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Numerical Solution of Fourth Order Linear Differential Equations by Adomian Decomposition Method

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Authors' contributions

This work was carried out with joint efforts of all authors. Author EUA designed the study and wrote the manuscript supervised and edited by author FOO. Author AT gave the analytical solution to all sample problems used in the article.

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Abstract

In this paper, we state the general Adomian Decomposition Method (ADM) for Fourth order linear differential equations. And applied it to obtain analytic solution in a rapidly convergent series to this class of equations. The concept of ADM was further applied to physical problems and the result showed excellent potentials of applying this method.

Keywords: Adomian decomposition method; fourth order linear differential equation.

1 Introduction

Application of fourth order linear differential equations occurs in various physical problems. Some of these problems describe certain phenomena related to theory of Elastic Stability. A classical fourth order differential equations arising in Beam-Column theory is a useful tool for modeling and studying naturally

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occurring phenomena. Such as determining when a uniform cross section beam may break, as well as predicting future outcomes. These models are function of time.

A general fourth order differential equation is given as

$$\phi^{(iv)}(t) = g(t, \phi(t), \dot{\phi}(t), \ddot{\phi}(t), \dddot{\phi}(t)) \tag{1}$$

With initial values given as

$$\phi(t) = \alpha_1, \quad \dot{\phi}(t) = \alpha_2, \quad \ddot{\phi}(t) = \alpha_3, \quad \dddot{\phi}(t) = \alpha_4$$

The ADM introduces the solution, $\phi(t)$ and the nonlinear function, $g(t,\phi)$ by the infinite series

$$\phi(t) = \sum_{n=0}^{\infty} \phi_n(t) \tag{2}$$

and

$$g(t,\phi) = \sum_{n=0}^{\infty} A_n \tag{3}$$

where the component in equation $\phi_n(t)$ in equation (2) is determined recurrently. A_n is the Adomian polynomial with algorithms given in [1].

2 The Concept of ADM

The theory of ADM has been given by many researchers, see [2-6,1,7]. The concept writes equation (1) as

$$D(\phi) = e \tag{4}$$

where D is a differential operator, ϕ and e are functions of t. Equation (4) is transform into operator form as

$$A\phi + B\phi + C\phi = e \tag{5}$$

where A is a linear term which is easily invertible, B is the remainder of the linear operator and C represent the nonlinear term. Solving for $A\phi$, we have

$$A\phi = e - B\phi - C\phi \tag{6}$$

Since A is invertible, an equivalent expression is given as

$$A^{-1}A\phi = A^{-1}e - A^{-1}B\phi - A^{-1}C\phi \tag{7}$$

where A^{-1}, in this paper, is a four-fold integral operator. Consequently, equation (7) becomes,

$$\phi = \alpha_1 + \alpha_2 t + \alpha_3 \frac{t^2}{2} + \alpha_4 \frac{t^3}{6} + A^{-1}e - A^{-1}B\phi - A^{-1}C\phi \qquad (8)$$

ϕ is decomposed into a series as given in equation (2) with ϕ_0 identified as the first five terms on then right hand side of equation (8). The nonlinear term which is decomposed into the Adomian polynomial is considered as zero in this paper. Consequently, we can write

$$\begin{aligned}
\phi_1 &= -A^{-1}B\phi_0 \\
\phi_2 &= -A^{-1}B\phi_1 \\
\phi_3 &= -A^{-1}B\phi_2 \\
&\vdots \\
\phi_{n+1} &= -A^{-1}B\phi_n
\end{aligned}$$

This series converges when the nth partial sum $\varphi_n = \sum_{i=0}^{n-1} \phi_i$ will be the approximate solution [8-11].

3 Results and Discussion

Example 1.

