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Nozzle Shape Optimization to Achieve the Maximum Gain in the GDLs

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

The geometry of a divergent nozzle of the gas dynamic laser is optimized to achieve the maximum possible gain in an optical cavity. The problem is numerically studied using two-dimensional computation of the governing equations beside the conjugate gradient method for the shape optimization. The system of governing equations is solved with a finite volume approach using a structured grid in which the advection upstream splitting method is used to calculate the convective numerical fluxes. The finite difference approximation approach is used to calculate the sensitivity matrix coefficient of the optimization procedure. The results show that the optimum geometry of the considered problem can improve the maximum small signal gain as much as about 9 percent in comparison with the simple linear nozzle. The steady state solution of the equations is considered here.

Keywords: Conjugate gradient; divergent nozzle; gain; gas dynamic laser; non-equilibrium; numerical simulation; optical cavity; shape optimization; supersonic; vibrational energy.

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NOMENCLATURE

A	=	*the lasing gas symbol*
a	=	*constant in gain equation*
B	=	*the symbol of the gas stores the vibrational energy*
b	=	*constant in gain equation*
C	=	*catalyst gas symbol*
c	=	*mass fraction*
e	=	*specific internal energy*
e_{vib}	=	*specific vibrational energy*
h	=	*Plank's constant*
k	=	*Boltzmann constant*
N	=	*population*
R	=	*gas constant*
T	=	*temperature*
X	=	*mole fraction*
λ	=	*wavelength*
$\tau_{I, II}$	=	*characteristic relaxation time of vibrational modes*
τ_{12}	=	*radiative lifetime*
$\nu_{1, 2, 3}$	=	*vibrational mode's frequency of gas A*
ν_c	=	*collision frequency*

1. INTRODUCTION

The laser technology is available for different situations like research, medical, industrial and commercial uses. There are different types of lasers that have wide range of wavelengths and powers, such as solid state lasers or semiconductor lasers. One of the most useful types of these technologies is the gas laser which can be divided in three different categories: Electric discharge, chemical and gas dynamic lasers.

The invention of the Gas Dynamic Lasers (GDL) dates back about 50 years. The population inversions in molecular systems could be created by rapid cooling of the system. Therefore, such inversion could be obtained in the fast non-equilibrium expansion of an initially hot gaseous mixture through a supersonic nozzle. During this expansion, the working gas is turned into a supersonic laser medium and passes into the optical cavity (acts as a laser resonator) where using proper mirrors may extract a beam of laser perpendicular to the flow.

The GDLs can be scaled to large sizes without major physical complications. So, they can produce the highest continuous wave powers up to hundreds of kilowatts [1]. Different numerical and experimental studies have been performed in the past decades to analyze the gas dynamic lasers' operation, performance and design procedure [2-9]. There are also some reports on the parametric studies and optimization of the performance of the GDLs [10,11].

The theoretical analysis of the gas dynamic laser is based on the consecutive computation of the vibrational temperatures, population inversions, gain and finally the power extraction. But such calculations generally provide results on the raw power output available from the optical cavity, because the total picture of power extraction is more than that. One reason for this difference is the inhomogeneities of the flowing gas in the cavity that cause the phase distortions in the laser beam. Such inhomogeneities are due to the pattern of supersonic field due to the flow and geometry interactions. The author [12] has recently studied the variation of the gas dynamic laser's gain along the optical cavity to analyze the effects of the flow field's diamond pattern.

In the present study, the shape of the divergent nozzle is optimized to achieve the maximum possible gain in the optical cavity.

2. GOVERNING EQUATIONS AND NUMERICAL PROCEDURE

Here, the laser mixture contains the A-B-C gases which A is the active lasing molecule, B stores the vibrational energy and C has the role of catalyst for augmentation of the population inversions. The two-dimensional compressible equations governing the inviscid flow representing the conservation of mass, momentum and energy are used as the basic equations of the flow field.

The laser media under consideration has two modes of vibrational energy, so:

$$e = e(T) + \frac{V^2}{2} + e_{vib\,I} + e_{vib\,II} \tag{1}$$

The rate equations representing the relaxation of vibrational energies of these modes are:

$$\begin{aligned} \frac{\partial e_{vib\,I}}{\partial t} + u\frac{\partial e_{vib\,I}}{\partial x} + v\frac{\partial e_{vib\,I}}{\partial y} &= \frac{1}{\tau_I}\left(e_{vib\,I}{}^{eq} - e_{vib\,I}\right) \\ \frac{\partial e_{vib\,II}}{\partial t} + u\frac{\partial e_{vib\,II}}{\partial x} + v\frac{\partial e_{vib\,II}}{\partial y} &= \frac{1}{\tau_{II}}\left(e_{vib\,II}{}^{eq} - e_{vib\,II}\right) \end{aligned} \tag{2}$$

