Advances in Research
9(4): 1-21, 2017; Article no.AIR.33186
ISSN: 2348-0394, NLM ID: 101666096

SCIENCEDOMAIN ***international***
www.sciencedomain.org

Nonlinear H∞ Guidance Design for Missile against Maneuvering Target

Jacqueline Munene Makena[1*] and Solomon Omwoma[2*]

[1]Department of Aerospace and Aviation Engineering, Technical University of Kenya, Haile Salassie Avenue, P.O.Box 52428, Nairobi, 00200, Kenya.
[2]Department of Physical Sciences, Jaramogi Oginga Odinga University of Science and Technology, P.O.Box 210-40601, Bondo, Kenya.

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AIR/2017/33186
Editor(s):
(1) Emre Dil, Department of Physics, Sinop University, Turkey.
Reviewers:
(1) G. Y. Sheu, Chang-Jung Christian University, Tainan, Taiwan.
(2) Francisco Bulnes, Technological Institute of High Studies of Chalco, Mexico.
Complete Peer review History: http://www.sciencedomain.org/review-history/18859

Original Research Article

Received 1st April 2017
Accepted 27th April 2017
Published 1st May 2017

ABSTRACT

A new guidance law is derived for missile against maneuvering target by adopting nonlinear H_∞ control theory. The guidance law is based on three dimension (3D) nonlinear kinematics described by modified polar coordinate (MPC). In MPC, only three differential equations are used to describe the 3D relative motion between missile and target. The new guidance law is designed by solving the Hamilton-Jacobi-Isaacs (HJI) equation by simultaneous policy update algorithm (SPUA). In SPUA a sequence of Lyapunov function equations (LFEs) are used in direct successive approximation of HJI equation resulting to one interactive loop instead of two loops. Gelerkin's method is used to solve the LFEs and to develop Galerkin-based SPUA. Computationally efficient SPUA (CESPUA) based on Galerkin's method was subsequently used to solve the LFE in each iterative loop of SPUA. The proposed guidance law does not require the information of the target accelerations and avoids control of relative velocity in the direction of line of sight. In comparison to sliding mode guidance law, the developed law utilizes less control energy, has smaller interception time, and offers better tracking performance against uncertain target accelerations.

**Corresponding author: E-mail: solomwoma@yahoo.com, jacquena88@gmail.com;*

Keywords: Nonlinear H_∞ control theory; modified polar coordinate; Hamilton-Jacobi-Isaacs equation; simultaneous policy update algorithm; Lyapunov function equation; Galerkin's method.

1. INTRODUCTION

Proportional navigation (PN) guidance law is widely used in practical applications because of its ease in implementation and simple form [1]. PN provides optimal performance for non-maneuvering targets. In an event where the target's acceleration can be soundly estimated continuously during the flight, the augmented proportional navigation (APN) is used [1]. For targets with high maneuver, an optimal guidance law and nonlinear approximation for capture of the decelerating target [2] and accelerating target [3] have been studied. A comprehensive literature analysis on optimal guidance laws (OGLs) is evaluated in [4,5]. Both theory and extensive simulation studies indicate the performance of classical guidance laws and OGLs deteriorate for unpredictable maneuvering targets because these two guidance laws operate on perfect knowledge of target maneuvers.

To handle system uncertainties and exogenous disturbances, advanced control techniques, have been proposed in recent years such as, μ-synthesis control, H_∞ control and sliding-mode control etc. In sliding-mode control, Zhou et al. [6] derived an adaptive sliding-mode guidance law using linearized equations, Babu et al. [7], using the sliding surface of zero line-of-sight rate based on the Lyapunov method, proposed a guidance law for highly maneuvering targets, Moon et al. [8] studied the missile guidance law by variable-control structure. Brierly and Longchamp [9] applied a sliding mode control to a nonlinear system representing an air-to-air missile target engagement scenario. A sliding mode terminal guidance of missiles against maneuvering targets in the three-dimensional space is designed considering non-decoupling three-dimensional engagement geometry in [10]. Even though the sliding-mode control meets robust stability requirements, it cannot design the system performance boundedness.

Several papers have been proposed adopting nonlinear H_∞ control to solve the guidance problem. Liao et al. [11] derived a three-dimensional law based on nonlinear robust H_∞ filtering and input-to-state stability (ISS) for interception of maneuvering targets in the presence of input saturation. Yang and Chen [12,13] derived a nonlinear H_∞ guidance law for a worst-case target with an H_∞ evasive strategy. In [12,13] the H_∞ Guidance law includes accelerations in both directions along the line of sight (LOS) and perpendicular to the LOS. However, acceleration control in the direction of along the LOS is very difficult and in most cases the missile's acceleration is just controlled in the direction perpendicular to the missile body. Chen et al. [14] derived a new nonlinear fuzzy H_∞ guidance law with a saturation of actuators against maneuvering targets. The model was acquired by interpolating numerous linearized systems at different operating points through fuzzy certainty functions. Then, based on this, they constructed a H_∞ control to handle the approximation error and external disturbances. The constructed nonlinear fuzzy H_∞ guidance law was with control constraints against target maneuvers without solving the complicated HJI equation. It is a known fact that the approximation error between the fuzzy model and the true one is difficult to obtain exactly [15]. They assumed that the model error was included in a function called upper bound. But, if the approximation error cannot be evaluated in advance, their proposed fuzzy H_∞ guidance law only guarantees robust performance of the missile–target pursuit dynamics approximated by the linear fuzzy model. The steady state error always exists.

There are two major setbacks in the designing H_∞ guidance law for homing missile at present. One is how to easily find the solution to HJI equation, while the other is how to avoid control of the relative velocity between missile and target. In this paper, we present a nonlinear H_∞ guidance law for a homing missile against a maneuvering target. The guidance law is based on three dimension (3D) nonlinear kinematics described by modified polar coordinate (MPC). In MPC, only three differential equations are used to describe the three-dimension relative motion between missile and target instead of six differential equations in the polar coordinates [16]. In addition, control of acceleration along the LOS is avoided. The HJI equation is solved by Galerkin Simultaneous policy update algorithm. This is the first time, to the best of our knowledge; SPUA technique has been applied to solve the nonlinear missile guidance problem.

2. NONLINEAR MISSILE- KINEMATICS MODEL

In this study, a 3D engagement geometry is considered as shown in Fig. 1. The relative position vector (LOS) $\boldsymbol{r}$ between the missile and the target is

$$\boldsymbol{r} = r_T - r_M = re_r \quad (1)$$

By taking the first-order derivative and second-order derivative of equation (1) with respect to time, we obtained the relative dynamics between target and missile written in the form of,

$$\dot{\boldsymbol{r}} = \dot{r}e_r + r\dot{e}_r = \boldsymbol{V_T} - \boldsymbol{V_M} \quad (2)$$

$$\ddot{\boldsymbol{r}} = \ddot{r}e_r + 2\dot{r}\dot{e}_r + r\ddot{e}_r = \boldsymbol{a_T} - \boldsymbol{a_M} \quad (3)$$

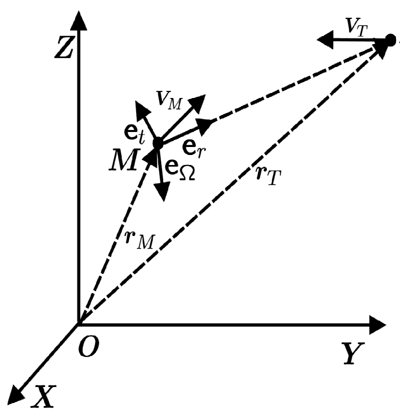

Fig. 1. Three-dimension relative motion model in MPC

Where

1. r_T and r_M are the position of target and missile, respectively in an inertial coordinates OXYZ.
2. r is the range between the missile and the target.
3. e_r is the unit vector in the direction of LOS.
4. $\boldsymbol{V_T}$, $\boldsymbol{a_T}$ and $\boldsymbol{V_M}$, $\boldsymbol{a_M}$ are target and missile velocity and acceleration vectors, respectively.

Let Ω be the angular velocity vector of the LOS. From [15] we have $\dot{e}_r = \Omega \times e_r$ and $\Omega = e_r \times \dot{e}_r$. Then the unit vectors are defined as, $e_t = \dot{e}_r/\|\Omega\|$ and $e_\Omega = \Omega/\|\Omega\|$. e_t, and e_Ω are the unit vectors in the direction of $\dot{e}_r$ and Ω respectively. $(e_r\ e_t, e_\Omega)$ Constructs an orthogonal coordinate for three-dimension space, which is the MPC compared with classical polar coordinate $(e_r\ e_\theta, e_\phi)$ [13]. However, we should note that MPC and classical polar coordinate cannot be mutual transformed completely [16]. Accelerations of the missile and the target can be expressed in MPC $(e_r\ e_t, e_\Omega)$ as

$$\boldsymbol{a_M} = a_{M_r}e_r + a_{M_t}e_t + a_{M_\Omega}e_\Omega \quad (4a)$$

$$\boldsymbol{a_T} = a_{T_r}e_r + a_{T_t}e_t + a_{T_\Omega}e_\Omega \quad (4b)$$

Assume state variables

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \dot{r} \\ r\|\Omega\| \\ r \end{bmatrix}$$

From literatures [17,18], the system equations of relative kinematics between the missile and the target are

$$\frac{dx_1}{dt} = \frac{x_2^2}{x_3} + a_{Tr} - a_{Mr}, \quad x_1(t_0) = x_{10} \quad (5a)$$

$$\frac{dx_2}{dt} = -\frac{x_1 x_2}{x_3} + a_{Tt} - a_{Mt}, \quad x_2(t_0) = x_{20} \quad (5b)$$

$$\frac{dx_3}{dt} = x_1 \quad , \quad x_3(t_0) = x_{30} \quad (5c)$$

Comparing the system equations (5) with equations used to describe missile-target kinematics in [13], we can conclude that the system equations in MPC are simpler and more concise compared to those in classical polar coordinates (PC).