Consider

$$\phi^{(iv)} - 3\ddot{\phi} - 4\phi = 0, \qquad \phi(0) = 1, \quad \dot{\phi}(0) = \frac{1}{3}, \quad \ddot{\phi}(0) = 0, \quad \dddot{\phi}(0) = 0 \qquad (9)$$

The exact solution of equation (9) is

$$\phi = \frac{1}{12}\left(\frac{7}{5}e^{2t} + e^{-2t}\right) + \frac{4}{5}\left(\frac{1}{3}\sin t + \cos t\right) \qquad (10)$$

In series form equation (10) is given as

$$\phi = 1 + \frac{1}{3}t + \frac{1}{6}t^4 + \frac{1}{90}t^5 + \frac{1}{60}t^6 + \frac{1}{1260}t^7 + \frac{13}{10080}t^8 + \frac{13}{272160}t^9 + \frac{17}{302400}t^{10} + \ \ldots \qquad (11)$$

Applying equations (4) to (8) of ADM on equation (9), we obtain

$$\begin{aligned}
\phi_0 &= 1 + \frac{1}{3}t \\
\phi_1 &= \frac{1}{6}t^4 + \frac{1}{90}t^5 \\
\phi_2 &= \frac{1}{60}t^6 + \frac{1}{1260}t^7 + \frac{1}{2520}t^8 + \frac{1}{68040}t^9
\end{aligned}$$

Sum of ϕ_0, ϕ_1 and the first two terms of ϕ_2 is equivalent to the first six terms of equation (11) which is the exact solution of the differential equation (9). We further show the similarities in the two results in Figs. 1 and 2. In Fig. 2 we only considered $\phi = \sum_{n=0}^{5} \phi_n$.

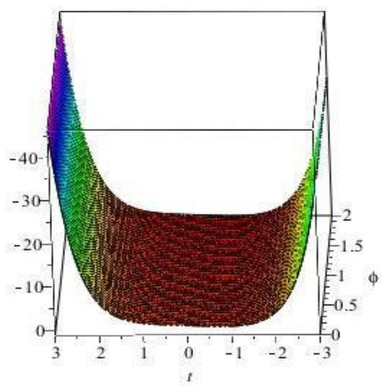

Fig. 1. Exact solution of Example 1

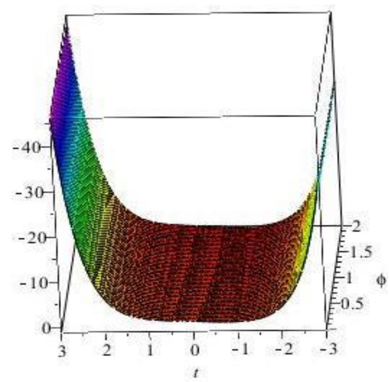

Fig. 2. ADM solution of Example 1

Example 2.

Consider

$$\phi^{(iv)} - 18\ddot{\phi} + 81\phi = 0, \qquad \phi(0) = 0, \quad \dot{\phi}(0) = -1, \quad \ddot{\phi}(0) = 0, \quad \dddot{\phi}(0) = 0 \tag{12}$$

The exact solution of equation (12) is

$$\phi = \frac{1}{4}\left[e^{-3t}(1+t) + e^{3t}(1-t)\right] \tag{13}$$

In series form equation (13) is given as

$$\phi = -t + \frac{27}{40}t^5 + \frac{81}{280}t^7 + \frac{243}{4480}t^9 + \frac{729}{123200}t^{11} + \frac{2187}{5125120}t^{13} + \frac{19683}{896896000}t^{15} + \ \ldots \tag{14}$$

Applying also equations (4) to (8) of ADM on equation (12), we obtain

$$\phi_0 = -t$$

$$\phi_1 = \frac{27}{40}t^5$$

$$\phi_2 = \frac{81}{280}t^7 - \frac{81}{4480}t^9$$

$$\phi_3 = \frac{81}{1120}t^9 - \frac{729}{123200}t^{11} + \frac{2187}{25625600}t^{13}$$

$$\phi_4 = \frac{729}{61600}t^{11} - \frac{6561}{6406400}t^{13} + \frac{19683}{896896000}t^{15} - \frac{59049}{4879114240\ 00}t^{17}$$

Approximating $\phi = \sum_{n=0}^{5} \phi_n$ we have

$$\phi = -t + \frac{27}{40}t^5 + \frac{81}{280}t^7 + \frac{243}{4480}t^9 + \frac{729}{123200}t^{11} + \frac{2187}{5125120}t^{13} - \frac{6561}{68992000}t^{15} + \ \ldots \quad (15)$$

Comparing equations (14) and (15) we see that all terms are the same except for the last term on the right hand side of each equation. So further evaluation of ϕ_n gives better accuracy. The similarities between the two solutions are shown in Figs. 3 and 4.

