}^{-q_{SC}}\right) \end{array} \right) \right)} \tag{15}$$

For any two CIVQRO2-TLNs Z_{CIVT-1} and Z_{CIVT-1}:

(1) If $\Gamma_{SF}(Z_{CIVT-1}) > \Gamma_{SF}(Z_{CIVT-2})$, then $\Gamma_{SF}(Z_{CIVT-1}) > \Gamma_{SF}(Z_{CIVT-2})$;
(2) If $Z_{CIVT-1} = Z_{CIVT-2}$, then
  - If $\Psi_{AF}(Z_{CIVT-1}) > \Psi_{AF}(Z_{CIVT-1})$, then $\Psi_{AF}(Z_{CIVT-1}) > \Psi_{AF}(Z_{CIVT-1})$;
  - If $\Psi_{AF}(Z_{CIVT-1}) = \Psi_{AF}(Z_{CIVT-1})$, then $\Psi_{AF}(Z_{CIVT-1}) = \Psi_{AF}(Z_{CIVT-1})$.

Example 1: For any two CIVQRO2-TLNs $Z_{CIVT-1} = \left((\check{s}_3, 0.4), \left([0.4, 0.6]e^{i2\pi[0.4,0.6]}, [0.1, 0.4]e^{i2\pi[0.1,0.4]}\right)\right)$ and $Z_{CIVT-2} = \left((\check{s}_4, 0.2), \left([0.3, 0.5]e^{i2\pi[0.3,0.5]}, [0.2, 0.4]e^{i2\pi[0.2,0.4]}\right)\right)$ with $Z_{CIVT-2} = \left((\check{s}_4, 0.2), \left([0.3, 0.5]e^{i2\pi[0.3,0.5]}, [0.2, 0.4]e^{i2\pi[0.2,0.4]}\right)\right)$ by using the values of scalers $q_{SC} = 3$ and $q_{SC} = 3$, then by using Eq. (10) to Eq. (13), we have

$$Z_{CIVT-1} \oplus_{CIVT} Z_{CIVT-2} = \left(\begin{array}{c} \Delta_{LT}\left(\Delta_{LT}^{-1}(\check{s}_3, 0.4) + \Delta_{LT}^{-1}(\check{s}_4, 0.2) - \frac{\Delta_{LT}^{-1}(\check{s}_3,0.4)\Delta_{LT}^{-1}(\check{s}_4,0.2)}{6}\right), \\ \left(\begin{array}{c} \left[\begin{array}{c} (0.4^3 + 0.3^3 - 0.4^3 \times 0.3^3)^{\frac{1}{3}}, \\ (0.6^3 + 0.5^3 - 0.6^3 \times 0.5^3)^{\frac{1}{3}} \end{array}\right] e^{i2\pi\left[\begin{array}{c} (0.4^3 + 0.3^3 - 0.4^3 \times 0.3^3)^{\frac{1}{3}}, \\ (0.6^3 + 0.5^3 - 0.6^3 \times 0.5^3)^{\frac{1}{3}} \end{array}\right]}, \\ [0.1 \times 0.2, 0.4 \times 0.4]e^{i2\pi[0.1\times0.2,0.4\times0.4]} \end{array}\right) \end{array}\right)$$

$$= \left(\begin{array}{c} (\check{s}_5, 0.22), \\ \left(\begin{array}{c} [0.4469, 0.6797]e^{i2\pi[0.4469,0.6797]}, \\ [0.02, 0.16]e^{i2\pi[0.02,0.16]} \end{array}\right) \end{array}\right).$$

$$Z_{CIVT-1} \otimes_{CIVT} Z_{CIVT-2} = \left(\begin{array}{c} \Delta_{LT}\left(\frac{\Delta_{LT}^{-1}(\check{s}_3,0.4)\Delta_{LT}^{-1}(\check{s}_4,0.2)}{6}\right), \\ \left(\begin{array}{c} [0.4 \times 0.3, 0.6 \times 0.5]e^{i2\pi[0.4\times0.3,0.6\times0.5]}, \\ \left[\begin{array}{c} (0.1^3 + 0.2^3 - 0.1^3 \times 0.2^3)^{\frac{1}{3}}, \\ (0.4^3 + 4 - 0.4^3 \times 0.4^3)^{\frac{1}{3}} \end{array}\right] e^{i2\pi\left[\begin{array}{c} (0.1^3 + 0.2^3 - 0.1^3 \times 0.2^3)^{\frac{1}{3}}, \\ (0.4^3 + 4 - 0.4^3 \times 0.4^3)^{\frac{1}{3}} \end{array}\right]} \end{array}\right) \end{array}\right)$$

$$= \left(\begin{array}{c} (\check{s}_2, 0.38), \\ \left(\begin{array}{c} [0.12, 0.3]e^{i2\pi[0.12,0.3]}, \\ [0.2079, 0.4985]e^{i2\pi[0.2079,0.4985]} \end{array}\right) \end{array}\right).$$

$$2Z_{CIVT-1} = \left(\begin{array}{c} \Delta_{LT}\left(6\left(1 - \left(1 - \frac{3.4}{6}\right)^2\right)\right), \\ \left(\left[\begin{array}{c} \left(1 - (1 - 0.4^3)^2\right)^{\frac{1}{3}}, \\ \left(1 - (1 - 0.6^3)^2\right)^{\frac{1}{3}} \end{array}\right] e^{i2\pi\left[\begin{array}{c} \left(1 - (1 - 0.4^3)^2\right)^{\frac{1}{3}}, \\ \left(1 - (1 - 0.6^3)^2\right)^{\frac{1}{3}} \end{array}\right]}, [0.1^2, 0.4^2]e^{i2\pi[0.1^2,0.4^2]}\right) \end{array}\right)$$

$$= \left(\begin{array}{c} (\check{s}_5, -0.1267), \\ \left(\begin{array}{c} [0.5, 0.73]e^{i2\pi[0.5,0.73]}, \\ [0.01, 0.16]e^{i2\pi[0.01,0.16]} \end{array}\right) \end{array}\right).$$

$$Z^2_{CIVT-1} = \left(\begin{array}{c} \Delta_{LT}\left((3.4)^2 6^{1-2}\right), \\ \left([0.4^2, 0.6^2]e^{i2\pi[0.4^2,0.6^2]}, \begin{bmatrix} \left(1-(1-0.1^3)^2\right)^{\frac{1}{3}}, \\ \left(1-(1-0.4^3)^2\right)^{\frac{1}{3}} \end{bmatrix} e^{i2\pi \begin{bmatrix} \left(1-(1-0.1^3)^2\right)^{\frac{1}{3}}, \\ \left(1-(1-0.4^3)^2\right)^{\frac{1}{3}} \end{bmatrix}} \right) \end{array}\right)$$

$$= \left(\begin{array}{c} (\check{s}_2, -0.0733), \\ \left(\begin{array}{c} [0.16, 0.36]e^{i2\pi[0.16,0.36]}, \\ [0.13, 0.50]e^{i2\pi[0.13,0.50]} \end{array}\right) \end{array}\right)$$

As a new mixture of the CIVQROFS and 2-TLS that can easily handle awkward and complicated sort of data occurring in genuine life, the beneficial aspects of the CIVQRO2-TLS compared to with the existing fuzzy tools are discussed in the presence of Table 2 .

Moreover, some special cases of the CIVQRO2-TLS are deliberated as follows:

(1) Using $q_{sc} = 1$ in CIVQRO2-TLS, then we get complex interval-valued intuitionistic fuzzy 2-tuple linguistic set;
(2) Using $q_{SC} = 2$ in CIVQRO2-TLS, the complex interval-valued Pythagorean fuzzy 2-tuple linguistic set is obtained.
(3) Using $\alpha_{SC} = 0$ with $\alpha_{SC} = 0$ in CIVQRO2-TLS, the complex interval-valued q-rung orthopair fuzzy linguistic set is constructed.
(4) Using $\alpha_{SC} = 0$ with $\alpha_{SC} = 0$ in CIVQRO2-TLS, then we get complex interval-valued intuitionistic fuzzy linguistic set.
(5) Using $\alpha_{SC} = 0$ with $\alpha_{SC} = 0$ in CIVQRO2-TLS, then we get complex interval-valued Pythagorean fuzzy linguistic set.
(6) Using $\mu^-_{Z_{RP}} = \mu^+_{Z_{RP}}, \eta^-_{Z_{RP}} = \eta^+_{Z_{RP}}, \mu^-_{Z_{IP}} = \mu^+_{Z_{IP}}, \eta^-_{Z_{IP}} = \eta^+_{Z_{IP}}$ with $q_{SC} > 0$ in CIVQRO2-TLS, then we get CQROFS.

Table 2. Summary of the literary works.

Methods	Truth grade	Falsity grade	Interval-valued	2-tuple linguistic	Two-dimension
IFS	√	√	×	×	×
PFS	√	√	×	×	×
QROFS	√	√	×	×	×
IVIFS	√	√	√	×	×
IVPFS	√	√	√	×	×
IVQROFS	√	√	√	×	×
CIFS	√	√	×	×	√
CPFS	√	√	×	×	√
CQROFS	√	√	×	×	√
CIVIFS	√	√	√	×	√
CIVPFS	√	√	√	×	√
CIVQROFS	√	√	√	×	√
Proposed work	√	√	√	√	√

(7) Using $q_{SC} > 0$ with $q_{SC} = 1$ in CIVQRO2-TLS, then we get CIFS.
(8) Using $q_{SC} = 1$ with $q_{SC} > 0$ in CIVQRO2-TLS, then we get QROFS.
(9) Using $q_{SC} > 0$ with $q_{SC} = 1$ in CIVQRO2-TLS, then we get IFS.
(10) Ect ...

From the presence of the above analysis, we can see that most of existing fuzzy tools are the specific cases of the CIVQRO2-TLS, therefore the CIVQRO2-TLS is massive generalized and more dominant as compared to prevailing works.

Aggregation operators for CIVQRO2-TL information

In the occurrence of the above theory, in this section we discover the thought of CIVQRO2-TLWA, CIVQRO2-TLOWA, CIVQRO2-TLHA, CIVQRO2-TLWG, CIVQRO2-TLOWG and CIVQRO2-TLHG operators. Throughout this scenario, let $q_{SC} = 1$ be the weight vector with $\sum_{j=1}^{n} \varpi_{W-j} = 1,\ \varpi_{W-j} \in [0, 1]$ for the family of CIVQRO2-TLNs

$$\mathrm{Z}_{CIVT-j} = \left(\begin{array}{c} \left(\check{s}_{\check{S}_{LT-j}}, \alpha_{SC-j}\right), \\ \left(\begin{array}{c} \left[\mu^{-}_{\mathrm{Z}_{RP-j}}, \mu^{+}_{\mathrm{Z}_{RP-j}}\right] e^{i2\pi\left[\mu^{-}_{\mathrm{Z}_{IP-j}}, \mu^{+}_{\mathrm{Z}_{IP-j}}\right]}, \\ \left[\eta^{-}_{\mathrm{Z}_{RP-j}}, \eta^{+}_{\mathrm{Z}_{RP-j}}\right] e^{i2\pi\left[\eta^{-}_{\mathrm{Z}_{IP-j}}, \eta^{+}_{\mathrm{Z}_{IP-j}}\right]} \end{array} \right) \end{array} \right), j = 1, 2, 3, \ldots n.$$

Definition 4: The CIVQRO2-TLWA operator is originated by;

$$CIVQRO2 - TLWA(\mathrm{Z}_{CIVT-1}, \mathrm{Z}_{CIVT-2}, \ldots, \mathrm{Z}_{CIVT-n}) = \oplus_{j=1}^{n}\left(\varpi_{W-j}\mathrm{Z}_{CIVT-j}\right) \tag{16}$$

In the consideration of Eq. (16), we exposed several results.