For the mixtures as A-B-C system, the vibrational energies are defined as:

$$\begin{aligned} e_{vib I} &= c_A R_A \left\{ \left(\frac{hv_1}{k}\right)\left[\exp\left(\frac{hv_1}{kT_{vibI}} - 1\right)\right]^{-1} + \left(\frac{2hv_2}{k}\right)\left[\exp\left(\frac{hv_2}{kT_{vibI}} - 1\right)\right]^{-1} \right\} \\ e_{vib\,II} &= c_A R_A \left\{ \left(\frac{hv_3}{k}\right)\left[\exp\left(\frac{hv_3}{kT_{vib\,II}} - 1\right)\right]^{-1} \right\} \end{aligned} \tag{3}$$

The populations of two energy levels of the lasing molecule are [3]:

$$\begin{aligned} N_{001} &= \frac{N_A \exp\left(-\frac{hv_3}{kT_{vibII}}\right)}{\left[1-\exp\left(-\frac{hv_1}{kT_{vibI}}\right)\right]^{-1}\left[1-\exp\left(-\frac{hv_2}{kT_{vibI}}\right)\right]^{-2}\left[1-\exp\left(-\frac{hv_3}{kT_{vibII}}\right)\right]^{-1}} \\ N_{100} &= \frac{N_A \exp\left(-\frac{hv_1}{kT_{vib\,I}}\right)}{\left[1-\exp\left(-\frac{hv_1}{kT_{vib\,I}}\right)\right]^{-1}\left[1-\exp\left(-\frac{hv_2}{kT_{vib\,I}}\right)\right]^{-2}\left[1-\exp\left(-\frac{hv_3}{kT_{vib\,II}}\right)\right]^{-1}} \end{aligned} \tag{4}$$

Finally, these populations are used to compute the small signal gain using the relation [8]:

$$G_o = \left(\frac{\lambda^2}{4\pi\tau_{12}v_c}\right)\left(N_{001} - N_{100}\right)\left(\frac{a}{T}\right)\exp\left(-\frac{b}{T}\right) \tag{5}$$

Here, the cell-centered finite-volume method is used to discretize the governing equations. All equations are coupled and are discretized on the structured grids. The flux terms are treated using an AUSM+ (advection upstream splitting method) at cell faces [13]. The local time steps are used to speed up the computations and obtain the final steady state results.

For the shape optimization, the divergent nozzle geometry is defined such that it has five linear sections (as Fig. 1). So there are four points' lateral locations to be computed to achieve the maximum gain on the axis of the cavity.

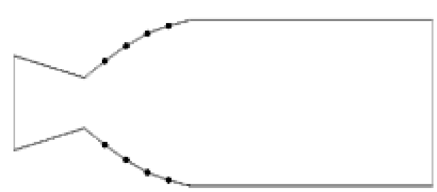

Fig. 1. Schematic geometry of the GDL for shape optimization

If P is assigned as an unknown parameters vector (here is lateral position of nodes) with N components, the objective of the chosen algorithm is to minimize the ordinary least squares norm which is defined below:

$$S(P) = \sum_{i=1}^{I} \left[Y_i - T_i(P)\right]^2 \tag{6}$$

Where Y is the vector of line variation of required small signal gain on an axis and $T(P)$ is the calculated gain variation. The iterative procedure of the conjugate gradient method for the minimization of the above norm $S(P)$ is given by:

$$P^{k+1} = P^k - \beta^k d^k \tag{7}$$

Where β^k is the search step size, d^k is the direction of descent and the superscript k is the number of iterations in the optimization loop. The direction of descent is a conjugation of the gradient direction: $\nabla S(P^k)$ and the direction of descent of the previous iteration, d^{k-1}. It is given as:

$$d^k = \nabla S\left(P^k\right) + \gamma^k d^{k-1} \tag{8}$$

The Polak-Ribiere [14] expression is used for determination of conjugation coefficient γ^k:

$$\gamma^k = \frac{\sum_{j=1}^{N}\left\{\left[\nabla S\left(P^k\right)\right]_j \left[\nabla S\left(P^k\right) - \nabla S\left(P^{k-1}\right)\right]_j\right\}}{\sum_{j=1}^{N}\left\{\left[\nabla S\left(P^k\right) - \nabla S\left(P^{k-1}\right)\right]_j^2\right\}} \tag{9}$$