3. GUIDANCE LAW DESIGN BASED ON $\mathbf{H}_\infty$ CONTROL

In the terminal phase of guidance, the gravity on missile and target are almost equal since the distance between the missile and target is much smaller than the radius of the earth. Generally, the acceleration in the direction of the LOS cannot be controlled. Theoretically, it is assumed that acceleration components along the LOS satisfy $a_{M_r} = a_{T_r} = g_r$ and components perpendicular to the LOS satisfy $a_{M_t} = \hat{a}_{M_t} + g_t$, $a_{T_t} = \hat{a}_{T_t} + g_t$. Thus system equations (10) can be rewritten as

$$\frac{dx_1}{dt} = \frac{x_2^2}{x_3} \quad , \quad x_1(t_0) = x_{10} \quad (6a)$$

$$\frac{dx_2}{dt} = -\frac{x_1 x_2}{x_3} + \hat{a}_{Tt} - \hat{a}_{Mt}\ , \ x_2(t_0) = x_{20} \quad (6b)$$

$$\frac{dx_3}{dt} = x_1\ , \qquad x_3(t_0) = x_{30} \quad (6c)$$

We rewrite (5) as (6) in the standard forms of system [19],

$$\dot{x} = f(x) + g_1(x)w + g_2(x)u \quad (7a)$$

$$z(x) = \begin{bmatrix} h_1(x) \\ u \end{bmatrix} \quad (7b)$$

Where x is the state vector, $x \in R^n$ with control input $u \in R^m$ and subjected to a set of exogenous input disturbance variable $w \in R^r$ and $z \in R^s$ is penalized output. The terms $f(x), g_1(x), g_2(x)$ and $h_1(x)$ are smooth (Differentiable for all degrees of differentiation i.e. C^∞) mapping defined in the neighbourhood of the region in R^n and $f(0) = h_1(0) = 0$ .The penalty function is chosen as $h_1(x) = x_2^2/x_3 = r\|\Omega\|^2$, which is a weighting function with respect to the turning rate of the LOS $\|\Omega\| = x_2/x_3$. For this guidance problem, the functions and variables are given as,

$$f(x) = \begin{bmatrix} \frac{x_2^2}{x_3} \\ -\frac{x_1 x_2}{x_3} \\ x_1 \end{bmatrix}\ , \ g_1(x) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\ ,$$

$$g_2(x) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}\ , \ h_1(x) = \frac{x_2^2}{x_3}\ , \ u(x) = \hat{a}_{Mt}\ ,$$

$$w(x) = \hat{a}_{Tt}$$

Substitute (7) into the Hamilton-Jacobi-Isaac inequality (8) [19],

$$\left(\frac{\partial V}{\partial x}\right) f + \frac{1}{2}\left(\frac{\partial V}{\partial x}\right)\left(\frac{1}{\gamma^2} g_1 g_1^T - g_2 g_2^T\right)\left(\frac{\partial V}{\partial x}\right)^T + \frac{1}{2} h_1^T h_1 \le 0 \quad (8)$$

The desired Hamilton-Jacobi-Isaac inequality for the guidance system is:

$$\frac{\partial V}{\partial x_1}\frac{x_2^2}{x_3} - \frac{\partial V}{\partial x_2}\frac{x_1 x_2}{x_3} + \frac{\partial V}{\partial x_3} x_1 + \frac{1}{2}\left(\frac{1}{\gamma^2} - 1\right)\left(\frac{\partial V}{\partial x_2}\right)^2 + \frac{1}{2}\frac{x_2^4}{x_3^2} \le 0 \quad (9)$$

The general solution of inequality (8) is very difficult to obtain, but a special solution can be found by successive Galerkin's approximation. A smooth positive definite solution $V(x)$ of inequality (8) will make states locally asymptotically stabilizing.

To design the terminal guidance law, it is required that the turning rate of the LOS goes to zero asymptotically; the relative velocity along the LOS converges to a negative value because its only then that the relative distance between the missile and target (r) decreases to zero. For the interception, it is sufficient that r becomes zero in an instance ($r(t_f) = 0$, where t_f is interception time) and there is no need for r to asymptotically converge to zero. Therefore, if the solution $V(x)$ is smooth, nonnegative, and positive definite with respect to the turning rate of the LOS, the above conditions can be satisfied for proper initial states.

4. NONLINEAR H_∞ CONTROL THEORY

In this section, we introduce the nonlinear control standard results for later use. Take into account a nonlinear state space system [17]:

$$\dot{x} = f(x) + g(x)w\ , \ w \in R^s \quad (10a)$$

$$z = h(x)\ , \ z \in R^s \quad (10b)$$

When control u is applied to the system, we consider the nonlinear system in equation (7). The purpose of control is to stabilize the closed loop and to attenuate the effect of disturbance input w to the output to be regulated z. The nonlinear H_∞ control problem of the system (7) is to evaluate a control u that makes $\mathcal{L}_2$ gain of system (10) equal or less than a positive constant γ^2 when initial states satisfy $x(t_0) = 0$, that is,

$$\int_{t_0}^{T} z^T(t) z(t) dt \le \gamma^2 \int_{t_0}^{T} w^T(t) w(t) dt\ , \ \forall w \in \mathcal{L}_2[t_0, T] \quad (11)$$

Since $\int_{t_0}^{T} w^T(t) w(t) dt$ is the energy input of the system (target's acceleration energy cost) and $\int_{t_0}^{T} z^T(t)\, z(t) dt$ is the energy output of the system (missile's weighting energy cost), then $\int_{t_0}^{T} z^T(t)\, z(t)dt \Big/ \int_{t_0}^{T} w^T(t) w(t)dt$ is known as the $\mathcal{L}_2$ gain of the system. The smallest γ satisfying equation (10) is called the H_∞- norm of the system. The solutions to H_∞ problem are related to Hamilton-Jacobi inequality. Consider the nonlinear system (7) with disturbances. Let $\gamma > 0$, suppose there exists a smooth solution $V(x) \geq 0$ to the Hamiltonian-Jacobi inequality (8). Where $x \neq 0$, $V(0) = 0$. As van der Schaft [19] shows, a sufficient condition for system (7) to have $\mathcal{L}_2$-gain less than or equal to $\gamma > 0$ is that:

$$u(x) = -\frac{1}{2} g_2^T(x) \left(\frac{\partial V}{\partial x}\right)^T \tag{12}$$

Once V has been derived, the H_∞ control law in equation (12) can be determined. Hence, solving the nonlinear H_∞ control problem is equivalent to finding positive function $V(x)$ satisfying the HJI equation (11). Nonlinear H_∞ control can guarantee that the closed-loop system (7a) is internally stable. This is verified by showing that if $V(x) > 0$ is a solution of the HJI then; $V(x)$ is a qualified Lyapunov function of system (7a). To be a Lyapunov function $V(x)$ must satisfy the condition $\dot{V}(x) \leq 0$. From the HJI equation (11), the properties of $V(x) > 0$ and $\dot{V}(x) \leq 0$ show that $V(x)$ is a qualified Lyapunov function, and hence the closed-loop system (7a) in the sense of Lyapunov is stable. To summarize, nonlinear H_∞ control theory [19] guarantees: (1) state boundedness (Lyapunov stability) and (2) performance boundedness: $\|z\|_2 \leq \gamma \|w\|_2$.

5. SOLUTION OF HJI VIA GALERKIN'S APPROXIMATION

In this section, the simultaneous policy update algorithm (SPUA) developed in [20,21] is used to solve the HJI equation. In SPUA the Lyapunov function equations are used directly to approximate the HJI equation. In this algorithm, the control and disturbance were updated simultaneously as shown in algorithm 1. The convergence of SPUA is verified in [20].

5.1 Simultaneous Policy Update Algorithm

Algorithm 1: given an initial function $V^{(0)} \in \mathbb{V}_0$.

Let $u^{(0)} = -\frac{1}{2} g_2^T \frac{\partial V^{(0)}}{\partial x}$ and

$w^{(0)} = \frac{1}{2\gamma^2} g_1^T \frac{\partial V^{(0)}}{\partial x}$ and $i = 0$

Solve for $V^{(i+1)}$ *from:*

$$\left(\frac{\partial V^{(i+1)}}{\partial x}\right)\left(f + g_1 w^{(i)} + g_2 u^{(i)}\right) + h^T h + \left\|u^{(i)}\right\|^2 - \gamma^2 \left\|w^{(i)}\right\|^2 = 0 \tag{13}$$

Update control and disturbances policies with

$$u^{(i)} = -\frac{1}{2} g_2^T \frac{\partial V^{(i+1)}}{\partial x} \tag{14}$$

$$w^{(i)} = \frac{1}{2\gamma^2} g_1^T \frac{\partial V^{(i+1)}}{\partial x} \tag{15}$$

Set $i = i + 1$. *If* $\left\|V^{(i)} - V^{(i-1)}\right\|_\Theta \leq \varepsilon$ *(*ε *is a small positive real number), stop and output* $V^{(i)}$ *as the solution of HJI equation (4). Else go back and solve for* $V^{(i+1)}$ *and continue.*

The solution of equation (13) converges to solution of HJI equation when *i* goes to infinity, as established in [22]. The essence of algorithm1 is to reduce the HJI equation to an infinite sequence of linear partial differential equation (13) which can be rewritten as;

$$\left(\frac{\partial V}{\partial x}\right)^T \left(f + g_1 w + g_2 u\right) + h^T h + \|u\|^2 - \gamma^2 \|w\|^2 = 0 \tag{16}$$

With a boundary condition V(0)=0 equation (16) Generalized-Hamilton-Jacobi-Isaacs (GHJI) equation which is a Lyapunov function [22]. In order to obtain an implementable algorithm we must approximate the solution of the GHJI equations such that the control $u^{(i)}$ can be practically implemented in feedback form. The GHJI is difficult to solve analytically. The challenge with algorithm 1 is that equation (13) is also difficult to solve. To approximate this equation, we employ a global Galerkin's approximate discussed in the following section.