Theorem 1: Constructed on Eq. (16), we get;

$$CIVQRO2-TLWA(Z_{CIVT-1},Z_{CIVT-2},\ldots,Z_{CIVT-n})=\left(\begin{array}{c}\Delta_{LT}\left(t\left(1-\prod_{j=1}^{n}\left(1-\frac{\Delta_{LT}^{-1}\left(\check{s}_{\check{S}_{LT-j}},\alpha_{SC-j}\right)}{t}\right)^{\varpi_{W-j}}\right)\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\left(1-\prod_{j=1}^{n}\left(1-\mu_{Z_{RP-j}}^{-}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}},\\ \left(1-\prod_{j=1}^{n}\left(1-\mu_{Z_{RP-j}}^{+}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}}\end{array}\right]e^{i2\pi\left[\begin{array}{c}\left(1-\prod_{j=1}^{n}\left(1-\mu_{Z_{IP-j}}^{-}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}},\\ \left(1-\prod_{j=1}^{n}\left(1-\mu_{Z_{IP-j}}^{+}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}}\end{array}\right]},\\ \left[\prod_{j=1}^{n}\eta_{Z_{RP-j}}^{-\varpi_{W-j}},\prod_{j=1}^{n}\eta_{Z_{RP-j}}^{+\varpi_{W-j}}\right]e^{i2\pi\left[\prod_{j=1}^{n}\eta_{Z_{IP-j}}^{-\varpi_{W-j}},\prod_{j=1}^{n}\eta_{Z_{IP-j}}^{+\varpi_{W-j}}\right]}\end{array}\right)\end{array}\right) \tag{17}$$

Using $q_{SC}=1$ and $q_{SC}=1$ in Eq. (17), we get the CIVI2-TLS and CIVP2-TLS, respectively. Furthermore, the idempotency, boundedness, and monotonicity in the presence of the CIVQRO2-TLWA operator are described as follows.

Proposition 1: If $Z_{CIVT-j}=Z_{CIVT}$, then

$$CIVQRO2-TLWA(Z_{CIVT-1},Z_{CIVT-2},\ldots,Z_{CIVT-n})=Z_{CIVT} \tag{18}$$

Proposition 2: If $min\ Z_{CIVT-j}=Z_{CIVT}^{--}$ and $min\ Z_{CIVT-j}=Z_{CIVT}^{--}$, then

$$Z_{CIVT}^{--}\leq CIVQRO2-TLWA(Z_{CIVT-1},Z_{CIVT-2},\ldots,Z_{CIVT-n})\leq Z_{CIVT}^{++} \tag{19}$$

Proposition 3: If $Z_{CIVT-j}\geq Z'_{CIVT-j}$, then

$$\begin{aligned}&CIVQRO2-TLWA(Z_{CIVT-1},Z_{CIVT-2},\ldots,Z_{CIVT-n})\\&\quad\geq CIVQRO2-TLWA\left(Z'_{CIVT-1},Z'_{CIVT-2},\ldots,Z'_{CIVT-n}\right)\end{aligned} \tag{20}$$

Definition 6: The CIVQRO2-TLOWA operator is originated by:

$$\begin{aligned}&CIVQRO2-TLOWA(Z_{CIVT-1},Z_{CIVT-2},\ldots,Z_{CIVT-n})\\&\quad=\oplus_{j=1}^{n}\left(\varpi_{W-j}Z_{CIVT-o(j)}\right)\end{aligned} \tag{21}$$

where $(o(1),o(2),..,o(n))$ is a permutation with a condition that is $Z_{CIVT-o(j-1)}\geq Z_{CIVT-o(j)}$. In the consideration of Eq. (21), we exposed following several results.

Theorem 2: Constructed on Eq. (21), we get:

$$CIVQRO2-TLOWA(Z_{CIVT-1},Z_{CIVT-2},\ldots,Z_{CIVT-n})=\left(\begin{array}{c}\Delta_{LT}\left(t\left(1-\prod_{j=1}^{n}\left(1-\frac{\Delta_{LT}^{-1}\left(\check{s}_{\check{S}_{LT-o(j)}},\alpha_{SC-o(j)}\right)}{t}\right)^{\varpi_{W-j}}\right)\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\left(1-\prod_{j=1}^{n}\left(1-\mu_{Z_{RP-o(j)}}^{-}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}},\\ \left(1-\prod_{j=1}^{n}\left(1-\mu_{Z_{RP-o(j)}}^{+}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}}\end{array}\right]e^{i2\pi\left[\begin{array}{c}\left(1-\prod_{j=1}^{n}\left(1-\mu_{Z_{IP-o(j)}}^{-}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}},\\ \left(1-\prod_{j=1}^{n}\left(1-\mu_{Z_{IP-o(j)}}^{+}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}}\end{array}\right]},\\ \left[\prod_{j=1}^{n}\eta_{Z_{RP-o(j)}}^{-\varpi_{W-j}},\prod_{j=1}^{n}\eta_{Z_{RP-o(j)}}^{+\varpi_{W-j}}\right]e^{i2\pi\left[\prod_{j=1}^{n}\eta_{Z_{IP-o(j)}}^{-\varpi_{W-j}},\prod_{j=1}^{n}\eta_{Z_{IP-o(j)}}^{+\varpi_{W-j}}\right]}\end{array}\right)\end{array}\right) \tag{22}$$

For $q_{SC}=1$ and $q_{SC}=1$ in Eq. (22), we get CIVI2-TLS and CIVP2-TLS. Furthermore, the idempotency, boundedness, and monotonicity of the CIVQRO2-TLOWA operator are analyzed below.

Proposition 4: If $Z_{CIVT-j}=Z_{CIVT}$, then

$$CIVQRO2-TLOWA(Z_{CIVT-1},Z_{CIVT-2},\ldots,Z_{CIVT-n})=Z_{CIVT} \tag{23}$$

Proposition 5: If $CIVQRO2-TLOWA(Z_{CIVT-1},Z_{CIVT-2},\ldots,Z_{CIVT-n})=Z_{CIVT}$ and $max\ Z_{CIVT-j}=Z_{CIVT}^{++}$, then

$$Z_{CIVT}^{--}\le CIVQRO2-TLOWA(Z_{CIVT-1},Z_{CIVT-2},\ldots,Z_{CIVT-n})\le Z_{CIVT}^{++} \tag{24}$$

Proposition 6: If $Z_{CIVT-j}\ge Z'_{CIVT-j}$, then

$$\begin{aligned}&CIVQRO2-TLWA(Z_{CIVT-1},Z_{CIVT-2},\ldots,Z_{CIVT-n})\\&\quad\ge CIVQRO2-TLWA\left(Z'_{CIVT-1},Z'_{CIVT-2},\ldots,Z'_{CIVT-n}\right)\end{aligned} \tag{25}$$

Definition 7: The CIVQRO2-TLHA operator is originated by:

$$CIVQRO2-TLHA(Z_{CIVT-1},Z_{CIVT-2},\ldots,Z_{CIVT-n})=\oplus_{j=1}^{n}\left(\varpi_{W-j}\hat{Z}_{CIVT-o(j)}\right) \tag{26}$$

Where $(o(1),o(2),..,o(n))$ is a permutation with a condition that is $(o(1),o(2),..,o(n))$. In the consideration of Eq. (26), we exposed several results.

Theorem 3: Constructed on Eq. (26), we get

$$CIVQRO2-TLHA(Z_{CIVT-1},Z_{CIVT-2},\ldots,Z_{CIVT-n})=\left(\begin{array}{c}\Delta_{LT}\left(t\left(1-\prod_{j=1}^{n}\left(1-\frac{\Delta_{LT}^{-1}\left(\check{s}_{\check{S}_{LT-o(j)}},\hat{\alpha}_{SC-o(j)}\right)}{t}\right)^{\varpi_{W-j}}\right)\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\left(1-\prod_{j=1}^{n}\left(1-\hat{\mu}^{-}_{Z_{RP-o(j)}}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}},\\ \left(1-\prod_{j=1}^{n}\left(1-\hat{\mu}^{+}_{Z_{RP-o(j)}}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}}\end{array}\right]e^{i2\pi\left[\begin{array}{c}\left(1-\prod_{j=1}^{n}\left(1-\hat{\mu}^{-}_{Z_{IP-o(j)}}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}},\\ \left(1-\prod_{j=1}^{n}\left(1-\hat{\mu}^{+}_{Z_{IP-o(j)}}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}}\end{array}\right]},\\ \left[\prod_{j=1}^{n}\hat{\eta}^{-\varpi_{W-j}}_{Z_{RP-o(j)}},\prod_{j=1}^{n}\hat{\eta}^{+\varpi_{W-j}}_{Z_{RP-o(j)}}\right]e^{i2\pi\left[\prod_{j=1}^{n}\hat{\eta}^{-\varpi_{W-j}}_{Z_{IP-o(j)}},\prod_{j=1}^{n}\hat{\eta}^{+\varpi_{W-j}}_{Z_{IP-o(j)}}\right]}\end{array}\right)\end{array}\right) \tag{27}$$

For $q_{SC}=1$ and $q_{SC}=2$ in Eq. (27), we get CIVI2-TLS and CIVP2-TLS, which shows the specific cases of the explored ideas.

Definition 7: The CIVQRO2-TLWG operator is originated by:

$$CIVQRO2-TLWG(Z_{CIVT-1},Z_{CIVT-2},\ldots,Z_{CIVT-n})=\otimes_{j=1}^{n}\left(Z_{CIVT-j}\right)^{\varpi_{W-j}} \tag{28}$$

Theorem 4: Constructed on Eq. (28), we get:

$$CIVQRO2-TLWG(Z_{CIVT-1},Z_{CIVT-2},\ldots,Z_{CIVT-n})=\left(\begin{array}{c}\Delta_{LT}\left(t\left(\prod_{j=1}^{n}\left(\frac{\Delta_{LT}^{-1}\left(\check{s}_{\check{S}_{LT-j}},\alpha_{SC-j}\right)}{t}\right)^{\varpi_{W-j}}\right)\right),\\ \left(\begin{array}{c}\left[\prod_{j=1}^{n}\mu^{-\varpi_{W-j}}_{Z_{RP-j}},\prod_{j=1}^{n}\mu^{+\varpi_{W-j}}_{Z_{RP-j}}\right]e^{i2\pi\left[\prod_{j=1}^{n}\mu^{-\varpi_{W-j}}_{Z_{IP-j}},\prod_{j=1}^{n}\mu^{+\varpi_{W-j}}_{Z_{IP-j}}\right]},\\ \left[\begin{array}{c}\left(1-\prod_{j=1}^{n}\left(1-\eta^{-}_{Z_{RP-j}}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}},\\ \left(1-\prod_{j=1}^{n}\left(1-\eta^{+}_{Z_{RP-j}}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}}\end{array}\right]e^{i2\pi\left[\begin{array}{c}\left(1-\prod_{j=1}^{n}\left(1-\eta^{-}_{Z_{IP-j}}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}},\\ \left(1-\prod_{j=1}^{n}\left(1-\eta^{+}_{Z_{IP-j}}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}}\end{array}\right]}\end{array}\right)\end{array}\right) \tag{29}$$

If $q_{SC}=1$ and $q_{SC}=1$ in Eq. (29), then we have the CIVI2-TLS and CIVP2-TLS. Furthermore, the idempotency, boundedness, and monotonicity in the presence of the CIVQRO2-TLWG operator are described below.

Definition 8: The CIVQRO2-TLOWG operator is originated by:

$$CIVQRO2 - TLOWG(Z_{CIVT-1}, Z_{CIVT-2}, \ldots, Z_{CIVT-n}) = \otimes_{j=1}^{n} \left(Z_{CIVT-o(j)}\right)^{\varpi_{W-j}} \tag{30}$$

where $(o(1), o(2), .., o(n))$ is a permutation with a condition that is $(o(1), o(2), .., o(n))$.