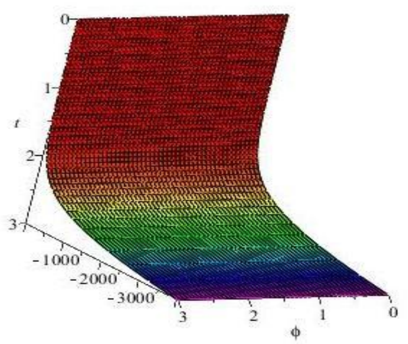

Fig. 3. Exact solution of Example 2

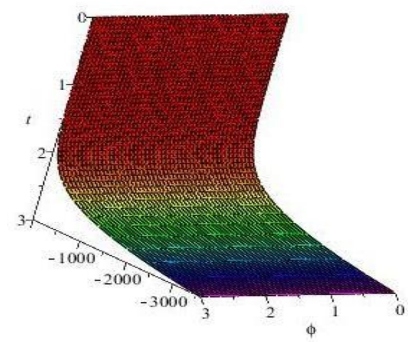

Fig. 4. ADM solution of Example 2

Example 3.

Consider

$$\phi^{(iv)} - 10\ddot{\phi} + 9\phi = 0, \qquad \phi(0) = 5, \ \ \dot{\phi}(0) = -1, \ \ \ddot{\phi}(0) = 21, \ \ \dddot{\phi}(0) = -49 \quad (16)$$

The exact solution of equation (9) is

$$\phi = 2e^{-3t} - e^{-t} + 4e^{t} \quad (17)$$

The series form equation (17) is given as,

$$\phi = 5 - t + \frac{21}{2}t^2 - \frac{49}{6}t^3 + \frac{55}{8}t^4 - \frac{481}{120}t^5 + \frac{487}{240}t^6 - \frac{4369}{5040}t^7 + \frac{125}{384}t^8 - \frac{5623}{51840}t^9 + \ \ldots (18)$$

Similarly, applying equations (4) to (8) of ADM on equation (16), we obtain

$$\phi_0 = 5 - t + \frac{21}{2}t^2 - \frac{49}{6}t^3$$

$$\phi_1 = \frac{55}{8}t^4 - \frac{481}{120}t^5 - \frac{21}{80}t^6 + \frac{7}{80}t^7$$

$$\phi_2 = \frac{55}{24}t^6 - \frac{481}{504}t^7 - \frac{75}{896}t^8 + \frac{971}{40320}t^9 + \frac{3}{6400}t^{10} - \frac{7}{70400}t^{11}$$

Continuing in this order, we have

$$\sum_{n=0}^{5}\phi_n = 5 - t + \frac{21}{2}t^2 - \frac{49}{6}t^3 + \frac{55}{8}t^4 - \frac{481}{120}t^5 + \frac{487}{240}t^6 - \frac{4369}{5040}t^7 + \frac{125}{384}t^8 + \Re \quad (19)$$

where

$$\Re = -\frac{5623}{51840}t^9 + \frac{39367}{1209600}t^{10} - \frac{354289}{39916800}t^{11} + \quad . \quad . \quad .$$

The first ten terms of equation (19) are the same those of equation (18) which is the exact solution of the given differential equation (16). The solutions of the exact method and ADM of $\phi = \sum_{n=0}^{5}\phi_n$ are shown in Figs. 5 and 6.