5.2 Galerkin Approximation

Let the partial differential equation $\mathcal{A}(V) = 0$ with boundary conditions $V(0) = 0$. Galerkin method assumes that we can find a complete set of basis functions $\{\phi_j\}_{j=1}^{\infty}$ so that $\phi_j(0) = 0, \forall j$ and

$V(x) = \sum_{j=1}^{\infty} c_j \phi_j(x)$, where the sum is presumed to converge point-wise in some set Θ [22]. An approximation of *V(x)* is formed by truncating the series to $V(x) = \sum_{j=1}^{N} c_j \phi_j(x)$. The coefficients c_j are obtained by solving the algebraic equation

$$\int_{\Theta} \mathrm{A}\left(V_N\left(x\right)\right)\phi_l\left(x\right)dx = 0 \text{ Where } l = 1, \dots \dots, N \tag{17}$$

We use Galerkin's method to approximate GHJI equation and show that, when this approximation is combined with algorithm 1, the result is a practical algorithm for approximating nonlinear H_∞ control laws.

Assume that $u: \Theta \to R^m$ is a feedback control law that asymptotically stabilizes system (2) over a bounded domain of state space Θ. Also assume that the set $\{\phi_j\}_{j=1}^{\infty}$ is a complete basis set for the domain of the GHJI equation (16). Then, according to equation (17), an approximate solution to equation (16) is given by an infinite series of known functions $\phi_j(x)$ that are continuous and defined everywhere on the domain of state space Θ as $V^{(i+1)}(x) = \sum_{j=1}^{\infty} c_j^{(i+1)} \phi_j(x)$ [22]. The unknown coefficients $c_j^{(i+1)}$ are found using Galerkin solution strategy. For implementation purpose and practical issue, an infinite number of terms cannot be used to express $V(x)$, so an approximation to the assumed solution having the desired degree of accuracy can be formed by considering the first N terms of the infinite series: $V^{(i+1)}(x) = \sum_{j=1}^{N} c_j^{(i+1)} \phi_j(x)$, where the sum is assumed to converge point wise in Θ. The approximation to the assumed solution $V^{(i+1)}(x)$ is dependent on both the number and characteristics of the basis functions chosen to form the approximation. The coefficients $c_j^{(i+1)}$ are calculated by solving the following algebraic equation:

$$\int_{\Theta}\left[\left(\frac{\partial \hat{V}^{(i+1)}}{\partial x}\right)^T \left(f + g_1 w^{(i)} + g_2 u^{(i)}\right) + \left\|u^{(i)}\right\|^2 - \gamma^2 \left\|w^{(i)}\right\|^2 + h^T h\right]\phi_j dx = 0$$

$$j = 1, \dots, N, \tag{18}$$

$$\Rightarrow \int_{\Theta}\left[\frac{\partial\left(\sum_{j=1}^{N} c_j^{(i+1)}\phi_j\right)^T}{\partial x}\left(f + g_1 w^{(i)} + g_2 u^{(i)}\right) + \left\|u^{(i)}\right\|^2 - \gamma^2 \left\|w^{(i)}\right\|^2 + h^T h\right]\phi_j dx = 0$$

$$\Rightarrow \int_{\Theta}\left[\left(\sum_{j=1}^{N} c_j^{(i+1)} \frac{\partial \phi_j}{\partial x}\right)\left(f + g_1 w^{(i)} + g_2 u^{(i)}\right) + \left\|u^{(i)}\right\|^2 - \gamma^2 \left\|w^{(i)}\right\|^2 + h^T h\right]\phi_j dx = 0$$

Defining $\boldsymbol{c}_N^{(i+1)} = \left(c_1^{(i+1)}, \dots \dots c_N^{(i+1)}\right)^T$, $\Phi_N(x) = (\phi_1, . \ . \ . \ . \ . \ \phi_N)^T$. In which $\nabla\Phi_N(x) = \left(\left(\frac{\partial \phi_1}{\partial x}\right) \dots \dots \dots \left(\frac{\partial \phi_N}{\partial x}\right)\right)^T$ is the Jacobian of Φ_N.

$$\Rightarrow \int_{\Theta}\left[\left(\mathrm{c}_N^{(i+1)}\right)^T \nabla\Phi_N \left(f + g_1 w^{(i)} + g_2 u^{(i)}\right) + \left\|u^{(i)}\right\|^2 - \gamma^2 \left\|w^{(i)}\right\|^2 + h^T h\right]\Phi_N dx = 0$$

$$\Rightarrow \left(\mathrm{c}_N^{(i+1)}\right)\int_{\Theta}\left[\nabla\Phi_N \left(f + g_1 w^{(i)} + g_2 u^{(i)}\right)\right]\Phi_N dx = -\int_{\Theta}\left(\left\|u^{(i)}\right\|^2 - \gamma^2 \left\|w^{(i)}\right\|^2 + h^T h\right)\Phi_N dx$$

Since $\left(\boldsymbol{c}_N^{(i+1)}\right)^T \Phi_N(x) = \Phi_N^T(x)\boldsymbol{c}_N^{(i+1)}$

$$\Rightarrow \int_{\Theta}\left[\Phi_N \left(f + g_1 w^{(i)} + g_2 u^{(i)}\right)^T \nabla\Phi_N^T dx\right]\mathrm{c}_N^{(i+1)} = -\int_{\Theta}\left(\left\|u^{(i)}\right\|^2 - \gamma^2 \left\|w^{(i)}\right\|^2 + h^T h\right)\Phi_N dx$$

$$\Rightarrow\left[\int_\Theta \Phi_N f^T \nabla\Phi_N^T dx + \int_\Theta \Phi_N \left(g_2 u^{(i)} + g_1 w^{(i)}\right)^T \nabla\Phi_N^T dx\right] c_N^{(i+1)} = -\int_\Theta \Phi_N \left(\left\|u^{(i)}\right\|^2 - \gamma^2 \left\|w^{(i)}\right\|^2\right) dx - \int_\Theta \Phi_N h^T h dx$$

That is

$$A^{(i)} \boldsymbol{c}_N^{(i+1)} = -b^{(i)} \tag{19}$$

Where

$$A^{(i)} = A_1 + A_2^{(i)}\ ,\ \ A_1 = \int_\Theta \Phi_N f^T \nabla\Phi_N^T dx\ ,\ A_2^{(i)} = \int_\Theta \Phi_N (g_2 u^{(i)} + g_1 w^{(i)})^T \nabla\Phi_N^T dx \tag{20}$$

$$b^{(i)} = b_1 + b_2^{(i)}\ ,\ b_1 = \int_\Theta \Phi_N h^T h dx\ ,\ \ b_2^{(i)} = \int_\Theta \Phi_N (\left\|u^{(i)}\right\|^2 - \gamma^2 \left\|w^{(i)}\right\|^2) dx \tag{21}$$

From equation (19), the coefficients $\boldsymbol{c}_N$ can be computed with

$$\boldsymbol{c}_N^{(i+1)} = -\left(A^{(i)}\right)^{-1} b^{(i)} \tag{22}$$

Where A^i is invertible [22]. Then the solution of Lyapunov function equation (13) is obtained by;

$$V^{(i+1)}(x) = (c^{(i+1)})^T \Phi_N(x) = \Phi_N^T(x) c^{(i+1)} \tag{23}$$

Continuous function can be uniformly estimated to any degree of precision via a complete set of linear independent basis functions. This can be done by application of the famous high-order Weierstrass approximation theorem [23]. Therefore by using any infinite-dimensional linear independent basis function set $\Phi\,(x) = \{\phi_j(x)\}_{j=1}^{\infty}$ the solution of HJI equation can be uniformly estimated to any degree of precision.

5.3 Galerkin Simultaneous Policy Update Algorithm

In this section we used Galerkin simultaneous policy update algorithm (GSPUA) developed in [20] to solve the GHJI equation (13). Using equations (14) and (15), to update the control law equation and disturbance policy, we obtain

$$\hat{u}^{(i)} = -\frac{1}{2} g_2^T \frac{\partial \hat{V}^{(i+1)}}{\partial x} = -\frac{1}{2} g_2^T \sum\nolimits_{j=1}^{N} c^{(i+1)} \frac{\partial \phi_j}{\partial x} = -\frac{1}{2} g_2^T \nabla\Phi_N^T\, c_N^{(i+1)} \tag{24}$$

$$\hat{w}^{(i)} = \frac{1}{2\gamma^2} g_1^T \frac{\partial \hat{V}^{(i+1)}}{\partial x} = \frac{1}{2\gamma^2} g_1^T \sum\nolimits_{j=1}^{N} c^{(i+1)} \frac{\partial \phi_j}{\partial x} = \frac{1}{2\gamma^2} g_1^T \nabla\Phi_N^T\, c_N^{(i+1)} \tag{25}$$

Then the GSPUA for the solution of HJI equation is presented as follows.