Theorem 5: Constructed on Eq. (30), we get

$$CIVQRO2 - TLOWG(Z_{CIVT-1}, Z_{CIVT-2}, \ldots, Z_{CIVT-n}) = \left(\begin{array}{c} \Delta_{LT}\left(t\left(\prod_{j=1}^{n}\left(\frac{\Delta_{LT}^{-1}\left(\check{s}_{\check{S}_{LT-o(j)}}, \alpha_{SC-o(j)}\right)}{t}\right)^{\varpi_{W-j}}\right)\right), \\ \left(\begin{array}{c} \left[\prod_{j=1}^{n}\mu_{Z_{RP-o(j)}}^{-\varpi_{W-j}}, \prod_{j=1}^{n}\mu_{Z_{RP-o(j)}}^{+\varpi_{W-j}}\right] e^{i2\pi\left[\prod_{j=1}^{n}\mu_{Z_{IP-o(j)}}^{-\varpi_{W-j}}, \prod_{j=1}^{n}\mu_{Z_{IP-o(j)}}^{+\varpi_{W-j}}\right]}, \\ \left[\begin{array}{c} \left(1-\prod_{j=1}^{n}\left(1-\eta_{Z_{RP-o(j)}}^{-\ q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}}, \\ \left(1-\prod_{j=1}^{n}\left(1-\eta_{Z_{RP-o(j)}}^{+\ q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}} \end{array} \right] e^{i2\pi\left[\begin{array}{c} \left(1-\prod_{j=1}^{n}\left(1-\eta_{Z_{IP-o(j)}}^{-\ q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}}, \\ \left(1-\prod_{j=1}^{n}\left(1-\eta_{Z_{IP-o(j)}}^{+\ q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}} \end{array} \right]} \end{array} \right) \end{array} \right) \tag{31}$$

For $q_{SC} = 1$ and $q_{SC} = 2$, in Eq. (31), we get CIVI2-TLS and CIVP2-TLS.

Definition 10: The CIVQRO2-TLHG operator is originated by:

$$CIVQRO2 - TLHG(Z_{CIVT-1}, Z_{CIVT-2}, \ldots, Z_{CIVT-n}) = \otimes_{j=1}^{n} \left(\hat{Z}_{CIVT-o(j)}\right)^{\varpi_{W-j}} \tag{32}$$

where $(o(1), o(2), .., o(n))$ is a permutation with a condition that is $(o(1), o(2), .., o(n))$. In the consideration of Eq. (32), we exposed several results.

Theorem 6: Constructed on Eq. (32), we get

$$CIVQRO2-TLHG(Z_{CIVT-1}, Z_{CIVT-2}, \ldots, Z_{CIVT-n}) = \left(\begin{array}{c} \Delta_{LT}\left(t\left(\prod_{j=1}^{n}\left(\frac{\Delta_{LT}^{-1}\left(\check{s}_{\hat{\check{S}}_{LT-o(j)}}, \hat{\alpha}_{SC-o(j)}\right)}{t}\right)^{\varpi_{W-j}}\right)\right), \\ \left(\begin{array}{c} \left[\prod_{j=1}^{n}\hat{\mu}_{Z_{RP-o(j)}}^{-\varpi_{W-j}}, \prod_{j=1}^{n}\hat{\mu}_{Z_{RP-o(j)}}^{+\varpi_{W-j}}\right] e^{i2\pi\left[\prod_{j=1}^{n}\hat{\mu}_{Z_{IP-o(j)}}^{-\varpi_{W-j}}, \prod_{j=1}^{n}\hat{\mu}_{Z_{IP-o(j)}}^{+\varpi_{W-j}}\right]}, \\ \left[\begin{array}{c} \left(1-\prod_{j=1}^{n}\left(1-\hat{\eta}_{Z_{RP-o(j)}}^{-}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}}, \\ \left(1-\prod_{j=1}^{n}\left(1-\hat{\eta}_{Z_{RP-o(j)}}^{+}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}} \end{array} \right] e^{i2\pi\left[\begin{array}{c} \left(1-\prod_{j=1}^{n}\left(1-\hat{\eta}_{Z_{IP-o(j)}}^{-}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}}, \\ \left(1-\prod_{j=1}^{n}\left(1-\hat{\eta}_{Z_{IP-o(j)}}^{+}{}^{q_{SC}}\right)^{\varpi_{W-j}}\right)^{\frac{1}{q_{SC}}} \end{array} \right]} \end{array} \right) \end{array} \right) \quad (33)$$

Note that, the merits of idempotency, boundedness, and monotonicity of the proposed CIVQRO2-TLOWG and CIVQRO2-TLHG operators can be described as the previous analysis.

Application in MADM problems

In this study, we present a MADM method based on the proposed operators to solve complicated decision-making problem with CIVQRO2-TL information. To settle the above issues, we choose the family of alternatives and the family of attributes with respect to weight vectors to examine the reliability and proficiency of the explored approaches, whose expressions are follow as: $Z_{CIVT} = \{Z_{CIVT-1}, Z_{CIVT-2}, Z_{CIVT-3}, \ldots, Z_{CIVT-n}\}$ and $\mathcal{L}_{AT} = \{\mathcal{L}_{AT-1}, \mathcal{L}_{AT-2}, \ldots, \mathcal{L}_{AT-m}\}$ with $\varpi_W = \{\varpi_{W-1}, \varpi_{W-2}, \ldots, \varpi_{W-n}\}$ by using the CIVQRO2-TL information $Z_{CIVT-jk} = \left(\begin{array}{c} \left(\check{s}_{\check{S}_{LT-jk}}, \alpha_{SC-jk}\right), \\ \left(\begin{array}{c} \left[\mu_{Z_{RP-jk}}^{-}, \mu_{Z_{RP-jk}}^{+}\right] e^{i2\pi\left[\mu_{Z_{IP-jk}}^{-}, \mu_{Z_{IP-jk}}^{+}\right]}, \\ \left[\eta_{Z_{RP-jk}}^{-}, \eta_{Z_{RP-jk}}^{+}\right] e^{i2\pi\left[\eta_{Z_{IP-jk}}^{-}, \eta_{Z_{IP-jk}}^{+}\right]} \end{array} \right) \end{array} \right)$. Then, the steps of the explored procedure are summarized are follow as:

Stage 1: Stabilize the data given in the form of decision framework, in the presence of the idea, if necessary, then

$$r_{ij} = \begin{pmatrix} \left(\check{s}_{\check{S}_{LT-jk}}, \alpha_{SC-jk}\right), \\ \begin{pmatrix} \left[\mu^-_{Z_{RP-jk}}, \mu^+_{Z_{RP-jk}}\right] e^{i2\pi\left[\mu^-_{Z_{IP-jk}}, \mu^+_{Z_{IP-jk}}\right]}, \\ \left[\eta^-_{Z_{RP-jk}}, \eta^+_{Z_{RP-jk}}\right] e^{i2\pi\left[\eta^-_{Z_{IP-jk}}, \eta^+_{Z_{IP-jk}}\right]} \end{pmatrix} \end{pmatrix} = \begin{cases} \begin{pmatrix} \left(\check{s}_{\check{S}_{LT-jk}}, \alpha_{SC-jk}\right), \\ \begin{pmatrix} \left[\mu^-_{Z_{RP-jk}}, \mu^+_{Z_{RP-jk}}\right] e^{i2\pi\left[\mu^-_{Z_{IP-jk}}, \mu^+_{Z_{IP-jk}}\right]}, \\ \left[\eta^-_{Z_{RP-jk}}, \eta^+_{Z_{RP-jk}}\right] e^{i2\pi\left[\eta^-_{Z_{IP-jk}}, \eta^+_{Z_{IP-jk}}\right]} \end{pmatrix} \end{pmatrix} & \text{for benefit types of attributes} \\ \begin{pmatrix} \left(\check{s}_{\check{S}_{LT-jk}}, \alpha_{SC-jk}\right), \\ \begin{pmatrix} \left[\eta^-_{Z_{RP-jk}}, \eta^+_{Z_{RP-jk}}\right] e^{i2\pi\left[\eta^-_{Z_{IP-jk}}, \eta^+_{Z_{IP-jk}}\right]}, \\ \left[\mu^-_{Z_{RP-jk}}, \mu^+_{Z_{RP-jk}}\right] e^{i2\pi\left[\mu^-_{Z_{IP-jk}}, \mu^+_{Z_{IP-jk}}\right]} \end{pmatrix} \end{pmatrix} & \text{for cost types of attributes} \end{cases} \quad (34)$$

Otherwise, go to stage 2.

Stage 2: In the presence of the qualitative idea of CIVQRO2-TLWA and CIVQRO2-TLWG operators, we demonstrate the single value with the help of accumulated values of the given data.

Stage 3: The score value is diagnosed with the help of SF.

Stage 4: With the help of SF, we find the ranking results to choose the beneficial one.

Numerical example

Example 2: The owners of the enterprise want to buildup some new branch of the enterprise. Therefore, a group of experts selected to evaluate four possible organizations in the shape of attributes: $\mathcal{L}_{AT-1}$: Development situation; $\mathcal{L}_{AT-1}$: Public influence; $\mathcal{L}_{AT-3}$: Eco-friendly influence; $\mathcal{L}_{AT-3}$: Advancement of civilization. Moreover, some alternatives in the shape of five different aspects/ opinions: Z_{CSF-1}: Cost inspection; Z_{CSF-1}: Enhancement situation; Z_{CSF-3}: Political influence; Z_{CSF-3}: Ecological power; Z_{CSF-5}: General community. Let $\varpi_W = (0.21, 0.09, 0.31, 0.39)^T$ be the weight vectors of attributes. This analysis includes some stages, which help investigate the beneficial optimal.

Stage 1: Construct the decision information in the form of CIVQRO2-TLNs, stated in Table 3.

Table 3 includes all the beneficial data, thus there is no need for standardization.

Table 3. CIVQRO2-TL decision matrix.

