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Existence and Solution of Wind Speed Equation of a Point in Air

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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Short Research Article

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Abstract

This paper proves the existence of solution of wind speed equation of a point in air by fixed point theorem of periodic g-contrastive mapping. Further more, the solution of the wind speed equation is obtained by method of separating variables.

Keywords: Banach contraction mapping theorem (or Banach fixed point theorem); periodic g-contractive mapping theorem; method of separating variables; wind speed equation.

1 Introduction

In [1], we obtain wind speed equation and approximate wind speed equation (neglected the term of weight of air in wind speed equation) of a point in air on relationship between wind speed **u**, pressure **p** and temperature **T**. In this paper, we prove the existence of solution of wind speed equation by fixed point theorem of periodic g-contrastive mapping. Further more, we find the solution of wind speed equation by method of separating variables.

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Fixed point theory is one of the active mathematical branch. Fixed Point Theory play an important role in proving the existence of solution of algebra equation, differential equation and integral equation, etc.

A lot of researches on contractive mapping have been done [2]. Among these works, the Banach contraction mapping theorem (or Banach fixed point theorem) [3] is the most famous, important and widely used theorem. However, the latter developed g-contractive mapping theorems [4-7] have improved and over the Banach fixed point theorem in two aspects: looser constraint and wider use for the latter. The Banach fixed point theorem needs the contractive ratio less than a constant lesser than 1. While, the g-contractive mapping theorem [4] allows some of contractive ratios equal to or greater than 1, if the geometric mean of the contractive ratio is less than a constant lesser than 1. Further more, the g-contraction mapping theorem suits for periodic mapping with k-related fixed points, while the Banach fixed point theorem only suits for single mapping and get one fixed point.

The idea of g-contraction mapping was originally proposed by the author in 1983 [4]. In it, the author first pointed out the restriction of Banach fixed point theorem and then proved a more general g-contractive mapping theorem with lesser restriction and has a unique set of k related fixed points suited for describing periodic phenomenon. The concept of g-contractive mapping issued by an iteration method with the quickest convergence [8], which had been used by others [9] and it showed that "this method converges quicker than the Lamweber's method, and agrees with experimental data well". In it , different rules (functions or mappings) are used for different steps of the iteration process, so as to obtain the quickest convergence, while in Banach's iteration only one mapping is used in all steps of iteration process.

The g-contraction mapping theorems have applications in the fields of mechanics [5], prey-predator system, stock price [6,10,11], and the analysis of equilibrium state [7]. A new limit of periodic function and periodic mapping (g-contractive mapping) at infinity is defined [12]. Here, the application of g-contractive mapping theorem to wind speed equation is added. Related problem with fractional order can be referred to [13]. Some important applications can be seen in [14–17].

2 Proof of the Existence of Solution of the Wind Speed Equation (2-1)

2.1 Transferring the wind speed equation to a cycling iterations

At first, we transfer (2-1) into a set of cycling iterations.

$$\frac{\partial u}{\partial t} + g = k\frac{\partial T}{\partial s}, \tag{2-1}$$

$$\frac{\partial u}{\partial t} = k\frac{\partial T}{\partial s}, \tag{2-1$_a$}$$

Where (2-1) is called the "wind speed equation", $(2\text{-}1)_a$ is called the approximate "wind speed equation". $u = u(s,t)$ is the wind speed, $T = T(s,t)$ is the temperature, u and T are unknown functions. s is the trace of the point in air, t is the time, g and k are constants.

Let

$$u_n = u_{n-1} + g = F(T_n) = k\frac{\partial}{\partial s} T_n, \quad n \in N = \{1,2,\dots\}, \ u_0 = u_0(s,t), \tag{2-2}$$

$$T_{n+1} = H(u_n) = \frac{\partial}{\partial t} u_n, \tag{2-3}$$

Where u_0 is an arbitrary given function.