Algorithm 2 (GSPUA)

Step 1: *select the independent basis function set* $\Phi_N(x)$, *and evaluate* A_1 *and* b_1. *Give the initial coefficients* $c^{(0)}$ *such that* $\hat{V}^{(0)} \in \mathbb{V}_0$.

Let $\hat{u}^{(0)} = -\frac{1}{2} g_2^T \nabla\Phi_N^T c^{(0)}$, $\hat{w}^{(0)} = \frac{1}{2\gamma^2} g_1^T \nabla\Phi_N^T c^{(0)}$ and *i=0*

Step 2: *compute $A^{(i)}$ and $b^{(i)}$ according to equations (20) and (21). And then solve equation (22) for $c^{(i+1)}$*

Step 3: *update the control and disturbance policies*

$$\hat{u}^{(i+1)} = -\frac{1}{2} g_2^T \nabla \Phi_N^T c^{(i+1)}$$

$$\hat{w}^{(i+1)} = \frac{1}{2\gamma^2} g_1^T \nabla \Phi_N^T c^{(i+1)}$$

Step 4: *set $i = i + 1$, $\|c^{(i)} - c^{(i-1)}\| \le \varepsilon$ (ε is a small positive real number), stop and use $c^{(i)}$ as the coefficient for V^* (i.e. V^* is approximated expressed with $V^* = \left(c^{(i)}\right)^T \Phi_N$), else go back to step 2 and continue.*

The convergence of algorithm 2 is proved in [20]. In order to solve for the coefficients $\boldsymbol{c}^{(i+1)}$ from equation (22) in the GSPUA, we are required to calculate the integrals $A^{(i)}$ and $b^{(i)}$ in each iterative step. This is often time-consuming and the number of basis elements needed to form a complete basis grows exponentially with the dimension of system states. As the number of basis elements grows, an increasing amount of memory is needed to store the coefficients in the estimation and this causes run-time execution problems. To overcome this problem, we use computationally efficient simultaneous policy update algorithm (CESPUA) derived in [20]. CESPUA improves the estimation efficiency in two ways: (1) it evaluates all integrals altogether, this reduces the number of integral evaluations. (2) The use Monte Carlo method together with the Latin hypercube sampling (LHS) to evaluate integrals, which improves the efficiency of integral evaluations. We write $\hat{u}^{(i)}$ and $\hat{w}^{(i)}$ as

$$\hat{u}^{(i)} = -\frac{1}{2} g_2^T \left(\sum_{j=1}^{N} c_j^{(i)} \frac{\partial \phi_j}{\partial x} \right) = -\frac{1}{2} \sum_{j=1}^{N} c_j^{(i)} \left(g_2^T \frac{\partial \phi_j}{\partial x} \right) \tag{26}$$

$$\hat{w}^{(i)} = \frac{1}{2\gamma^2} g_1^T \left(\sum_{j=1}^{N} c_j^{(i)} \frac{\partial \phi_j}{\partial x} \right) = \frac{1}{2\gamma^2} \sum_{j=1}^{N} c_j^{(i)} \left(g_1^T \frac{\partial \phi_j}{\partial x} \right) \tag{27}$$

Substituting equations (26) and (27) into $A_2^{(i)}$ in equation (20) yields;

$$\begin{aligned} A_2^{(i)} &= \int_\Theta \Phi_N \left(g_2 \hat{u}^{(i)} + g_1 \hat{w}^{(i)} \right)^T \nabla \Phi_N^T dx \\ &= \int_\Theta \Phi_N \left[g_2 \left(-\frac{1}{2} \sum_{j=1}^{N} c_j^{(i)} \left(g_2^T \frac{\partial \phi_j}{\partial x} \right) \right) + g_1 \left(\frac{1}{2\gamma^2} \sum_{j=1}^{N} c_j^{(i)} \left(g_1^T \frac{\partial \phi_j}{\partial x} \right) \right) \right]^T \nabla \Phi_N^T dx \\ &= -\frac{1}{2} \sum_{j=1}^{N} c_j^{(i)} \int_\Theta \Phi_N \frac{\partial \phi_j^T}{\partial x} g_2 g_2^T \nabla \Phi_N^T dx + \frac{1}{2\gamma^2} \sum_{j=1}^{N} c_j^{(i)} \int_\Theta \Phi_N \frac{\partial \phi_j^T}{\partial x} g_1 g_1^T \nabla \Phi_N^T dx \end{aligned} \tag{28}$$

Since,

$$\Phi_N \frac{\partial \phi_j^T}{\partial x} g_2 g_2^T \nabla \Phi_N^T = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_N \end{bmatrix} g_2 g_2^T \begin{bmatrix} \frac{\partial \phi_1}{\partial x} & \cdots & \frac{\partial \phi_N}{\partial x} \end{bmatrix}$$

$$= \begin{bmatrix} \phi_1 \frac{\partial \phi_j^T}{\partial x} g_2 g_2^T \frac{\partial \phi_1}{\partial x} & \cdots & \phi_1 \frac{\partial \phi_j^T}{\partial x} g_2 g_2^T \frac{\partial \phi_l}{\partial x} & \cdots & \phi_1 \frac{\partial \phi_j^T}{\partial x} g_2 g_2^T \frac{\partial \phi_N}{\partial x} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ \phi_k \frac{\partial \phi_j^T}{\partial x} g_2 g_2^T \frac{\partial \phi_1}{\partial x} & \cdots & \phi_k \frac{\partial \phi_j^T}{\partial x} g_2 g_2^T \frac{\partial \phi_l}{\partial x} & \cdots & \phi_k \frac{\partial \phi_j^T}{\partial x} g_2 g_2^T \frac{\partial \phi_N}{\partial x} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ \phi_N \frac{\partial \phi_j^T}{\partial x} g_2 g_2^T \frac{\partial \phi_1}{\partial x} & \cdots & \phi_N \frac{\partial \phi_j^T}{\partial x} g_2 g_2^T \frac{\partial \phi_l}{\partial x} & \cdots & \phi_N \frac{\partial \phi_j^T}{\partial x} g_2 g_2^T \frac{\partial \phi_N}{\partial x} \end{bmatrix}$$

And,

$$\Phi_N \frac{\partial \phi_j^T}{\partial x} g_1 g_1^T \nabla \Phi_N^T = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_N \end{bmatrix} g_1 g_1^T \begin{bmatrix} \frac{\partial \phi_1}{\partial x} & \cdots & \frac{\partial \phi_N}{\partial x} \end{bmatrix}$$

$$= \begin{bmatrix} \phi_1 \frac{\partial \phi_j^T}{\partial x} g_1 g_1^T \frac{\partial \phi_1}{\partial x} & \cdots & \phi_1 \frac{\partial \phi_j^T}{\partial x} g_1 g_1^T \frac{\partial \phi_l}{\partial x} & \cdots & \phi_1 \frac{\partial \phi_j^T}{\partial x} g_1 g_1^T \frac{\partial \phi_N}{\partial x} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ \phi_k \frac{\partial \phi_j^T}{\partial x} g_1 g_1^T \frac{\partial \phi_1}{\partial x} & \cdots & \phi_k \frac{\partial \phi_j^T}{\partial x} g_1 g_1^T \frac{\partial \phi_l}{\partial x} & \cdots & \phi_k \frac{\partial \phi_j^T}{\partial x} g_1 g_1^T \frac{\partial \phi_N}{\partial x} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ \phi_N \frac{\partial \phi_j^T}{\partial x} g_1 g_1^T \frac{\partial \phi_1}{\partial x} & \cdots & \phi_N \frac{\partial \phi_j^T}{\partial x} g_1 g_1^T \frac{\partial \phi_l}{\partial x} & \cdots & \phi_N \frac{\partial \phi_j^T}{\partial x} g_1 g_1^T \frac{\partial \phi_N}{\partial x} \end{bmatrix}$$

Therefore, we can rewrite equation (28) as

$$A_2^{(i)} = -\frac{1}{2}\sum_{j=1}^{N} c_j^{(i)} X_j + \frac{1}{2\gamma^2}\sum_{j=1}^{N} c_j^{(i)} Y_j$$

$$= \frac{1}{2}\sum_{j=1}^{N} c_j^{(i)} (-X_j + \frac{1}{\gamma^2} Y_j) \tag{29}$$

Where,

$$X_j = \int_\Theta \Phi_N \frac{\partial \phi_j^T}{\partial x} g_2 g_2^T \nabla \Phi_N^T dx \tag{30}$$

$$Y_j = \int_\Theta \Phi_N \frac{\partial \phi_j^T}{\partial x} g_1 g_1^T \nabla \Phi_N^T dx \tag{31}$$

$X_j = \left(X_{j(k,l)}\right)_{N\times N} \in \mathbf{R}^{N\times N}$, $Y_j = \left(Y_{j(k,l)}\right)_{N\times N} \in \mathbf{R}^{N\times N}$, where $j = 1, \cdots, N$

$X_{j(k,l)} = \int_\Theta \phi_k \frac{\partial \phi_j^T}{\partial x} g_2 g_2^T \frac{\partial \phi_l}{\partial x} dx$ and $Y_{j(k,l)} = \int_\Theta \phi_k \frac{\partial \phi_j^T}{\partial x} g_1 g_1^T \frac{\partial \phi_l}{\partial x} dx$.