	$\mathcal{L}_{AT-1}$	$\mathcal{L}_{AT-2}$
Z_{CSF-1}	$\left((\check{s}_2,0),\left(\begin{matrix}[0.1,0.2]e^{i2\pi[0.11,0.21]},\\ [0.4,0.5]e^{i2\pi[0.41,0.51]}\end{matrix}\right)\right)$	$\left((\check{s}_5,0),\left(\begin{matrix}[0.2,0.4]e^{i2\pi[0.21,0.41]},\\ [0.5,0.6]e^{i2\pi[0.51,0.47]}\end{matrix}\right)\right)$
Z_{CSF-2}	$\left((\check{s}_3,0),\left(\begin{matrix}[0.4,0.5]e^{i2\pi[0.41,0.51]},\\ [0.2,0.3]e^{i2\pi[0.21,0.31]}\end{matrix}\right)\right)$	$\left((\check{s}_2,0),\left(\begin{matrix}[0.3,0.4]e^{i2\pi[0.31,0.41]},\\ [0.2,0.3]e^{i2\pi[0.21,0.31]}\end{matrix}\right)\right)$
Z_{CSF-3}	$\left((\check{s}_4,0),\left(\begin{matrix}[0.4,0.5]e^{i2\pi[0.41,0.51]},\\ [0.3,0.5]e^{i2\pi[0.31,0.48]}\end{matrix}\right)\right)$	$\left((\check{s}_3,0),\left(\begin{matrix}[0.2,0.5]e^{i2\pi[0.21,0.51]},\\ [0.1,0.3]e^{i2\pi[0.11,0.31]}\end{matrix}\right)\right)$
Z_{CSF-4}	$\left((\check{s}_2,0),\left(\begin{matrix}[0.4,0.5]e^{i2\pi[0.41,0.51]},\\ [0.1,0.2]e^{i2\pi[0.11,0.21]}\end{matrix}\right)\right)$	$\left((\check{s}_3,0),\left(\begin{matrix}[0.1,0.3]e^{i2\pi[0.11,0.31]},\\ [0.3,0.5]e^{i2\pi[0.31,0.51]}\end{matrix}\right)\right)$
Z_{CSF-5}	$\left((\check{s}_4,0),\left(\begin{matrix}[0.2,0.4]e^{i2\pi[0.21,0.41]},\\ [0.3,0.6]e^{i2\pi[0.31,0.57]}\end{matrix}\right)\right)$	$\left((\check{s}_1,0),\left(\begin{matrix}[0.2,0.4]e^{i2\pi[0.21,0.41]},\\ [0.3,0.4]e^{i2\pi[0.31,0.41]}\end{matrix}\right)\right)$
	$\mathcal{L}_{AT-3}$	$\mathcal{L}_{AT-4}$
Z_{CSF-1}	$\left((\check{s}_1,0),\left(\begin{matrix}[0.2,0.3]e^{i2\pi[0.21,0.31]},\\ [0.4,0.5]e^{i2\pi[0.41,0.51]}\end{matrix}\right)\right)$	$\left((\check{s}_3,0),\left(\begin{matrix}[0.1,0.3]e^{i2\pi[0.11,0.31]},\\ [0.5,0.6]e^{i2\pi[0.51,0.61]}\end{matrix}\right)\right)$
Z_{CSF-2}	$\left((\check{s}_3,0),\left(\begin{matrix}[0.4,0.6]e^{i2\pi[0.41,0.61]},\\ [0.3,0.4]e^{i2\pi[0.31,0.37]}\end{matrix}\right)\right)$	$\left((\check{s}_5,0),\left(\begin{matrix}[0.4,0.5]e^{i2\pi[0.41,0.51]},\\ [0.1,0.2]e^{i2\pi[0.11,0.21]}\end{matrix}\right)\right)$
Z_{CSF-3}	$\left((\check{s}_2,0),\left(\begin{matrix}[0.2,0.3]e^{i2\pi[0.21,0.31]},\\ [0.5,0.6]e^{i2\pi[0.51,0.61]}\end{matrix}\right)\right)$	$\left((\check{s}_3,0),\left(\begin{matrix}[0.1,0.2]e^{i2\pi[0.11,0.21]},\\ [0.6,0.7]e^{i2\pi[0.61,0.71]}\end{matrix}\right)\right)$
Z_{CSF-4}	$\left((\check{s}_4,0),\left(\begin{matrix}[0.1,0.4]e^{i2\pi[0.11,0.41]},\\ [0.3,0.5]e^{i2\pi[0.31,0.51]}\end{matrix}\right)\right)$	$\left((\check{s}_5,0),\left(\begin{matrix}[0.4,0.5]e^{i2\pi[0.41,0.51]},\\ [0.1,0.3]e^{i2\pi[0.11,0.31]}\end{matrix}\right)\right)$
Z_{CSF-5}	$\left((\check{s}_5,0),\left(\begin{matrix}[0.1,0.4]e^{i2\pi[0.11,0.41]},\\ [0.4,0.5]e^{i2\pi[0.41,0.51]}\end{matrix}\right)\right)$	$\left((\check{s}_2,0),\left(\begin{matrix}[0.4,0.6]e^{i2\pi[0.41,0.61]},\\ [0.1,0.3]e^{i2\pi[0.11,0.31]}\end{matrix}\right)\right)$

Stage 2: In the presence of the qualitative idea of CIVQRO2-TLWA and CIVQRO2-TLWG operators, we demonstrate the single value with the help of accumulated values of the given data for $\varpi_W = (0.21, 0.09, 0.31, 0.39)^T$, respectively:

$$Z_{CIVT-1} = ((\hat{s}_3, -0.32), ([0.17, 0.31]e^{i2\pi[0.18,0.32]}, [0.45, 0.55]e^{i2\pi[0.46,0.56]}));$$

$$Z_{CIVT-1} = ((\hat{s}_3, -0.32), ([0.17, 0.31]e^{i2\pi[0.18,0.32]}, [0.45, 0.55]e^{i2\pi[0.46,0.56]}));$$

$$Z_{CIVT-3} = \left((\check{s}_3, -0.02), \left([0.28, 0.39]e^{i2\pi[0.29,0.39]}, [0.41, 0.57]e^{i2\pi[0.42,0.58]}\right)\right);$$

$$Z_{CIVT-3} = \left((\check{s}_3, -0.02), \left([0.28, 0.39]e^{i2\pi[0.29,0.39]}, [0.41, 0.57]e^{i2\pi[0.42,0.58]}\right)\right);$$

$$Z_{CIVT-5} = \left((\check{s}_4, -0.35), \left([0.32, 0.51]e^{i2\pi[0.33,0.52]}, [0.21, 0.41]e^{i2\pi[0.22,0.42]}\right)\right);$$

and

$$Z_{CIVT-1} = \left((\check{s}_2, 0.1), \left([0.13, 0.29]e^{i2\pi[0.131,0.291]}, [0.46, 0.56]e^{i2\pi[0.461,0.561]}\right)\right);$$

$$Z_{CIVT-1} = \left((\check{s}_2, 0.1), \left([0.13, 0.29]e^{i2\pi[0.131,0.291]}, [0.46, 0.56]e^{i2\pi[0.461,0.561]}\right)\right);$$

$$Z_{CIVT-3} = \left((\check{s}_3, -0.19), \left([0.17, 0.30]e^{i2\pi[0.18,0.31]}, [0.52, 0.43]e^{i2\pi[0.521,0.44]}\right)\right);$$

$$Z_{CIVT-3} = \left((\check{s}_3, -0.19), \left([0.17, 0.30]e^{i2\pi[0.18,0.31]}, [0.52, 0.43]e^{i2\pi[0.521,0.44]}\right)\right);$$

$$Z_{CIVT-5} = \left((\check{s}_3, -0.18), \left([0.21, 0.47]e^{i2\pi[0.21,0.48]}, [0.32, 0.48]e^{i2\pi[0.33,0.49]}\right)\right);$$

Stage 3: The score values are calculated with the help of SF (for CIVQRO2-TLWA operator), we have:

$$\begin{aligned}\Gamma_{SF}(Z_{CIVT-1}) &= \check{s}_{-0.05778}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.09924}, \Gamma_{SF}(Z_{CIVT-3}) \\ &= \check{s}_{-0.09409}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.03913}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.05779}\end{aligned}$$

Some rules for CIVQRO2-TLWG operator, we get:

$$\begin{aligned}\Gamma_{SF}(Z_{CIVT-1}) &= \check{s}_{-0.05778}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.09924}, \Gamma_{SF}(Z_{CIVT-3}) \\ &= \check{s}_{-0.09409}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.03913}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.05779}\end{aligned}$$

Stage 4: With the help of SF, we find the ranking values is to find the beneficial one:

$$Z_{CIVT-2} \geq Z_{CIVT-5} \geq Z_{CIVT-4} \geq Z_{CIVT-1} \geq Z_{CIVT-3},$$

$$Z_{CIVT-2} \geq Z_{CIVT-5} \geq Z_{CIVT-4} \geq Z_{CIVT-1} \geq Z_{CIVT-3}.$$

Therefore, Z_{CIVT-2} is the beneficial optimal for two operators.

Advantages of the explored approach

Keeping the advantages of the explored operators, in this study, we try to show the advantages of this investigated method through a more in-depth analysis. For this, we choose to use the complex interval-valued Pythagorean 2-tuple linguistic set (CIVP2-TLS) and CIVQRO2-TLS to represent the decision information and resolve them by using the WA and WG operators. The explored operators are more powerful and more capable to cope with awkward and complicated information in realistic decision issues. Firstly, the decision matrix in the form of CIVP2-TLNs is shown in Table 4. Then, the calculation results derived by proposed methods are discussed below.

In the presence of the qualitative idea of CIVQRO2-TLWA and CIVQRO2-TLWG operators, we demonstrate the single value with the help of accumulated values of the given data for Z_{CIVT-2}.

Table 4. Decision matrix in the form of CIVP2-TLNs.

	$\mathcal{L}_{AT-1}$	$\mathcal{L}_{AT-2}$
Z_{CSF-1}	$\left((\check{s}_2, 0), \left(\begin{array}{c}[0.1, 0.6]e^{i2\pi[0.11,0.21]}, \\ [0.4, 0.5]e^{i2\pi[0.41,0.51]}\end{array}\right)\right)$	$\left((\check{s}_5, 0), \left(\begin{array}{c}[0.2, 0.7]e^{i2\pi[0.21,0.41]}, \\ [0.5, 0.6]e^{i2\pi[0.51,0.47]}\end{array}\right)\right)$
Z_{CSF-2}	$\left((\check{s}_3, 0), \left(\begin{array}{c}[0.4, 0.8]e^{i2\pi[0.41,0.51]}, \\ [0.2, 0.3]e^{i2\pi[0.21,0.31]}\end{array}\right)\right)$	$\left((\check{s}_2, 0), \left(\begin{array}{c}[0.3, 0.8]e^{i2\pi[0.31,0.41]}, \\ [0.2, 0.3]e^{i2\pi[0.21,0.31]}\end{array}\right)\right)$
Z_{CSF-3}	$\left((\check{s}_4, 0), \left(\begin{array}{c}[0.4, 0.7]e^{i2\pi[0.41,0.51]}, \\ [0.3, 0.5]e^{i2\pi[0.31,0.48]}\end{array}\right)\right)$	$\left((\check{s}_3, 0), \left(\begin{array}{c}[0.2, 0.8]e^{i2\pi[0.21,0.51]}, \\ [0.1, 0.3]e^{i2\pi[0.11,0.31]}\end{array}\right)\right)$
Z_{CSF-4}	$\left((\check{s}_2, 0), \left(\begin{array}{c}[0.4, 0.9]e^{i2\pi[0.41,0.51]}, \\ [0.1, 0.2]e^{i2\pi[0.11,0.21]}\end{array}\right)\right)$	$\left((\check{s}_3, 0), \left(\begin{array}{c}[0.1, 0.7]e^{i2\pi[0.11,0.31]}, \\ [0.3, 0.5]e^{i2\pi[0.31,0.51]}\end{array}\right)\right)$
Z_{CSF-5}	$\left((\check{s}_4, 0), \left(\begin{array}{c}[0.2, 0.7]e^{i2\pi[0.21,0.41]}, \\ [0.3, 0.6]e^{i2\pi[0.31,0.57]}\end{array}\right)\right)$	$\left((\check{s}_1, 0), \left(\begin{array}{c}[0.2, 0.47]e^{i2\pi[0.21,0.41]}, \\ [0.3, 0.4]e^{i2\pi[0.31,0.41]}\end{array}\right)\right)$
	$\mathcal{L}_{AT-3}$	$\mathcal{L}_{AT-4}$
Z_{CSF-1}	$\left((\check{s}_1, 0), \left(\begin{array}{c}[0.2, 0.7]e^{i2\pi[0.21,0.31]}, \\ [0.4, 0.5]e^{i2\pi[0.41,0.51]}\end{array}\right)\right)$	$\left((\check{s}_3, 0), \left(\begin{array}{c}[0.1, 0.7]e^{i2\pi[0.11,0.31]}, \\ [0.5, 0.6]e^{i2\pi[0.51,0.61]}\end{array}\right)\right)$
Z_{CSF-2}	$\left((\check{s}_3, 0), \left(\begin{array}{c}[0.4, 0.8]e^{i2\pi[0.41,0.61]}, \\ [0.3, 0.4]e^{i2\pi[0.31,0.37]}\end{array}\right)\right)$	$\left((\check{s}_5, 0), \left(\begin{array}{c}[0.4, 0.8]e^{i2\pi[0.41,0.51]}, \\ [0.1, 0.2]e^{i2\pi[0.11,0.21]}\end{array}\right)\right)$
Z_{CSF-3}	$\left((\check{s}_2, 0), \left(\begin{array}{c}[0.2, 0.7]e^{i2\pi[0.21,0.31]}, \\ [0.5, 0.6]e^{i2\pi[0.51,0.61]}\end{array}\right)\right)$	$\left((\check{s}_3, 0), \left(\begin{array}{c}[0.1, 0.6]e^{i2\pi[0.11,0.21]}, \\ [0.6, 0.7]e^{i2\pi[0.61,0.71]}\end{array}\right)\right)$
Z_{CSF-4}	$\left((\check{s}_4, 0), \left(\begin{array}{c}[0.1, 0.7]e^{i2\pi[0.11,0.41]}, \\ [0.3, 0.5]e^{i2\pi[0.31,0.51]}\end{array}\right)\right)$	$\left((\check{s}_5, 0), \left(\begin{array}{c}[0.4, 0.7]e^{i2\pi[0.41,0.51]}, \\ [0.1, 0.3]e^{i2\pi[0.11,0.31]}\end{array}\right)\right)$
Z_{CSF-5}	$\left((\check{s}_5, 0), \left(\begin{array}{c}[0.1, 0.8]e^{i2\pi[0.11,0.41]}, \\ [0.4, 0.5]e^{i2\pi[0.41,0.51]}\end{array}\right)\right)$	$\left((\check{s}_2, 0), \left(\begin{array}{c}[0.4, 0.8]e^{i2\pi[0.41,0.61]}, \\ [0.1, 0.3]e^{i2\pi[0.11,0.31]}\end{array}\right)\right)$