Here, $\nabla S\left(P^k\right)_j$ is the j^{th} component of the gradient direction evaluated at iteration k. The expression for the gradient direction is obtained by differentiating equation (6) with respect to the unknown parameter vector P, the result is:

$$\nabla S\left(P^k\right) = -2\left(J^k\right)^T \left[Y - T\left(P^k\right)\right] \tag{10}$$

Where the sensitivity matrix J^k is defined by the following equation:

$$J(P) = \left(\frac{\partial T^T(P)}{\partial P}\right)^T = \begin{bmatrix} \frac{\partial T_1}{\partial P_1} & \frac{\partial T_1}{\partial P_2} & \frac{\partial T_1}{\partial P_3} & \cdots & \frac{\partial T_1}{\partial P_N} \\ \frac{\partial T_2}{\partial P_1} & \frac{\partial T_2}{\partial P_2} & \frac{\partial T_2}{\partial P_3} & \cdots & \frac{\partial T_2}{\partial P_N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial T_I}{\partial P_1} & \frac{\partial T_I}{\partial P_2} & \frac{\partial T_I}{\partial P_3} & \cdots & \frac{\partial T_I}{\partial P_N} \end{bmatrix} \tag{11}$$

With N is the total number of unknown parameters and I is the total number of axis nodes. The step size, β^k is obtained by minimizing the function $S(P^{k+1})$ with respect to β^k. The final expression after some mathematical operations is:

$$\beta^k = \frac{\sum_{i=1}^{I}\left\{\left[\left(\frac{\partial T_i}{\partial P^k}\right) d^k\right]\left[T_i\left(P^k\right) - Y_i\right]\right\}}{\sum_{i=1}^{I}\left\{\left[\left(\frac{\partial T_i}{\partial P^k}\right) d^k\right]^2\right\}} \tag{12}$$

Here, each component of the sensitivity matrix is defined by implementation of following forward difference:

$$J_{ij} = \frac{T_i\left(P_1, P_2, \ldots, P_j + \varepsilon P_j, \ldots, P_N\right) - T_i\left(P_1, P_2, \ldots, P_j, \ldots, P_N\right)}{\varepsilon P_j} \tag{13}$$

The iterative procedure is stopped when the following criterion is satisfied:

$$S\left(P^{k+1}\right) < \varepsilon' \tag{14}$$

Here, ε' is set 0.15 for convergence criteria. This value is just selected from the numerical process where the variation of maximum gain apparently vanishes.

3. RESULTS AND DISCUSSION

At first, to show the validity of this code for the considered problem, two different GDLs are considered. At first, the CO_2-N_2-H_2O flow of Ref. [3] is solved in convergent-divergent nozzle geometry to compare the distribution of the vibrational temperature as shown in Fig. 2. Then, the N_2O-N_2-He gas dynamic laser of Ref. [8] is calculated to compare the small signal gain distribution as illustrated in Fig. 3. The results show the accuracy of the present numerical simulation program.

After this validation, the optimization is considered. The considered gas dynamic laser is the converging-diverging nozzle which is continued by the constant area chamber as an optical cavity. The GDL which is studied here is N_2O-N_2-He laser with geometry and flow properties as shown schematically in Fig. 4. The required small signal gain is defined equal to unity in the optimization procedure and the iterative algorithm is continued to minimize *S*, Eq. (14).