Substituting (2-3) into (2-2), and (2-2) into (2-3), we have

$$u_{n+1} = F(T_{n+1}) = F[H(u_n)] = F°H\, u_n = P_1\, u_n, \quad (2\text{-}4)$$

$$T_{n+1} = H(u_n) = H[F(T_n)] = H°F\, T_n = P_2\, T_n, \quad (2\text{-}5)$$

Where partial differential operator $F = k\,\frac{\partial}{\partial s}$, $H = \frac{\partial}{\partial t}$, periodic operator $P_1 = F°H$, $P_2 = H°F$,

$F°H$ = composition of mapping F and mapping H.

Now we form two related series $\{u_n\}$and $\{T_n\}$. We need to prove these series convergent by periodic g-contractive mapping theorem.

2.2 Periodic g-contractive mapping theorem

Definition 1. The ratio of

$\mathbf{r_i^u} = \|\mathbf{u_{i+1}} - \mathbf{u_i}\|/\|\mathbf{u_i} - \mathbf{u_{i-1}}\|$, and $\mathbf{r_i^T} = \|\mathbf{T_{i+1}} - \mathbf{T_i}\|/\|\mathbf{T_i} - \mathbf{T_{i-1}}\|$,**for i≥1,** is called **contraction ratio** of u_i and T_i, respectively. Where $\|x\|$ is the norm of x.

Here, we choice supper norm as the distance on $\|u_{i+1} - u_i\|$, and $\|T_{i+1} - T_i\|$.

$\|u\| = \max|u(s,t)| = R_1 > 1$, $\|T\| = \max|T(s,t)| = R_2 > 1$. They are bounded.

Definition 2, A sequential composite mapping is called a **"g-contractive mapping"**, if for each $i \in N$, there exists a constant G,such that the geometric mean contraction ratio G_i satisfies (2-6) and (2-7).

$$0 \le G_1 = (r_1^u r_2^u \dots r_i^u)^{1/i} < G < 1, \quad (2\text{-}6)$$

$$0 \le G_2 = (r_1^T r_2^T \dots r_i^T)^{1/i} < G < 1, \quad (2\text{-}7)$$

Definition 3, A sequential mappings $\{u_n\}$ or $\{T_n\}$, related by (2-4), (2-5) is called a **periodic mapping with period k = 2.**

Periodic g-contractive mapping theorem [5]:

Any periodic g-contractive mapping of complete nonempty metric space M has a unique set of k related fixed points in M. That is $\exists x_j^* \in M$, such that $P_j\, x_j^* = x_j^*$, $x_{k+j}^* = x_j^*$, $U_j x_j^* = x_{j+1}^*$.

Now, in our case, j = 1,2. k = 2, $x_1^* = u^*$, $x_2^* = T^*$, $P_1 x_1^* = P_1 u^* = u^*$, $P_2 x_2^* = P_2 T^* = T^*$, $U_2 T^* = FT^* = u^*$, $U_1 u^* = Hu^* = T^*$.

2.3 Constructing G such that (2-6), (2-7) hold

By (2-6), (2-7), we have:

$$\|u_{i+1} - u_i\| \le G_1^i \|u_1 - u_0\| \le G^i \|u_1 - u_0\|, \quad (2\text{-}8)$$

$$\|T_{i+1} - T_i\| \le G_2^i \|T_1 - T_0\| \le G^i \|T_1 - T_0\|, \quad (2\text{-}9)$$

We choose $G_1 = 1/R_1 < 1$, $G_2 = 1/R_2, < 1$, $S = 2R$, $R = \max\{R_1, R_2\}$, $G = 1/S$, then $G_1 < G < 1$, and $G_2 < G < 1$.