$$Z_{CIVT-1} = \left((\check{s}_3, -0.32), \left([0.17, 0.61]e^{i2\pi[0.18,0.32]}, [0.45, 0.55]e^{i2\pi[0.46,0.56]}\right)\right);$$

$$Z_{CIVT-2} = \left((\check{s}_4, 0.02), \left([0.40, 0.83]e^{i2\pi[0.41,0.54]}, [0.17, 0.28]e^{i2\pi[0.18,0.29]}\right)\right);$$

$$Z_{CIVT-3} = \left((\check{s}_3, -0.02), \left([0.28, 0.79]e^{i2\pi[0.29,0.39]}, [0.41, 0.57]e^{i2\pi[0.42,0.58]}\right)\right);$$

$$Z_{CIVT-4} = \left((\check{s}_4, 0.19), \left([0.35, 0.86]e^{i2\pi[0.351,0.47]}, [0.16, 0.34]e^{i2\pi[0.17,0.346]}\right)\right);$$

$$Z_{CIVT-5} = \left((\check{s}_4, -0.35), \left([0.32, 0.81]e^{i2\pi[0.33,0.52]}, [0.21, 0.41]e^{i2\pi[0.22,0.42]}\right)\right);$$

and

$$Z_{CIVT-5} = \left((\check{s}_4, -0.35), \left([0.32, 0.81]e^{i2\pi[0.33,0.52]}, [0.21, 0.41]e^{i2\pi[0.22,0.42]}\right)\right);$$

$$Z_{CIVT-2} = \left((\check{s}_4, -0.47), \left([0.39, 0.82]e^{i2\pi[0.391,0.522]}, [0.231, 0.322]e^{i2\pi[0.24,0.33]}\right)\right);$$

$$Z_{CIVT-3} = \left((\check{s}_3, -0.19), \left([0.17, 0.80]e^{i2\pi[0.18,0.31]}, [0.52, 0.43]e^{i2\pi[0.521,0.44]}\right)\right);$$

$$Z_{CIVT-4} = \left((\check{s}_4, -0.3), \left([0.23, 0.74]e^{i2\pi[0.231,0.441]}, [0.24, 0.41]e^{i2\pi[0.25,0.342]}\right)\right);$$

$$\mathrm{Z}_{CIVT-5} = \left((\check{s}_3, -0.18), \left([0.21, 0.87]e^{i2\pi[0.21,0.48]}, [0.32, 0.48]e^{i2\pi[0.33,0.49]}\right)\right);$$

The score value is diagnosed with the help of SF (for CIVQRO2-TLWA), we have

$$\begin{aligned}\Gamma_{SF}(\mathrm{Z}_{CIVT-1}) = \check{s}_{0.0288}, \Gamma_{SF}(\mathrm{Z}_{CIVT-2}) = \check{s}_{0.4969}, \Gamma_{SF}(\mathrm{Z}_{CIVT-3}) \\ = \check{s}_{0.1788}, \Gamma_{SF}(\mathrm{Z}_{CIVT-4}) = \check{s}_{0.5652}, \Gamma_{SF}(\mathrm{Z}_{CIVT-5}) = \check{s}_{0.3888}\end{aligned}$$

Some rules on the basis of the CIVQRO2-TLWG operator, we get:

$$\begin{aligned}\Gamma_{SF}(\mathrm{Z}_{CIVT-1}) = \check{s}_{0.1516}, \Gamma_{SF}(\mathrm{Z}_{CIVT-2}) = \check{s}_{0.4092}, \Gamma_{SF}(\mathrm{Z}_{CIVT-3}) \\ = \check{s}_{0.3453}, \Gamma_{SF}(\mathrm{Z}_{CIVT-4}) = \check{s}_{0.2586}, \Gamma_{SF}(\mathrm{Z}_{CIVT-5}) = \check{s}_{0.3763}\end{aligned}$$

With the help of the SF, the rankings of alternatives are achieved, such that

$$\mathrm{Z}_{CIVT-4} \geq \mathrm{Z}_{CIVT-2} \geq \mathrm{Z}_{CIVT-5} \geq \mathrm{Z}_{CIVT-3} \geq \mathrm{Z}_{CIVT-1}$$

$$\mathrm{Z}_{CIVT-2} \geq \mathrm{Z}_{CIVT-5} \geq \mathrm{Z}_{CIVT-3} \geq \mathrm{Z}_{CIVT-4} \geq \mathrm{Z}_{CIVT-1}$$

Therefore, $\mathrm{Z}_{CIVT-2}, \mathrm{Z}_{CIVT-4}$, are the beneficial optimal for the CIVQRO2-TLWA operator and CIVQRO2-TLWG, respectively.

Additionally, if we choose the complex q-rung orthopair 2-tuple linguistic kind of information's, which is discussed in the form of Table 5.

In the presence of the qualitative idea of CIVQRO2-TLWA and CIVQRO2-TLWG operators, we aggregate the single values into a collective one for $q_{SC} = 4$.

Table 5. Decision matrix in the form of CIVQRO2-TLNs.

	$\mathcal{L}_{AT-1}$	$\mathcal{L}_{AT-2}$
Z_{CSF-1}	$\left((\check{s}_2, 0), \left([0.1, 0.9]e^{i2\pi[0.11,0.21]}, [0.4, 0.5]e^{i2\pi[0.41,0.51]}\right)\right)$	$\left((\check{s}_5, 0), \left([0.2, 0.9]e^{i2\pi[0.21,0.41]}, [0.5, 0.6]e^{i2\pi[0.51,0.47]}\right)\right)$
Z_{CSF-2}	$\left((\check{s}_3, 0), \left([0.4, 0.9]e^{i2\pi[0.41,0.51]}, [0.2, 0.3]e^{i2\pi[0.21,0.31]}\right)\right)$	$\left((\check{s}_2, 0), \left([0.3, 0.9]e^{i2\pi[0.31,0.41]}, [0.2, 0.3]e^{i2\pi[0.21,0.31]}\right)\right)$
Z_{CSF-3}	$\left((\check{s}_4, 0), \left([0.4, 0.9]e^{i2\pi[0.41,0.51]}, [0.3, 0.5]e^{i2\pi[0.31,0.48]}\right)\right)$	$\left((\check{s}_3, 0), \left([0.2, 0.9]e^{i2\pi[0.21,0.51]}, [0.1, 0.3]e^{i2\pi[0.11,0.31]}\right)\right)$
Z_{CSF-4}	$\left((\check{s}_2, 0), \left([0.4, 0.9]e^{i2\pi[0.41,0.51]}, [0.1, 0.2]e^{i2\pi[0.11,0.21]}\right)\right)$	$\left((\check{s}_3, 0), \left([0.1, 0.9]e^{i2\pi[0.11,0.31]}, [0.3, 0.5]e^{i2\pi[0.31,0.51]}\right)\right)$
Z_{CSF-5}	$\left((\check{s}_4, 0), \left([0.2, 0.9]e^{i2\pi[0.21,0.41]}, [0.3, 0.6]e^{i2\pi[0.31,0.57]}\right)\right)$	$\left((\check{s}_1, 0), \left([0.2, 0.49]e^{i2\pi[0.21,0.41]}, [0.3, 0.4]e^{i2\pi[0.31,0.41]}\right)\right)$
	$\mathcal{L}_{AT-3}$	$\mathcal{L}_{AT-4}$
Z_{CSF-1}	$\left((\check{s}_1, 0), \left([0.2, 0.9]e^{i2\pi[0.21,0.31]}, [0.4, 0.5]e^{i2\pi[0.41,0.51]}\right)\right)$	$\left((\check{s}_3, 0), \left([0.1, 0.9]e^{i2\pi[0.11,0.31]}, [0.5, 0.6]e^{i2\pi[0.51,0.61]}\right)\right)$
Z_{CSF-2}	$\left((\check{s}_3, 0), \left([0.4, 0.9]e^{i2\pi[0.41,0.61]}, [0.3, 0.4]e^{i2\pi[0.31,0.37]}\right)\right)$	$\left((\check{s}_5, 0), \left([0.4, 0.9]e^{i2\pi[0.41,0.51]}, [0.1, 0.2]e^{i2\pi[0.11,0.21]}\right)\right)$
Z_{CSF-3}	$\left((\check{s}_2, 0), \left([0.2, 0.9]e^{i2\pi[0.21,0.31]}, [0.5, 0.6]e^{i2\pi[0.51,0.61]}\right)\right)$	$\left((\check{s}_3, 0), \left([0.1, 0.9]e^{i2\pi[0.11,0.21]}, [0.6, 0.7]e^{i2\pi[0.61,0.71]}\right)\right)$
Z_{CSF-4}	$\left((\check{s}_4, 0), \left([0.1, 0.9]e^{i2\pi[0.11,0.41]}, [0.3, 0.5]e^{i2\pi[0.31,0.51]}\right)\right)$	$\left((\check{s}_5, 0), \left([0.4, 0.9]e^{i2\pi[0.41,0.51]}, [0.1, 0.3]e^{i2\pi[0.11,0.31]}\right)\right)$
Z_{CSF-5}	$\left((\check{s}_5, 0), \left([0.1, 0.9]e^{i2\pi[0.11,0.41]}, [0.4, 0.5]e^{i2\pi[0.41,0.51]}\right)\right)$	$\left((\check{s}_2, 0), \left([0.4, 0.9]e^{i2\pi[0.41,0.61]}, [0.1, 0.3]e^{i2\pi[0.11,0.31]}\right)\right)$

Table 6. Comparative analysis for Table 3.