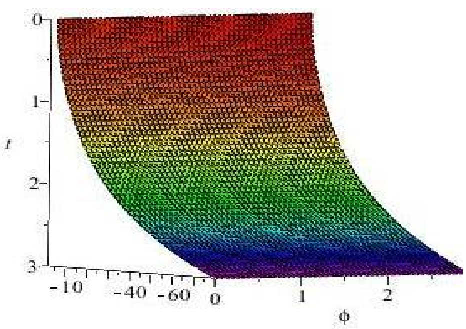

Fig. 5. Exact solution of Example 3

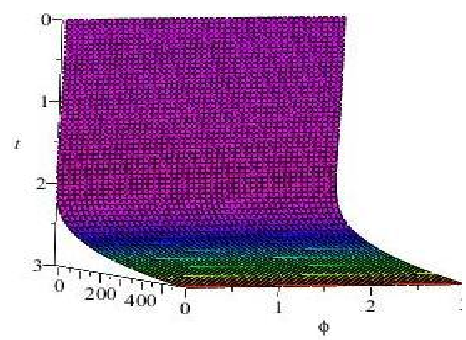

Fig. 6. ADM solution of Example 3

The nature of the curve in Fig. 6 is as a result of considering only a finite term of the series given in equation (19) as compared to an infinite series of equation (17).

Example 4.

Consider

$$\phi^{(iv)} + 32\ddot{\phi} + 256\phi = 0, \qquad \phi(0) = 0, \quad \dot{\phi}(0) = 1, \quad \ddot{\phi}(0) = 0, \quad \dddot{\phi}(0) = 1 \quad (20)$$

The exact solution of equation (20) is

$$\phi = \frac{49}{128}\sin 4t - \frac{17}{32}t\cos 4t \quad (21)$$

The series form equation (21) is given as

$$\phi = t + \frac{1}{6}t^3 - \frac{12}{5}t^5 + \frac{16}{9}t^7 - \frac{1664}{2835}t^9 + \frac{5888}{51975}t^{11} - \ ... \quad (22)$$

Similarly, applying equations (4) to (8) of ADM on equation (20), we obtain

$$\phi_0 = t + \frac{1}{6}t^3$$

$$\phi_1 = -\frac{12}{5}t^5 - \frac{16}{315}t^7$$

$$\phi_2 = \frac{64}{35}t^7 + \frac{128}{567}t^9 + \frac{256}{155925}t^{11}$$

Continuing in this order and summing the series we have equation (22) which is the exact solution of the given differential equation (20). The correlation of the exact solution and ADM solution of the given problem is given in Figs. 7 and 8.

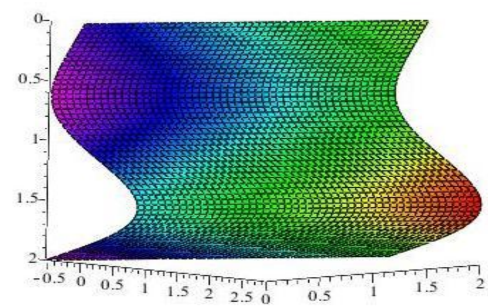

Fig. 7. Exact solution of Example 4

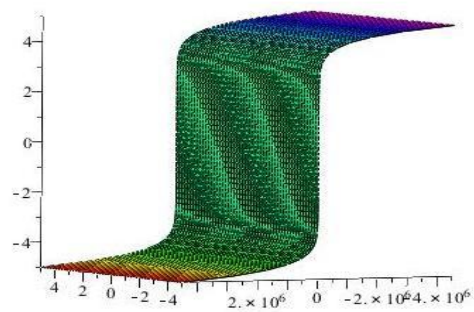

Fig. 8. ADM solution of Example 4

4 Conclusion

In this paper, we have successfully applied Adomian decomposition method to find numerical solution in fast convergent series to fourth order linear differential equations. We gave an introduction to physical areas where fourth order differential equations are used in real life situations. After the introduction of ADM, we gave the general concept of this method in fourth order differential equations. Four test problems were used to validate the concept and result showed great potential of this method event when finite terms of the series was considered in each case. We further demonstrated the reliability of this method in Figs. 1 to 8.

Competing Interests

Authors have declared that no competing interests exist.

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