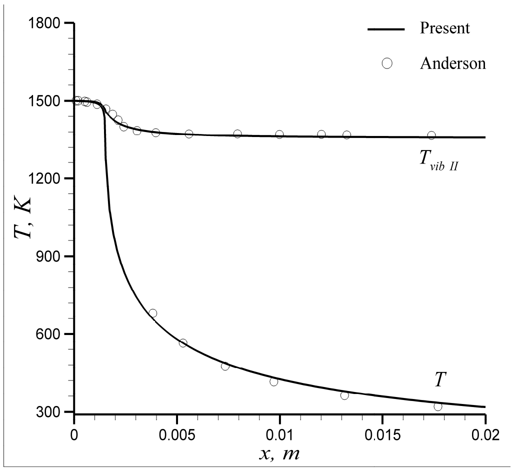

Fig. 2. Vibrational and translational temperature distribution

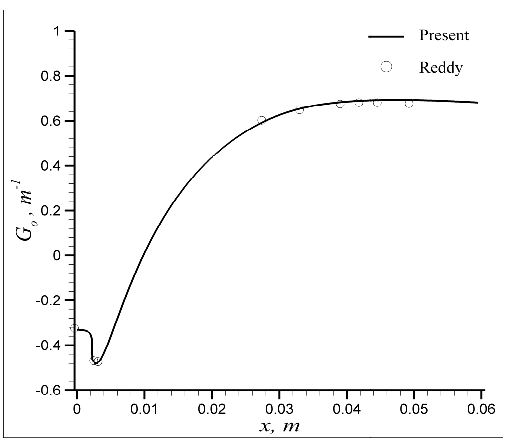

Fig. 3. Small signal gain distribution

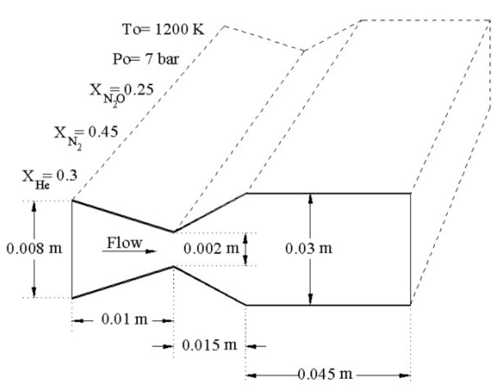

Fig. 4. Geometry and flow properties of the considered GDL

At first, the flow field is simulated for the two different divergent geometries, 1: linear and 2: quarter of cosine-function. Then, the optimization process is utilized to find the optimum divergent geometry for the maximum small signal gain in the resonator.

Variation of the *S* during the optimization process is shown in Fig. 5. The optimum geometry and the gain contours are shown in Fig. 6. The distribution of the small signal gain on the cavity's axis is illustrated in Fig. 7. The results show that the maximum gain is increased about 9 and 15 percent in comparison with the linear and cosine geometry, respectively. It should be noticed that the population inversion is a nonlinear function of all flow parameters and it is not possible to maximize the laser gain just using the optimum nozzle from the pure gas dynamic viewpoint and the full governing equations should be used in the optimization procedure, which has been done here.

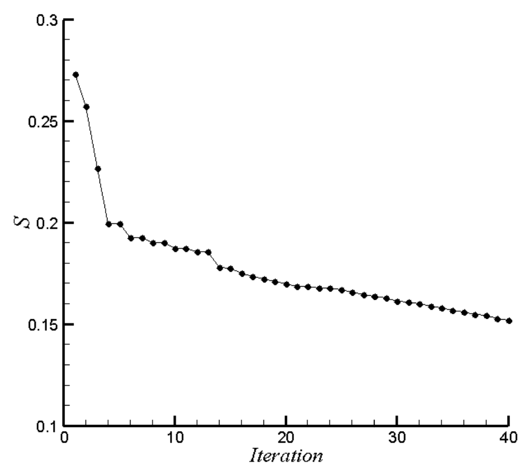

Fig. 5. Optimization process

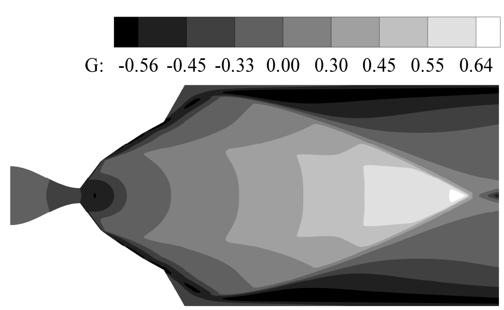

Fig. 6. The gain contours in the optimum geometry

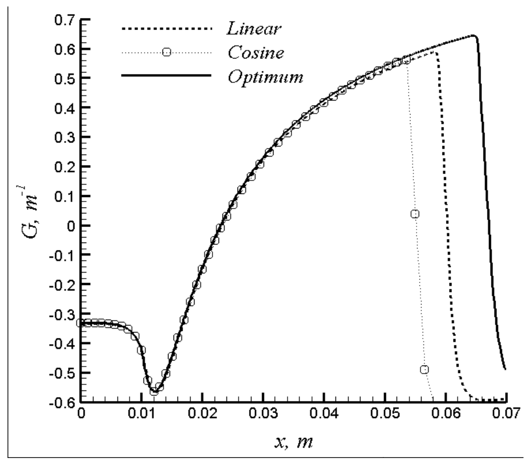

Fig. 7. Gain distribution on the axis

4. CONCLUSION

The shape optimization of a divergent nozzle of the gas dynamic laser is performed here using two-dimensional simulation of the flow field beside the conjugate gradient method in which the finite difference approximation approach is used to calculate the coefficient of the sensitivity matrix of the optimization procedure. The maximum small signal gain occurs in the cavity's centerline and it depends on the supersonic flow patterns as well as the flow field properties. Therefore, for the fixed inlet flow properties and the throat and exit dimensions, the laser gain can be varied by changing the divergent nozzle's geometry. The results show that the maximum gain may be increased about 9 percent using the optimum nozzle geometry in comparison with the simple linear-geometry divergent nozzle.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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