Methods	AOs	Score values	Ranking values
IVI2-TLSs	WA	Failed	Failed
	WG	Failed	Failed
IVP2-TLSs	WA	Failed	Failed
	WG	Failed	Failed
IVQRO2-TLS	WA	Failed	Failed
	WG	Failed	Failed
CIVI2-TLSs	WA	$\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{0.0536}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.0402}, \Gamma_{SF}(Z_{CIVT-3}) = \check{s}_{-0.08195}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{-0.0132}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{-0.01825}$	$Z_{CIVT-1} \geq Z_{CIVT-2} \geq Z_{CIVT-5} \geq Z_{CIVT-4} \geq Z_{CIVT-3}$
	WG	$\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{0.063}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.07148}, \Gamma_{SF}(Z_{CIVT-3}) = \check{s}_{0.30208}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.074}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.1480}$	$Z_{CIVT-3} \geq Z_{CIVT-5} \geq Z_{CIVT-4} \geq Z_{CIVT-2} \geq Z_{CIVT-1}$
CIVP2-TLSs	WA	$\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{-0.043}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.1439}, \Gamma_{SF}(Z_{CIVT-3}) = \check{s}_{-0.13045}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.007}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.06132}$	$Z_{CIVT-2} \geq Z_{CIVT-5} \geq Z_{CIVT-4} \geq Z_{CIVT-1} \geq Z_{CIVT-3}$
	WG	$\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{-0.03654}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.12096}, \Gamma_{SF}(Z_{CIVT-3}) = \check{s}_{0.2017}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.05273}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.07325}$	$Z_{CIVT-3} \geq Z_{CIVT-2} \geq Z_{CIVT-5} \geq Z_{CIVT-4} \geq Z_{CIVT-1}$
Proposed work	WA	$\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{-0.05778}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.09924}, \Gamma_{SF}(Z_{CIVT-3}) = \check{s}_{-0.09409}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.03913}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.05779}$	$Z_{CIVT-2} \geq Z_{CIVT-5} \geq Z_{CIVT-4} \geq Z_{CIVT-1} \geq Z_{CIVT-3}$
	WG	$\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{-0.04912}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.0747}, \Gamma_{SF}(Z_{CIVT-3}) = \check{s}_{0.06323}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.01592}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.00677}$	$Z_{CIVT-2} \geq Z_{CIVT-3} \geq Z_{CIVT-4} \geq Z_{CIVT-5} \geq Z_{CIVT-1}$

$$Z_{CIVT-1} = \left((\check{s}_3, -0.32), \left([0.17, 0.91]e^{i2\pi[0.18,0.32]}, [0.45, 0.55]e^{i2\pi[0.46,0.56]}\right)\right)$$

;

$$Z_{CIVT-2} = \left((\check{s}_4, 0.02), \left([0.40, 0.93]e^{i2\pi[0.41,0.54]}, [0.17, 0.28]e^{i2\pi[0.18,0.29]}\right)\right);$$

$$Z_{CIVT-3} = \left((\check{s}_3, -0.02), \left([0.28, 0.92]e^{i2\pi[0.29,0.39]}, [0.41, 0.57]e^{i2\pi[0.42,0.58]}\right)\right);$$

$$Z_{CIVT-4} = \left((\check{s}_4, 0.19), \left([0.35, 0.9]e^{i2\pi[0.351,0.47]}, [0.16, 0.34]e^{i2\pi[0.17,0.346]}\right)\right);$$

$$Z_{CIVT-5} = \left((\check{s}_4, -0.35), \left([0.32, 0.91]e^{i2\pi[0.33,0.52]}, [0.21, 0.41]e^{i2\pi[0.22,0.42]}\right)\right);$$

and

$$Z_{CIVT-1} = \left((\check{s}_2, 0.1), \left([0.13, 0.9]e^{i2\pi[0.131,0.291]}, [0.46, 0.56]e^{i2\pi[0.461,0.561]}\right)\right);$$

$$Z_{CIVT-2} = \left((\check{s}_4, -0.47), \left([0.39, 0.92]e^{i2\pi[0.391,0.522]}, [0.231, 0.322]e^{i2\pi[0.24,0.33]}\right)\right);$$

$$Z_{CIVT-3} = \left((\check{s}_3, -0.19), \left([0.17, 0.90]e^{i2\pi[0.18,0.31]}, [0.52, 0.43]e^{i2\pi[0.521,0.44]}\right)\right);$$

$$Z_{CIVT-4} = \left((\check{s}_4, -0.3), \left([0.23, 0.94]e^{i2\pi[0.231,0.441]}, [0.24, 0.41]e^{i2\pi[0.25,0.342]}\right)\right);$$

$$Z_{CIVT-5} = \left((\check{s}_3, -0.18), \left([0.21, 0.907]e^{i2\pi[0.21,0.48]}, [0.32, 0.48]e^{i2\pi[0.33,0.49]}\right)\right)$$

;

The score value is diagnosed with the help of SF (for CIVQRO2-TLWA operator), we have

$$\begin{aligned}\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{0.3756}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.6793}, \Gamma_{SF}(Z_{CIVT-3}) \\ = \check{s}_{0.3774}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.6794}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.5947}\end{aligned}$$

And for the CIVQRO2-TLWG operator, we get:

$$\begin{aligned}\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{0.2916}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.5892}, \Gamma_{SF}(Z_{CIVT-3}) \\ = \check{s}_{0.5184}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.5182}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.4349}\end{aligned}$$

Therefore, we can find the ranking lists for all alternatives, such that:

$$Z_{CIVT-4} \geq Z_{CIVT-2} \geq Z_{CIVT-5} \geq Z_{CIVT-3} \geq Z_{CIVT-1}$$

$$Z_{CIVT-2} \geq Z_{CIVT-4} \geq Z_{CIVT-3} \geq Z_{CIVT-5} \geq Z_{CIVT-1}$$

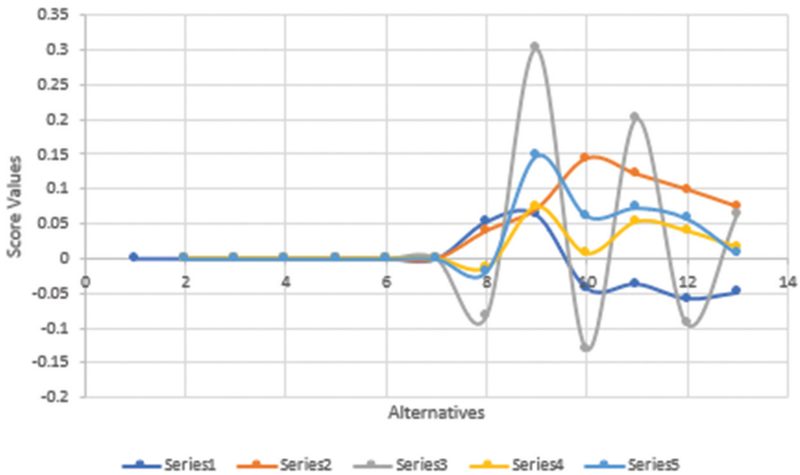

Figure 1. *Geometrical representation of the information of* .Table 6

Obviously, Z_{CIVT-2} is the beneficial optimal for the CIVQRO2-TLWA operator while Z_{CIVT-4} for the CIVQRO2-TLWG method.

Comparative analysis

To evaluate the proficiency and capability of the invented works, we try to compare the invented works with some prevailing works in the presence of the data given in Table 3, Table 4, and Table 5. The CIVI2-TLS, CIVP2-TLS, q-rung orthopair 2-tuple linguistic set (IVQRO2-TLS) (Wang, Garg, and Li 2019) and their special cases are used to

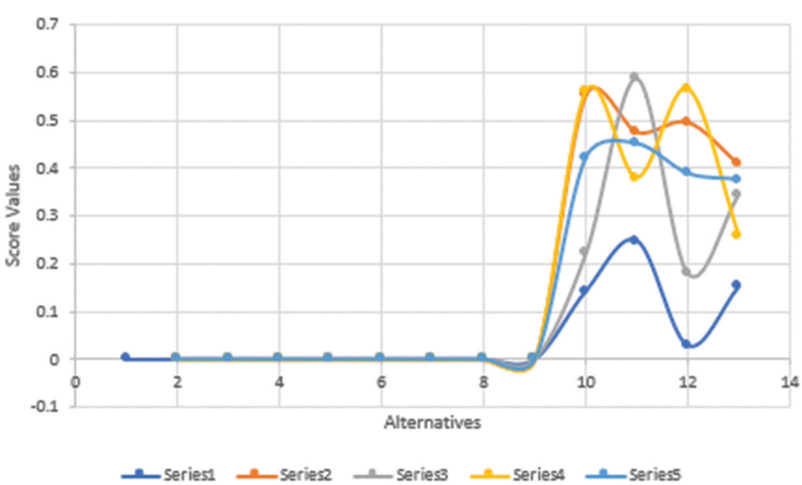

Figure 2. *Geometrical representation of the information of* .Table 7

Table 7. Comparative analysis for Table 4.

Methods	AOs	Score values	Ranking values
IVI2-TLSs	WA	Failed	Failed
	WG	Failed	Failed
IVP2-TLSs	WA	Failed	Failed
	WG	Failed	Failed
IVQRO2-TLS	WA	Failed	Failed
	WG	Failed	Failed
CIVI2-TLSs	WA	Failed	Failed
	WG	Failed	Failed
CIVP2-TLSs	WA	$\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{0.1415}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.5539}, \Gamma_{SF}(Z_{CIVT-3}) = \check{s}_{0.2212}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.5600}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.4227}$	$Z_{CIVT-4} \geq Z_{CIVT-2} \geq Z_{CIVT-5} \geq Z_{CIVT-3} \geq Z_{CIVT-1}$
	WG	$\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{0.247}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.4757}, \Gamma_{SF}(Z_{CIVT-3}) = \check{s}_{0.588}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.3802}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.4511}$	$Z_{CIVT-3} \geq Z_{CIVT-2} \geq Z_{CIVT-5} \geq Z_{CIVT-4} \geq Z_{CIVT-1}$
Proposed work	WA	$\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{0.0288}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.4969}, \Gamma_{SF}(Z_{CIVT-3}) = \check{s}_{0.1788}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.5652}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.3888}$	$Z_{CIVT-4} \geq Z_{CIVT-2} \geq Z_{CIVT-5} \geq Z_{CIVT-3} \geq Z_{CIVT-1}$
	WG	$\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{0.1516}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.4092}, \Gamma_{SF}(Z_{CIVT-3}) = \check{s}_{0.3453}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.2586}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.3763}$	$Z_{CIVT-2} \geq Z_{CIVT-5} \geq Z_{CIVT-3} \geq Z_{CIVT-4} \geq Z_{CIVT-1}$

Table 8. Comparative analysis for Table 5.

Methods	AOs	Score values	Ranking values
IVI2-TLSs	WA	Failed	Failed
	WG	Failed	Failed
IVP2-TLSs	WA	Failed	Failed
	WG	Failed	Failed
Wang, Garg, and Li (2019)	WA	Failed	Failed
	WG	Failed	Failed
CIVI2-TLSs	WA	Failed	Failed
	WG	Failed	Failed
CIVP2-TLSs	WA	Failed	Failed
	WG	Failed	Failed
Proposed work	WA	$\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{0.3756}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.6793}, \Gamma_{SF}(Z_{CIVT-3}) = \check{s}_{0.3774}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.6794}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.5947}$	$Z_{CIVT-4} \geq Z_{CIVT-2} \geq Z_{CIVT-5} \geq Z_{CIVT-3} \geq Z_{CIVT-1}$
	WG	$\Gamma_{SF}(Z_{CIVT-1}) = \check{s}_{0.2916}, \Gamma_{SF}(Z_{CIVT-2}) = \check{s}_{0.5892}, \Gamma_{SF}(Z_{CIVT-3}) = \check{s}_{0.5184}, \Gamma_{SF}(Z_{CIVT-4}) = \check{s}_{0.5182}, \Gamma_{SF}(Z_{CIVT-5}) = \check{s}_{0.4349}$	$Z_{CIVT-2} \geq Z_{CIVT-4} \geq Z_{CIVT-3} \geq Z_{CIVT-5} \geq Z_{CIVT-1}$

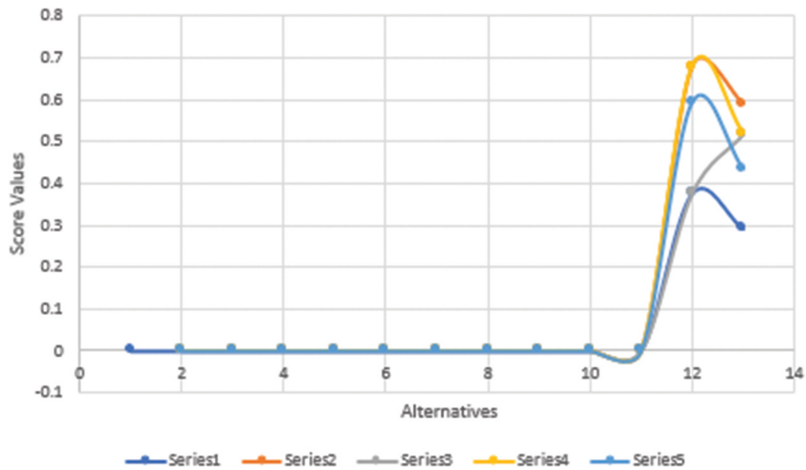

Figure 3. *Geometrical representation of the information of* .Table 6

calculated. Firstly, in the consideration of the data given in Table 3, the comparison of the invented and prevailing theories is diagnosed in Table 6.