In the same way, $b_2^{(i)}$ in equation (21) can be written as

$$b_2^{(i)} = \int_\Theta \Phi_N \left(\hat{u}^{(i)\mathrm{T}} \hat{u}^{(i)} - \gamma^2 \hat{w}^{(i)\mathrm{T}} \hat{w}^{(i)}\right) dx$$

$$= \int_\Theta \Phi_N \left(-\frac{1}{2}\sum_{j=1}^{N} c_j^{(i)} \left(g_2^T \frac{\partial \phi_j}{\partial x}\right)\right)^T \left(-\frac{1}{2} g_2^T \nabla \Phi_N^T c^{(i)}\right) dx$$

$$-\gamma^2 \int_\Theta \Phi_N \left(\frac{1}{2\gamma^2}\sum_{j=1}^{N} c_j^{(i)} \left(g_1^T \frac{\partial \phi_j}{\partial x}\right)\right)^T \left(\frac{1}{2\gamma^2} g_1^T \nabla \Phi_N^T c^{(i)}\right) dx$$

$$=\frac{1}{4}\sum_{j=i}^{N}c_j^{(i)}\int_{\Theta}\Phi_N\frac{\partial\phi_j^T}{\partial x}g_2g_2^T\nabla\Phi_N^Tc^{(i)}dx-\frac{1}{4\gamma^2}\sum_{j=i}^{N}c_j^{(i)}\int_{\Theta}\Phi_N\frac{\partial\phi_j^T}{\partial x}g_1g_1^T\nabla\Phi_N^Tc^{(i)}dx$$

$$=\frac{1}{4}\sum_{j=i}^{N}c_j^{(i)}\left(\int_{\Theta}\Phi_N\frac{\partial\phi_j^T}{\partial x}g_2g_2^T\nabla\Phi_N^Tdx\right)c^{(i)}-\frac{1}{4\gamma^2}\sum_{j=i}^{N}c_j^{(i)}\left(\int_{\Theta}\Phi_N\frac{\partial\phi_j^T}{\partial x}g_1g_1^T\nabla\Phi_N^Tdx\right)c^{(i)}$$

$$=\frac{1}{4}\left(\sum_{j=1}^{N}c_j^{(i)}\left(X_j-\frac{1}{\gamma^2}Y_j\right)\right)c^{(i)} \tag{32}$$

By substituting equations (29) and (32) into equation (22) yields;

$$c^{(i+1)}=-\left(A_1-\frac{1}{2}\sum_{j=1}^{N}c_j^{(i)}\left(X_j-\frac{1}{\gamma^2}Y_j\right)\right)^{-1}\left(b_1+\frac{1}{4}\left(\sum_{j=1}^{N}c_j^{(i)}\left(X_j-\frac{1}{\gamma^2}Y_j\right)\right)c^{(i)}\right) \tag{33}$$

Based on equation (33), we can perceive that X_j and Y_j are invariant in each iteration. They can be computed once and for all, therefore, we are not required to update $A^{(i)}$ and $b^{(i)}$ in each iteration as in algorithm 2. We use Monte Carlo method together with Latin hypercube sampling (LHS) to compute integrals A_1 , b_1 , X_j and Y_j $(j = 1,\cdots,N)$. To estimate integrals over multidimensional domains, Monte Carlo integration is generally applied. LHS is a uniform sampling technique that has small variance. McKay et al. [24] was the first to propose LHS as a Monte Carlo integration technique. We use X_j as an example to illustrate this technique. First, select H samples $x_i \in (i = 1,\cdots,H)$ with LHS. Then calculate X_j with,

$$X_j=\frac{1}{H}\sum_{i=1}^{H}\Phi_N(x_i)\frac{\partial\phi_j^T(x_i)}{\partial x}g_2(x_i)g_2^T(x_i)\nabla\Phi_N^T(x_i) \tag{34}$$

Based on equations (33) and (34), we use CESPUA.

Algorithm 3 (CESPUA)

Step 1: *select the independent basis function set $\Phi_N(x)$, and evaluate A_1, b_1, X_j, and Y_j $(j = 1, k, N)$ with the above Monte Carlo integration. Give the initial coefficients $c^{(0)}$ such that $\hat{V}^{(0)} \in \mathbb{V}_0$. Let i=0*

Step 2: *solve equation (33) for $c^{(i+1)}$.*

Step 3: *set $i = i + 1$.if $\left\|c^{(i)} - c^{(i-1)}\right\| \le \varepsilon$ (ε is a small positive real number), stop and use $c^{(i)}$ as the coefficient for V^* (i.e. V^* is approximated expressed with $V^* = \left(c^{(i)}\right)^T\Phi_N$), else go back to step 2 and continue.*

In CESPUA, so long as the basis functions are selected, the integrals A_1 , b_1 , X_j and Y_j (j = $1,\cdots,N$ can be computed off-line directly and remain invariant during the iterative process. Hence, it is very advantageous in design of offline controller.

6. SIMULATION RESULTS

CESPUA technique is used to solve the nonlinear missile guidance problem. The model described in section 5.2 is used to synthesize feedback control laws based on the CESPUA algorithm. The variables $f(x)$ and $g(x)$ are derived from equations of motion (7a) and (7b), therefore the domain of the state values Θ and the basis functions $\{\phi_j(x)\}_{j=1}^{N}$ need to determined. For this missile guidance problem, Θ is defined as, $-1300\,m/s < x_1 < -300m/s$, $0 < x_2 < 300m/s$, $0 < x_3 < 5000m$. We select basis functions as:

$$\{\phi_j\}_{j=1}^{10} = \{x_1^2 \quad x_1x_2 \quad x_1x_3 \quad x_2^2 \quad x_2x_3 \quad x_3^2 \quad x_1x_2x_3 \quad x_1^2x_2 \quad x_2^2x_3 \quad x_1x_3^2\}$$

and $\gamma = 12$. The initial coefficients $c^{(0)} = 0$, while the stop criterion ε is set as 10^{-7}.

Figs. 2 to 6 show coefficients $c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9$ and c_{10} in every iterative step.

The coefficients respectively converge to

$$c = [0.3855 \;\; 0.0102 \;\; 0.0131 \;\; 0.8604 \;\; -0.0019 \;\; 0.0000 \;\; -83.4750 \;\; -17.8143 \;\; -71.3399 \;\; 0.0008\,]^T$$

From the above coefficients the solution of HJI equation is computed by equation (24). The corresponding H_∞ controller is designed by equation (12). Numerical simulations are performed to illustrate the effectiveness of the proposed guidance law.