According to periodic g-contractive mapping theorem, sequence $\{u_n\}$ and $\{T_n\}$ converge.i.e., $u_{n+1} = u_n = u^*$, $T_{n+1} = T_n = T^*$, $n \to \infty$. Substituting $u^* = u$, $T^* = T$ into (2-2), (2-3), we have

$$\frac{\partial u}{\partial t} + g = k\frac{\partial T}{\partial s}, \quad (2\text{-}1)$$

u^*, T^* is the solution of (2-1). □

3 Solution of the Wind Speed Equation by Method of Separating Variables

(2-1) shows the relation between u and T. There are choices on u and T. We can choose u or T as known (given) function. However, we do not know u or T actually in practice. In the following, we study the 1solution of wind speed equation (2-1), where u and T are unknown functions to be determined.

Let

$$u_1(s,t) = u(s,t) + gt, \quad (3\text{-}1)$$

Then, (2-1) reduces to

$$\frac{\partial u_1(s,t)}{\partial t} = k\frac{\partial T(s,t)}{\partial s}, \quad (3\text{-}2)$$

Suppose that the variables s and t of $u_1(s,t)$ and $T(s,t)$ can be separated. Let

$$u_1(s,t) = a(s)b(t), \quad (3\text{-}3)$$

$$T(s,t) = c(s)d(t), \quad (3\text{-}4)$$

Substituting (3-3) and (3-4) into (3-2), we have

$$\frac{db(t)}{dt}a(s) = k\frac{dc(s)}{ds}d(t), \quad (3\text{-}5)$$

Separating variables, we have

$$\frac{db(t)}{d(t)dt} = k\frac{dc(s)}{a(s)ds} \quad (3\text{-}6)$$

In order to make the solution simple, let $d(t) = b(t)$, and $a(s) = k\,c(s)$, then (3-6) becomes:

$$\frac{db(t)}{b(t)dt} = \frac{da(s)}{a(s)ds} = \text{const}, \quad (3\text{-}7)$$

If (3-7) holds, then, each side must be equal to a constant. i.e.,

$$\frac{db(t)}{b(t)dt} = A(s), \quad (3\text{-}8)$$

$$\frac{da(s)}{a(s)ds} = B(t), \quad (3\text{-}9)$$

By integrating both hand sides of (3-8), (3-9), we get:

$$b(t) = \exp[A(s)t] = d(t), \quad (3\text{-}10)$$

$$a(s) = \exp[B(t)s] = kc(s) \ , \tag{3-11}$$

where $B(t), A(s)$ are unknown functions to be determined by known conditions. A simple choice is $A(s) = A = \text{const.}\ B(t) = B = \text{const.}$

Substituting (3-10), (3-11) into (3-3), (3-4), we have:

$$u_1(s,t) = \exp[A\,t + B\,s] = kT(s,t), \tag{3-12}$$

Where constants A, B, k can be determined by recorded data.

Substituting (3-12) into (3-1), we have:

$$u(s,t) = \exp[A\,t + B\,s] - \ gt, \tag{3-13}$$

$$T(s,t) = \frac{1}{k}\exp[A\,t + B\,s], \tag{3-14}$$

(3-13) and (3-14) is the solution of wind speed equation (2-1).

The solution (3-12) of approximate wind speed equation $(2\text{-}1)_a$, shows that **wind speed** is proportional to **temperature**. They relate closely with each other. If the temperature changes rapidly then the wind speed is also changed rapidly.

4 Conclusion

The wind speed equation (2-1) has two unknown functions u and T . To prove the existence of solution of (2-1), at first, we transfer (2-1) into two cycling iteration series $\{u_n\}$ and $\{T_n\}$, then, we prove $\{u_n\}$ and $\{T_n\}$ convergent to two related fixed points u^* and T^* by the periodic g-contractive mapping theorem. Obviously, the Banach fixed point theorem can not be used for problems of fixed points more than one, since it just can prove a series convergent to one fixed point. While the periodic g-contractive mapping theorem can be used for problems with k (k>1) related fixed points. This again shows that the periodic g-contractive mapping theorem has wilder use than the Banach fixed point theorem.

Further more, the solution of wind speed equation (2-1) is found by method of separating variables. The solution of approximate wind speed equation (3-12) shows that the wind speed is proportional to the temperature.

Competing Interests

Author has declared that no competing interests exist.

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