Table 6 shows different ranking results different concepts used. The best alternatives are different from each other, which are follow as: Z_{CIVT-1}, Z_{CIVT-2} and Z_{CIVT-3}. We can draw the graphical expression, which is utilized in the form of Figure 1, to express the family of alternatives with five different colors. In Figure 1, the X-axis represents the family of alternatives, and the Y-axis the score values. Thus, the decision-maker can easily examine the best alternative with the help of Figure 1. For simplicity, the different series have different values; thus, we easily obtained the result that which one is the best alternative.

In the consideration of the data given in Table 4, the comparison of the invented and prevailing theories is diagnosed in Table 7.

Table 7 also provides the different ranking results, which are in the form of Z_{CIVT-2}, Z_{CIVT-3} and Z_{CIVT-4}. For simplicity, we can draw the graphical expression of the information in Table 7, which is utilized in the form of Figure 2.

In the consideration of the data given in Table 5, the comparison of the invented and prevailing theories are diagnosed in Table 8. Further, we can draw the graphical expression of Table 8, shown in Figure 3.

From above comparative analysis, we can see that the investigated operators based on CIVQRO2-TLS are extensively reliable and proficient to manage awkward and difficult information in realistic issues. Therefore, the explored approaches are more generalized than existing notions (Ali and Mahmood, 2020a; Akram and Naz 2019; Beg and Rashid 2016; Garg, Ali, and Mahmood 2020b; Herrera and Martinez 2001; Liu, Ali, and Mahmood 2019, 2020; Liu and Chen 2018).

Conclusion

The main contribution of this study is stated below: (1) We discussed the conception of the CIVQRO2-TLS and some of their important laws; (2) In the occurrence of the above theory, we discovered several aggregation methods for CIVQRO2-TLS, including the CIVQRO2-TLWA, CIVQRO2-TLOWA, CIVQRO2-TLHA, CIVQRO2-TLWG, CIVQRO2-TLOWG, and CIVQRO2-TLHG operators, and diagnosed their fundamental properties; (3) We demonstrated the beneficial features of the invented works, a MADM system is diagnosed and checked with the help of several examples; (4) In last, we elaborated on the advantages, comparative analysis, and graphical interpretation of the invented approaches.

There are also several limitations in the proposed methods. In many scenarios, the invented theory can't be working effectively, such as for the situation $\mu^{+\P sc}_{\mathfrak{Q}_{IP}} + \vartheta^{+\P sc}_{\mathfrak{Q}_{IP}} + \eta^{+\P sc}_{\mathfrak{Q}_{IP}} \geq 1$, the CIVQRO2-TLS cann't handle such information. Moreover, the calculation of aggregation process is also cumbersome due to the complex data structure of CIVQRO2-TLS. In the upcoming time, we expose several new theories like the TOPSIS method for PFSs (Bakioglu and Atahan 2021), improved composite relation (Ejegwa 2020), divergence mean (Verma 2020), MABAC method (Verma 2021), matrix game (Verma and Aggarwal 2021), decision-making strategy (Ali, Mahmood, and Yang 2020; Aydemir and Gündüz 2020; Jan et al. 2019; Mahmood et al. 2021; Ullah et al. 2020a, 2018; Verma and Merigó 2020, 2021; Zeng et al. 2020; 2022a; Zhang et al. 2012). Further, we will also try to invent some new theories, methods, and operators under the consideration of the above works are to enhance the quality of the diagnosed works.

Abbreviations

FS: Fuzzy set; **IFS**: Intuitionistic fuzzy set; **PFS**: Pythagorean fuzzy set; **QROFS**: q-rung orthopair fuzzy set; **CFS**: Complex fuzzy set; **CIFS**: Complex intuitionistic fuzzy set; **CPFS**: Complex Pythagorean fuzzy set; **CQROFS**: Complex q-rung orthopair fuzzy set; **IVFS**: Interval-valued fuzzy set; **IVIFS**: Interval-valued intuitionistic fuzzy set; **IVPFS**: Interval-valued Pythagorean fuzzy set; **IVQROFS**: Interval-valued q-rung orthopair fuzzy set; **CIVFS**: Complex interval-valued fuzzy set; **CIVIFS**: Complex interval-valued intuitionistic fuzzy set; **CIVPFS**: Complex interval-valued Pythagorean fuzzy set; **CIVQROFS**: Complex interval-valued q-rung orthopair fuzzy set; **CIVQRO2-TL**: Complex interval-valued q-rung orthopair 2-tuple linguistic; **CIVQRO2-TLS**: Complex interval-valued q-rung orthopair 2-tuple linguistic set; **CIVQRO2-TLWA**: Complex interval-valued q-rung orthopair 2-tuple linguistic weighted averaging; **CIVQRO2-TLOWA**: Complex interval-valued q-rung orthopair 2-tuple linguistic ordered weighted averaging; **CIVQRO2-TLHA**: Complex interval-valued q-rung orthopair 2-tuple linguistic hybrid averaging; **CIVQRO2-TLWG**: Complex interval-valued q-rung orthopair 2-tuple linguistic weighted geometric; **CIVQRO2-TLOWG**: Complex interval-valued q-rung orthopair 2-tuple linguistic ordered weighted geometric; **CIVQRO2-TLHG**: Complex interval-valued q-rung orthopair 2-tuple linguistic hybrid geometric; **MADM**: Multi-attribute decision-making

Acknowledgments

The authors are very grateful to the anonymous reviewers and the editor for their valuable comments and constructive suggestions that improve the previous versions of this paper. This work is supported by National Social Science Foundation of China (20CTJ016, 21ATJ010), the Social Sciences Planning Projects of Zhejiang (21QNYC11ZD), Statistical Scientific Key Research Project of China (2021LZ33), Statistical Scientific Key Research Project of Zhejiang (21TJZZ25), Fundamental Research Funds for the Provincial Universities of Zhejiang (SJWZ2020002), Longyuan Construction Financial Research Project of Ningbo University (LYYB2002), Statistical Scientifiic Key Research Project of China and the First-Class Discipline of Zhejiang-A (Zhejiang Gongshang University-Statistics).

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This work was supported by the Social Sciences Planning Projects of Zhejiang [21QNYC11ZD]; Statistical Scientific Key Research Project of Zhejiang [21TJZZ25]; Longyuan Construction Financial Research Project of Ningbo University [LYYB2002]; National Social Science Foundation of China [20CTJ016, 21ATJ010]; Fundamental Research Funds for the Provincial Universities of Zhejiang [SJWZ2020002]; Statistical Scientific Key Research Project of China [2021LZ33].

ORCID

Shouzhen Zeng http://orcid.org/0000-0002-3604-0843
Huanhuan Jin http://orcid.org/0000-0002-1004-9890

References

Akram, M., and S. Naz. 2019. A novel decision-making approach under complex Pythagorean fuzzy environment. *Mathematical and Computational Applications* 24 (3):73. doi:10.3390/mca24030073.

Ali, Z., T. Mahmood, and M. S. Yang. 2020. Complex T-spherical fuzzy aggregation operators with application to multi-attribute decision making. *Symmetry* 12 (8):1311. doi:10.3390/sym12081311.

Ali, Z., and T. Mahmood. 2020a. Complex neutrosophic generalized dice similarity measures and their application to decision making. *CAAI Transactions on Intelligence Technology* 5 (2):78–87. doi:10.1049/trit.2019.0084.

Ali, Z., and T. Mahmood. 2020b. Maclaurin symmetric mean operators and their applications in the environment of complex q-rung orthopair fuzzy sets. *Computational and Applied Mathematics* 39 (3):161. doi:10.1007/s40314-020-01145-3.

Alkouri, A. M. D. J. S., and A. R. Salleh 2012. Complex intuitionistic fuzzy sets. In AIP Conference Proceedings. American Institute of Physics, Beijing, China, 1482 (1): 464–70.

Atanassov, K. T. 1986. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems* 20 (1):87–96. doi:10.1016/S0165-0114(86)80034-3.

Atanassov, K. T., and G. Gargov. 1989. Interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems* 31 (3):343–49. doi:10.1016/0165-0114(89)90205-4.

Aydemir, S. B., and S. Y. Gündüz. 2020. Extension of multi-Moora method with some q-rung orthopair fuzzy Dombi prioritized weighted aggregation operators for multi-attribute decision making. *Soft Computing* 24 (24):18545–63. doi:10.1007/s00500-020-05091-4.

Bakioglu, G., and A. O. Atahan. 2021. AHP integrated TOPSIS and VIKOR methods with Pythagorean fuzzy sets to prioritize risks in self-driving vehicles. *Applied Soft Computing* 99 (1):106948. doi:10.1016/j.asoc.2020.106948.

Beg, I., and T. Rashid. 2016. An intuitionistic 2-tuple linguistic information model and aggregation operators. *International Journal of Intelligent Systems* 31 (6):569–92. doi:10.1002/int.21795.

Bouchet, A., S. Montes, V. Ballarin, and I. Díaz. 2020. Intuitionistic fuzzy set and fuzzy mathematical morphology applied to color leukocytes segmentation. *Signal, Image and Video Processing* 14 (3):557–64. doi:10.1007/s11760-019-01586-2.

Ejegwa, P. A. 2020. Improved composite relation for Pythagorean fuzzy sets and its application to medical diagnosis. *Granular Computing* 5 (2):277–86. doi:10.1007/s41066-019-00156-8.

Ejegwa, P. A., I. C. Onyeke, and V. Adah. 2021. A Pythagorean fuzzy algorithm embedded with a new correlation measure and its application in diagnostic processes. *Granular Computing* 6 (4):1037–46. doi:10.1007/s41066-020-00246-y.

Faizi, S., T. Rashid, and S. Zafar. 2018. A multicriteria decision-making approach based on fuzzy AHP with intuitionistic 2-tuple linguistic sets. *Advances in Fuzzy Systems* 7 (1):1–12. doi:10.1155/2018/5789192.

Faizi, S., S. Nawaz, and A. Ur-Rehman. 2020. Intuitionistic 2-tuple linguistic aggregation information based on Einstein operations and their applications in group decision making. *Artificial Intelligence Review* 53 (6):4625–50. doi:10.1007/s10462-020-09856-z.

Garg, H. 2017. A novel improved accuracy function for interval valued Pythagorean fuzzy sets and its applications in the decision-making process. *International Journal of Intelligent Systems* 32 (12):1247–60. doi:10.1002/int.21898.