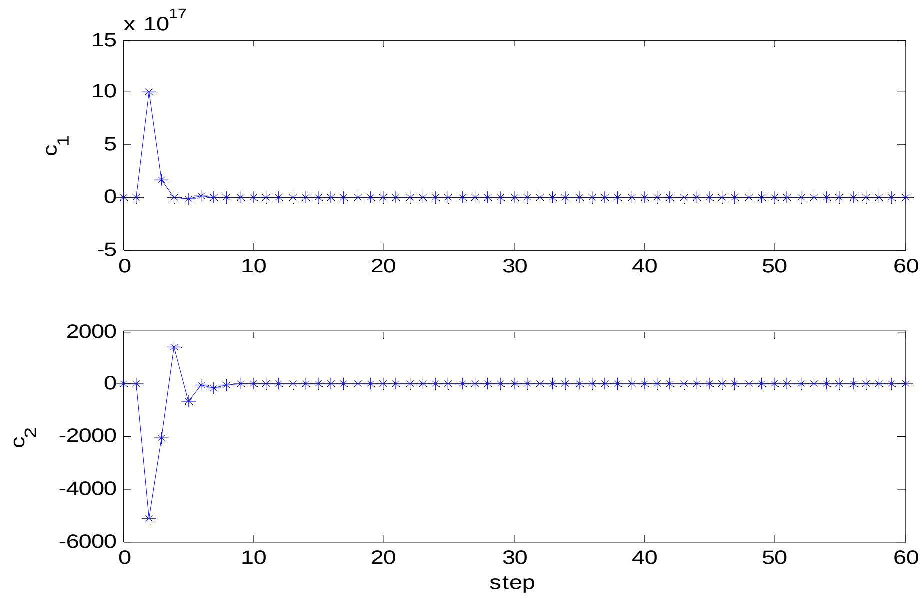

Fig. 2. Coefficients c1,c2 obtained by CESPUA

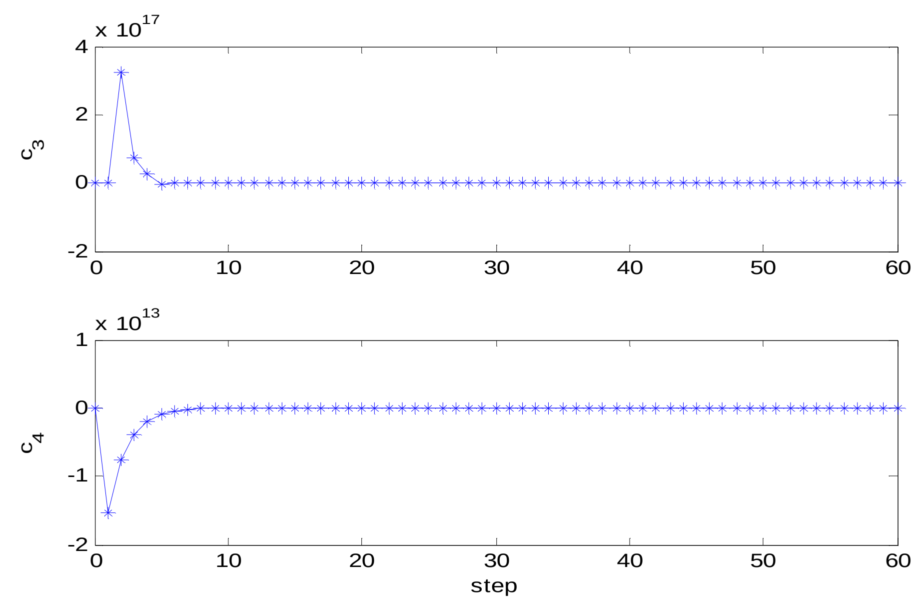

Fig. 3. Coefficients c3, c4 obtained by CESPUA

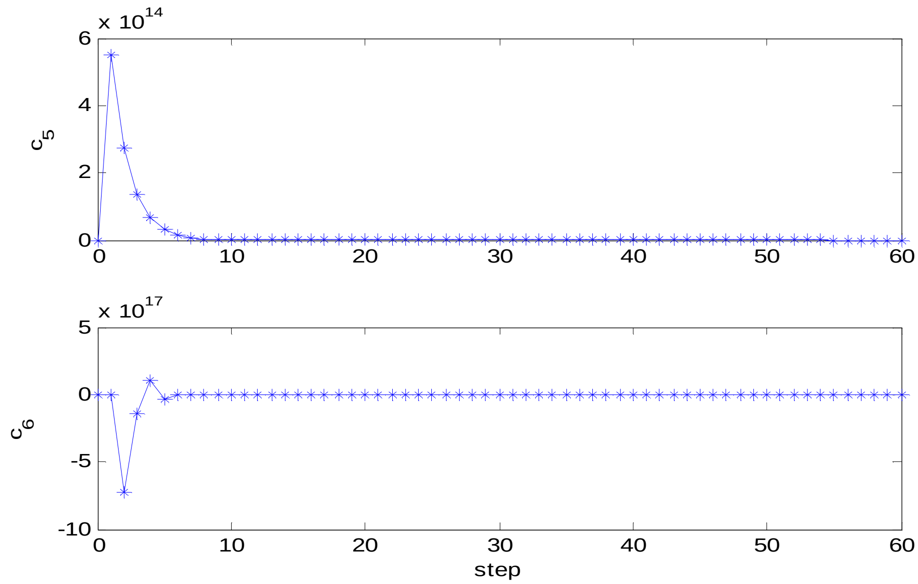

Fig. 4. Coefficients c5,c6 obtained by CESPUA

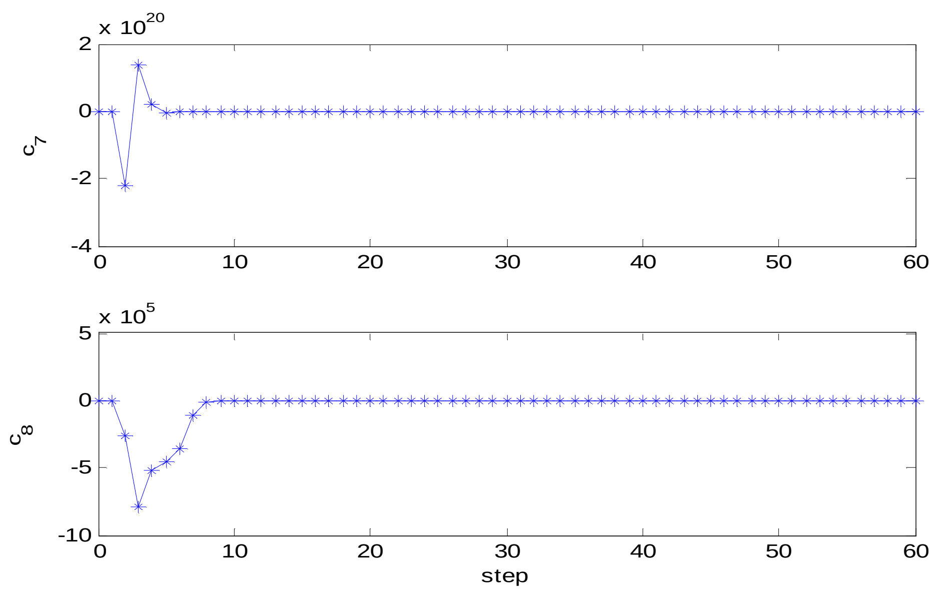

Fig. 5. Coefficients c7,c8 obtained by CESPUA

Case 1: The initial conditions for engagement are chosen as,

$$r_0 = 5000m$$
$$\dot{r}_0 = -300m/s$$
$$r_0 \|\Omega\|_0 = 300m/s$$

And highly maneuvering target with disturbance vector,

$$w(t) = \begin{bmatrix} 70\sin(0.5t) \\ 70\sin(0.5t + \pi/4) \\ 70\cos(0.5t) \end{bmatrix}$$

Fig. 7 shows the evolution of γ with time. It is shown that γ converges to $15.05\ (< \gamma^2 = 144)$ which satisfies the $\mathcal{L}_2 -$ gain requirement as $t \rightarrow \infty$.

From Fig. 8, it is shown that the relative distance becomes zero in an instant $r(t_f) = 0$ where t_f =6.69 sec is the interception time. It is also shown that there is no need for relative distance to asymptotically converge to zero. Its asymptotic convergence means that the missile initially approaches the target at a high speed; however, near the target; the relative speed reduces so slowly that the missile touches the target in an infinite time. Such behaviour is

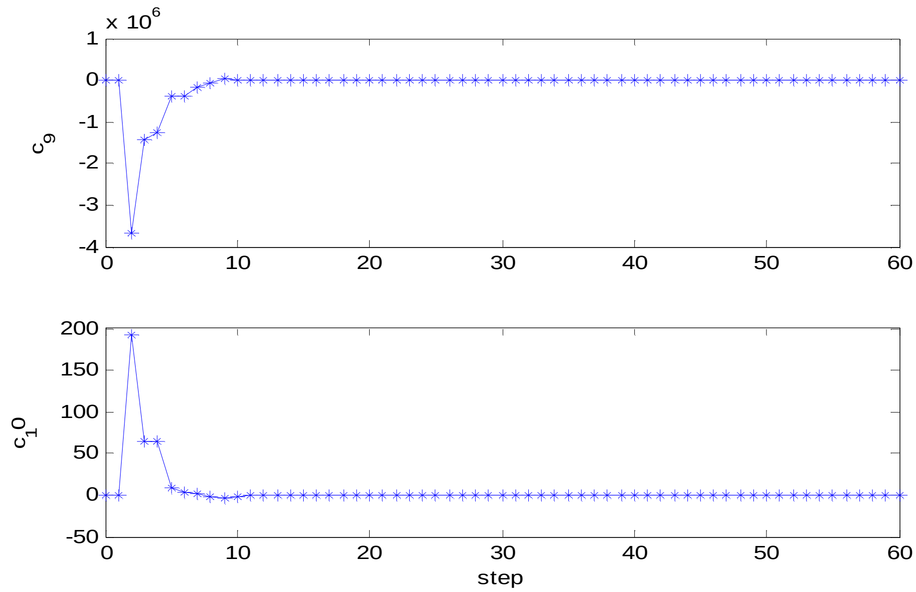

Fig. 6. Coefficients c9,c10 obtained by CESPUA

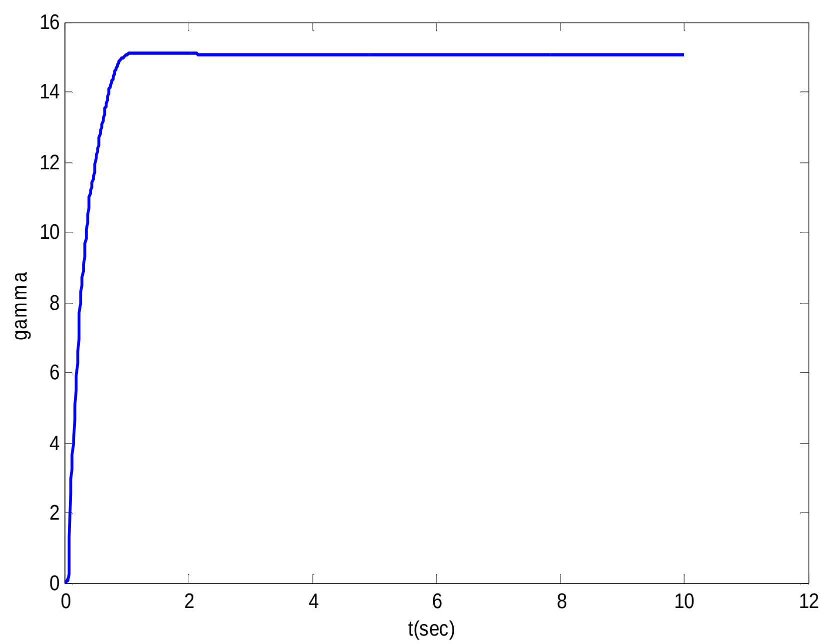

Fig. 7. Evolution of gamma (γ) with time

not desirable in missile guidance in which the missile should hit the target with non-zero speed in a finite time. Figs. 9 and 10 show the tangential relative velocity and radial relative velocity. The tangential relative velocity asymptotically converges to zero while the radial relative velocity asymptotically converges to a negative value ensuring the missile intercepts the target. Fig. 11 shows the guidance command (control command) of the missile.

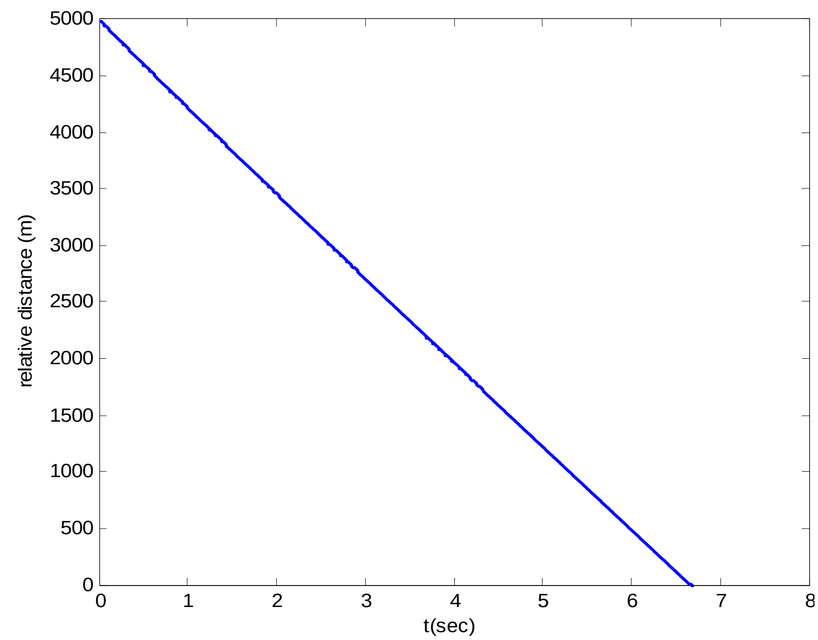

Fig. 8. Relative distance between the missile and the target

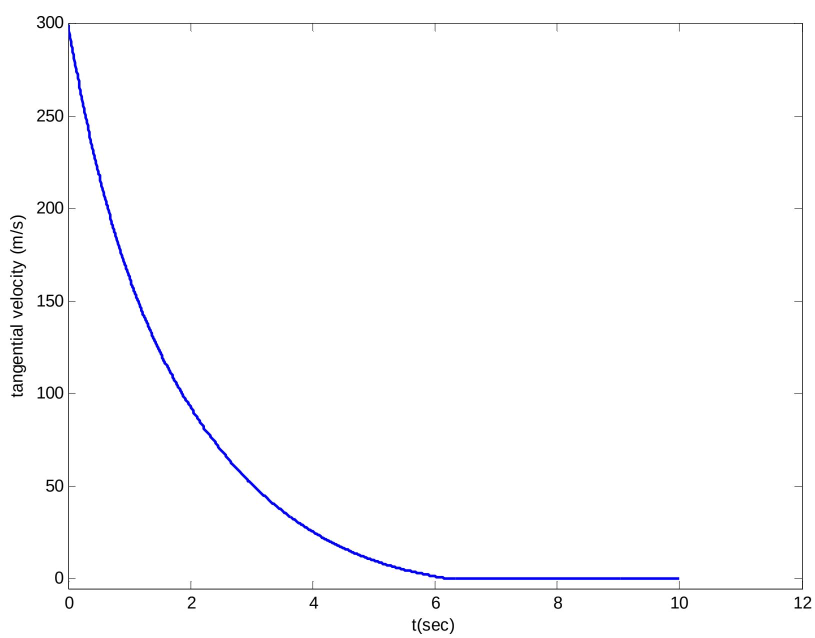

Fig. 9. Tangential relative velocity

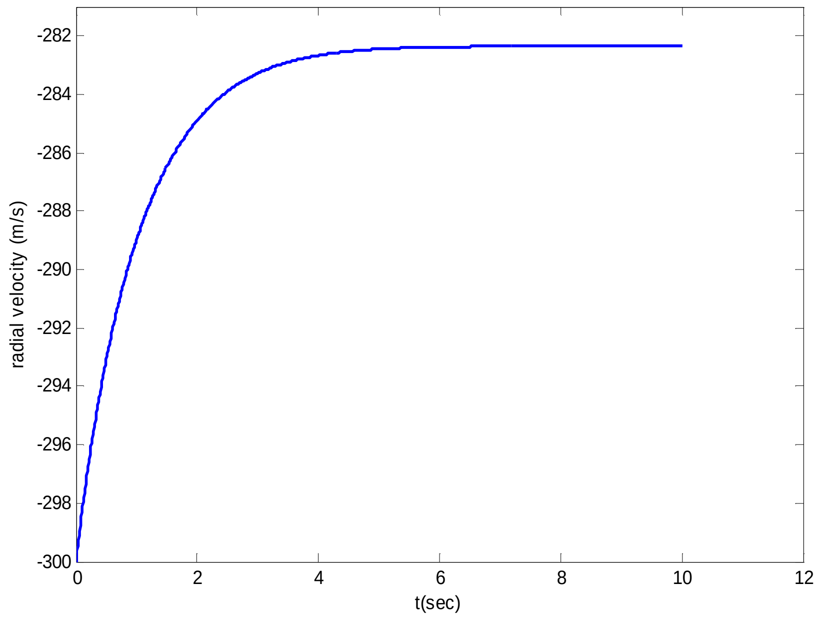

Fig. 10. Radial relative velocity

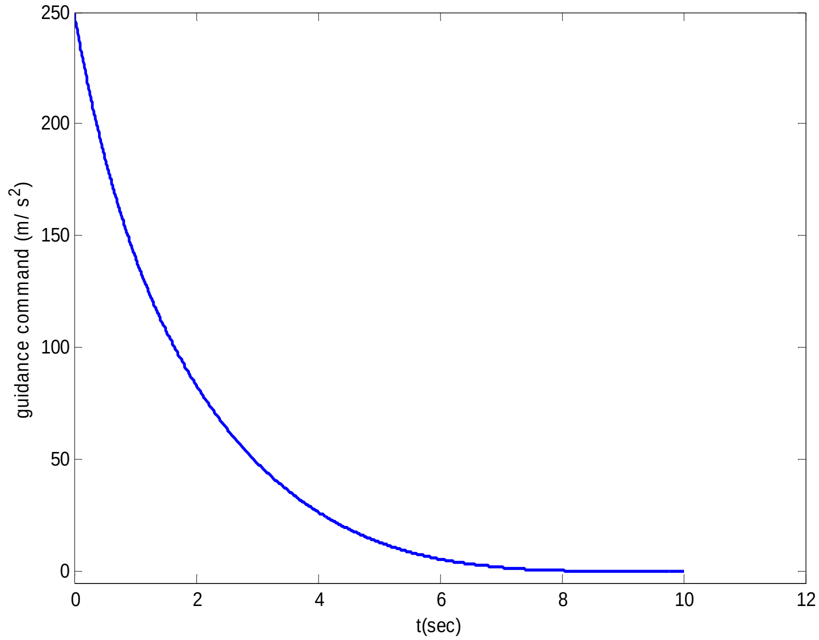

Fig. 11. Guidance command

Further simulations were conducted with variation of initial conditions chosen as,

Case 2:

$$r_0 = 8000m$$
$$\dot{r}_0 = -800m / s$$
$$r_0 \|\Omega\|_0 = 300m / s$$

Fig. 12 show the relative distance between the missile and target for initial condition in case 2.the interception time is 10.35 sec. Figs. 13 and 14 show the tangential relative velocity and radial relative velocity respectively. The proposed guidance law show robustness against variation of initial condition.

The proposed H_∞ guidance law is compared to the sliding mode guidance law presented by the following command [8]

$$u = -\frac{V_r V_\theta}{r} + asat\left(\frac{V_\theta}{\varepsilon}\right)$$

Where V_θ is tangential velocity, $a = 30$ and $\varepsilon = 1$.

a) Comparison of Tracking Errors and Final Time

Our design objective is to develop an effective guidance law to keep the LOS angular rate and