Garg, H., and D. Rani. 2019. Complex interval-valued intuitionistic fuzzy sets and their aggregation operators. *Fundamenta Informaticae* 164 (1):61–101. doi:10.3233/FI-2019-1755.

Garg, H., and K. Kumar. 2019. Linguistic interval-valued atanassov intuitionistic fuzzy sets and their applications to group decision making problems. *IEEE Transactions on Fuzzy Systems* 27 (12):2302–11. doi:10.1109/TFUZZ.2019.2897961.

Garg, H., and S. M. Chen. 2020. Multiattribute group decision making based on neutrality aggregation operators of q-rung orthopair fuzzy sets. *Information Sciences* 51 (7):427–47. doi:10.1016/j.ins.2019.11.035.

Garg, H., and K. Kumar. 2020. A novel exponential distance and its based TOPSIS method for interval-valued intuitionistic fuzzy sets using connection number of SPA theory. *Artificial Intelligence Review* 53 (1):595–624. doi:10.1007/s10462-018-9668-5.

Garg, H., J. Gwak, T. Mahmood, and Z. Ali. 2020a. Power aggregation operators and VIKOR methods for complex q-rung orthopair fuzzy sets and their applications. *Mathematics* 8 (4):538–67. doi:10.3390/math8040538.

Garg, H., Z. Ali, and T. Mahmood. 2020b. Algorithms for complex interval-valued q-rung orthopair fuzzy sets in decision making based on aggregation operators, AHP, and TOPSIS. *Expert Systems* 38 (1):12609–38.

Haktanır, E., and C. Kahraman. 2019. A novel interval-valued Pythagorean fuzzy QFD method and its application to solar photovoltaic technology development. *Computers & Industrial Engineering* 13 (2):361–72. doi:10.1016/j.cie.2019.04.022.

He, T., G. Wei, J. Lu, C. Wei, and R. Lin. 2019. Pythagorean 2-tuple linguistic taxonomy method for supplier selection in medical instrument industries. *International Journal of Environmental Research and Public Health* 16 (23):89–103. doi:10.3390/ijerph16234875.

Herrera, F., and L. Martinez. 2000. A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems* 8 (6):746–52. doi:10.1109/91.890332.

Herrera, F., and L. Martinez. 2001. The 2-tuple linguistic computational model: Advantages of its linguistic description, accuracy and consistency. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* 9 (1):33–48. doi:10.1142/S0218488501000971.

Hussain, A., M. I. Ali, and T. Mahmood. 2019. Covering based q-rung orthopair fuzzy rough set model hybrid with TOPSIS for multi-attribute decision making. *Journal of Intelligent & Fuzzy Systems* 37 (1):981–93. doi:10.3233/JIFS-181832.

Jan, N., Z. Ali, T. Mahmood, and K. Ullah. 2019. Some generalized distance and similarity measures for picture hesitant fuzzy sets and their applications in building material recognition and multi-attribute decision making. *Punjab University Journal of Mathematics* 51 (7):51–70.

Joshi, B. P., A. Singh, P. K. Bhatt, and K. S. Vaisla. 2018. Interval valued q-rung orthopair fuzzy sets and their properties. *Journal of Intelligent & Fuzzy Systems* 35 (5):5225–30. doi:10.3233/JIFS-169806.

Ju, Y., A. Wang, J. Ma, H. Gao, and G. Santibanez. 2020. Some q-rung orthopair fuzzy 2-tuple linguistic Muirhead mean aggregation operators and their applications to multiple-attribute group decision making. *International Journal of Intelligent Systems* 35 (1):184–213. doi:10.1002/int.22205.

Kumar, K., and H. Garg. 2018. Connection number of set pair analysis based TOPSIS method on intuitionistic fuzzy sets and their application to decision making. *Applied Intelligence* 48 (8):2112–19. doi:10.1007/s10489-017-1067-0.

Liang, D., A. P. Darko, and Z. Xu. 2018. Interval-valued Pythagorean fuzzy extended Bonferroni mean for dealing with heterogenous relationship among attributes. *International Journal of Intelligent Systems* 33 (7):1381–411. doi:10.1002/int.21973.

Liu, P., and S. M. Chen. 2018. Multiattribute group decision making based on intuitionistic 2-tuple linguistic information. *Information Sciences* 430 (2):599–619. doi:10.1016/j.ins.2017.11.059.

Liu, P., and P. Wang. 2018. Multiple-attribute decision-making based on Archimedean Bonferroni operators of q-rung orthopair fuzzy numbers. *IEEE Transactions on Fuzzy Systems* 27 (5):834–48. doi:10.1109/TFUZZ.2018.2826452.

Liu, P., and J. Liu. 2018. Some q-rung orthopai fuzzy Bonferroni mean operators and their application to multi-attribute group decision making. *International Journal of Intelligent Systems* 33 (2):315–47. doi:10.1002/int.21933.

Liu, P., Z. Ali, and T. Mahmood. 2019. A method to multi-attribute group decision-making problem with complex q-rung orthopair linguistic information based on heronian mean operators. *International Journal of Computational Intelligence Systems* 12 (2):1465–96. doi:10.2991/ijcis.d.191030.002.

Liu, P., Z. Ali, and T. Mahmood. 2020. The distance measures and cross-entropy based on complex fuzzy sets and their application in decision making. *Journal of Intelligent & Fuzzy Systems* 39 (3):3351–74. doi:10.3233/JIFS-191718.

Mahmood, T., U. Rehman, Z. Ali, and T. Mahmood. 2021. Hybrid vector similarity measures based on complex hesitant fuzzy sets and their applications to pattern recognition and medical diagnosis. *Journal of Intelligent & Fuzzy Systems* 40 (1):625–46. doi:10.3233/JIFS-200418.

Peng, X., and W. Li. 2019. Algorithms for interval-valued Pythagorean fuzzy sets in emergency decision making based on multiparametric similarity measures and WDBA. *IEEE ACCESS* 7 (4):7419–41. doi:10.1109/ACCESS.2018.2890097.

Ramot, D., R. Milo, M. Friedman, and A. Kandel. 2002. Complex fuzzy sets. *IEEE Transactions on Fuzzy Systems* 10 (2):171–86. doi:10.1109/91.995119.

Rani, D., and H. Garg. 2017. Distance measures between the complex intuitionistic fuzzy sets and their applications to the decision-making process. *International Journal for Uncertainty Quantification* 7 (5):423–39. doi:10.1615/Int.J.UncertaintyQuantification.2017020356.

Sajjad Ali Khan, M., S. Abdullah, M. Yousaf Ali, I. Hussain, and M. Farooq. 2018. Extension of TOPSIS method base on Choquet integral under interval-valued Pythagorean fuzzy environment. *Journal of Intelligent & Fuzzy Systems* 34 (1):267–82. doi:10.3233/JIFS-171164.

Ullah, K., T. Mahmood, N. Jan, and Z. Ali. 2018. A note on geometric aggregation operators in T-spherical fuzzy environment and their applications in multi-attribute decision making. *Journal of Engineering and Applied Sciences* 37 (2):75–86.

Ullah, K., H. Garg, T. Mahmood, N. Jan, and Z. Ali. 2020a. Correlation coefficients for T-spherical fuzzy sets and their applications in clustering and multi-attribute decision making. *Soft Computing* 24 (3):1647–59. doi:10.1007/s00500-019-03993-6.

Ullah, K., T. Mahmood, Z. Ali, and N. Jan. 2020b. On some distance measures of complex Pythagorean fuzzy sets and their applications in pattern recognition. *Complex & Intelligent Systems* 6 (1):15–27. doi:10.1007/s40747-019-0103-6.

Verma, R., and J. M. Merigó. 2020. Multiple attribute group decision making based on 2-dimension linguistic intuitionistic fuzzy aggregation operators. *Soft Computing* 24 (22):17377–400. doi:10.1007/s00500-020-05026-z.

Verma, R. 2020. Multiple attribute group decision-making based on order-α divergence and entropy measures under q-rung orthopair fuzzy environment. *International Journal of Intelligent Systems* 35 (4):718–50. doi:10.1002/int.22223.

Verma, R. 2021. On intuitionistic fuzzy order-α divergence and entropy measures with MABAC method for multiple attribute group decision-making. *Journal of Intelligent & Fuzzy Systems* 40 (1):1191–217. doi:10.3233/JIFS-201540.

Verma, R., and A. Aggarwal. 2021. On matrix games with 2-tuple intuitionistic fuzzy linguistic payoffs. *Iranian Journal of Fuzzy Systems* 18 (4):149–67.

Verma, R., and J. M. Merigó. 2021. On Sharma-Mittal's entropy under intuitionistic fuzzy environment. *Cybernetics and Systems* 52 (6):498–521. doi:10.1080/01969722.2021.1903722.

Wang, L., H. Garg, and N. Li. 2019. Interval-valued q-rung orthopair 2-tuple linguistic aggregation operators and their applications to decision making process. *IEEE Access* 7 (2):131962–77. doi:10.1109/ACCESS.2019.2938706.

Wei, G., M. Lu, F. E. Alsaadi, T. Hayat, and A. Alsaedi. 2017. Pythagorean 2-tuple linguistic aggregation operators in multiple attribute decision making. *Journal of Intelligent & Fuzzy Systems* 33 (2):1129–42. doi:10.3233/JIFS-16715.

Yager, R. R. 2013. Pythagorean membership grades in multicriteria decision making. *IEEE Transection and Fuzzy Systems* 22 (4):958–65. doi:10.1109/TFUZZ.2013.2278989.

Yager, R. R. 2016. Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems* 25 (5):1222–30. doi:10.1109/TFUZZ.2016.2604005.

Zadeh, L. A. 1965. Fuzzy sets. *Information and Control* 8 (3):338–53. doi:10.1016/S0019-9958(65)90241-X.

Zeng, S. Z., D. Luo, C. Zhang, and X. Li. 2020. A correlation-based TOPSIS method for multiple attribute decision making with single-valued neutrosophic information. *International Journal of Information Technology & Decision Making* 19 (1):343–58. doi:10.1142/S0219622019500512.

Zeng, S. Z., Z. Ali, and T. Mahmood. 2021. Novel complex T-spherical dual hesitant uncertain linguistic muirhead mean operators and their application in decision-making. *CMES-Computer Modeling in Engineering & Sciences* 129 (2):849–80. doi:10.32604/cmes.2021.016727.

Zeng, S. Z., S. Ali, M. K. Mahmood, K. Smarandache, and D. Ahmad. 2022a. Decision-making problems under the environment of m-polar diophantine neutrosophic N-soft set. *CMES-Computer Modeling in Engineering & Sciences* 130 (1):581–606.

Zeng, S. Z., N. Zhang, C. H. Zhang, W. Su, and L. A. Carlos. 2022b. Social network multiple-criteria decision-making approach for evaluating unmanned ground delivery vehicles under the Pythagorean fuzzy environment. *Technological Forecasting and Social Change* 175:121414–39. doi:10.1016/j.techfore.2021.121414.

Zeng, S. Z., J. M. Zhou, C. H. Zhang, and J. M. Merigó. 2022c. Intuitionistic fuzzy social network hybrid MCDM model for the assessment of digital reforms of manufacturing industry in China. *Technological Forecasting and Social Change* 176:121435. doi:10.1016/j.techfore.2021.121435.

Zhang, Y., H. Ma, B. Liu, and J. Liu. 2012. Group decision making with 2-tuple intuitionistic fuzzy linguistic preference relations. *Soft Computing* 16 (8):1439–46. doi:10.1007/s00500-012-0847-z.

Zhang, C. H., Q. Q. Hu, S. Z. Zeng, and W. H. Su. 2021. IOWLAD-based MCDM model for the site assessment of a household waste processing plant under a Pythagorean fuzzy environment. *Environmental Impact Assessment Review* 89:106579. doi:10.1016/j.eiar.2021.106579.