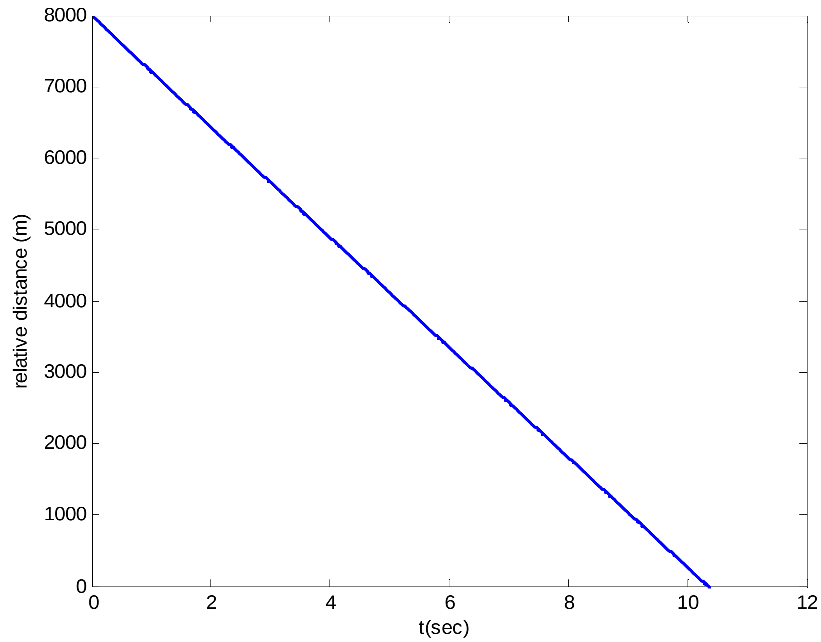

Fig. 12. Relative distance between the missile and the target

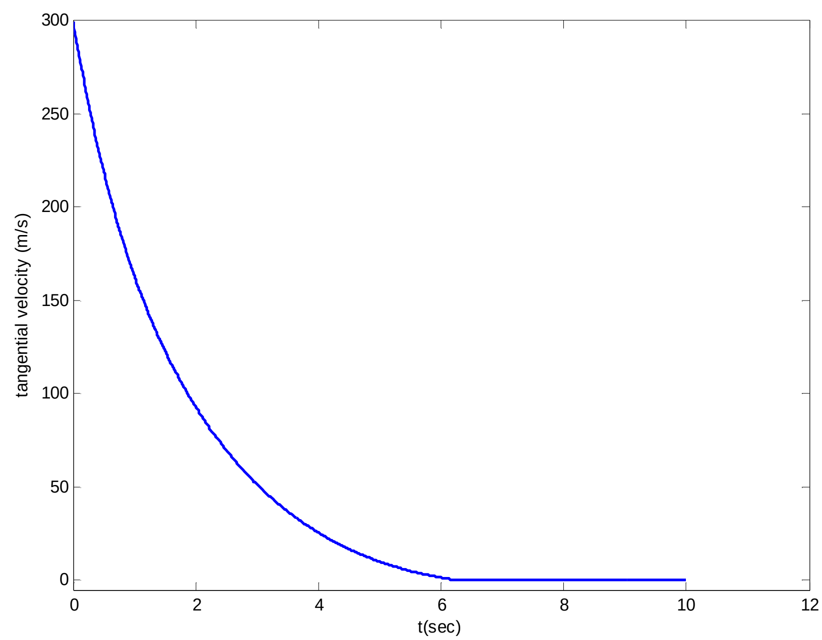

Fig. 13. Tangential relative velocity

relative distance as minimal as possible under uncertain target accelerations. In Fig. 15, the relative distances for the H_∞ guidance law converges to zero faster than that of sliding mode law.

In Fig. 15 the interception time for the two guidance laws are 6.69 sec and 6.98 sec respectively. While in Fig. 16, interception time is 10.35 sec and 11 sec respectively. From Fig. 17, it can be shown that the tangential velocity

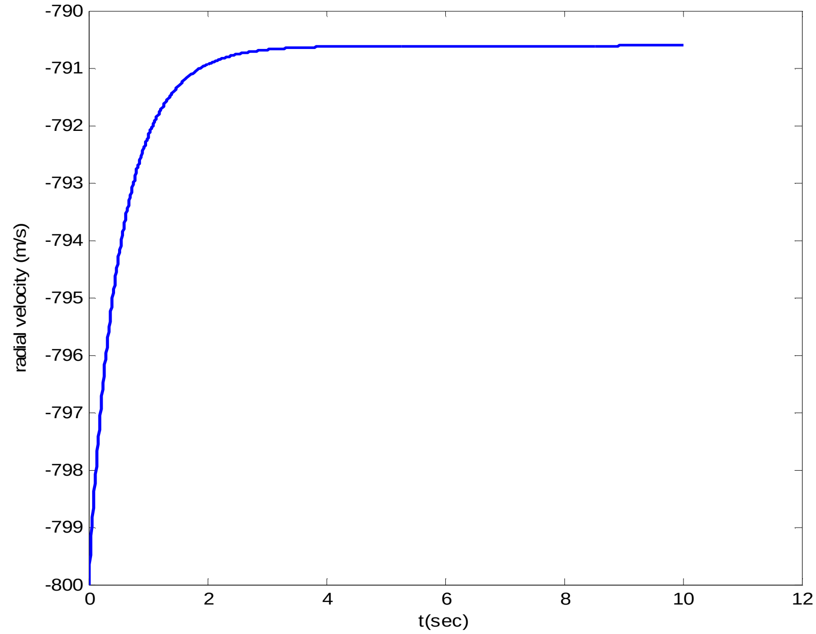

Fig. 14. Radial relative velocity

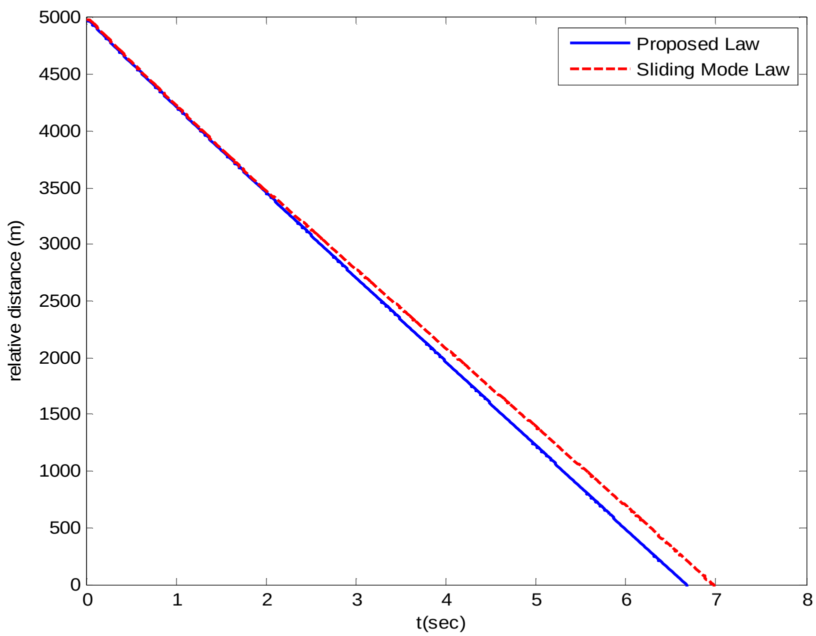

Fig. 15. Relative distances between the missile and target of H_∞ law and sliding mode law for initial conditions in case 1

converges to zero rapidly for the H_∞ law compared to the sliding mode law. This reveals that the H_∞ guidance law possesses excellent tracking ability and it is possible to attain smaller missed distances than that of sliding mode law.

b) Robustness

Two types of target maneuvers investigate robustness for the presented guidance law. In accordance with the definition of the performance robustness index, a robust guidance law ought to keep the engagement performance less sensitive to external disturbances. That is, the acceleration commands of the target.

In Fig. 15, the missile acceleration command is;

$$w(t) = \begin{bmatrix} 70\sin(0.5t) \\ 70\sin(0.5t + \pi/4) \\ 70\cos(0.5t) \end{bmatrix}$$

While in Fig. 16 the missile acceleration command is

$$w(t) = \begin{bmatrix} 200\sin(0.25t) \\ 200\sin(0.25t + \pi/6) \\ 200\cos(0.25t) \end{bmatrix}$$

From Figs. 15 and 16, the interception time for H_∞ guidance law is less than that of sliding mode law. Therefore, for different initial conditions, H_∞ guidance law is more robust to uncertain target maneuvers (accelerations) than sliding mode law.

c) Comparison of Control Efforts

The control commands for both guidance laws are shown in Fig. 18. The guidance command for sliding mode law is larger than that of H_∞ guidance law. Larger acceleration commands lead to higher energy (fuel) consumption. Therefore, concerning energy consumption, H_∞ guidance law yields better results.

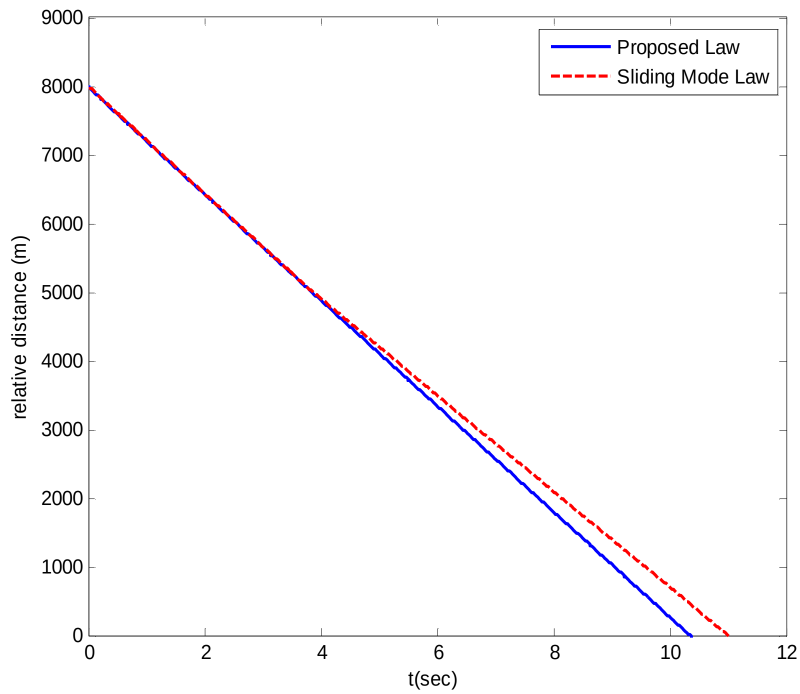

Fig. 16. Relative distances between the missile and target of H_∞ law and sliding mode law for initial conditions in case 2

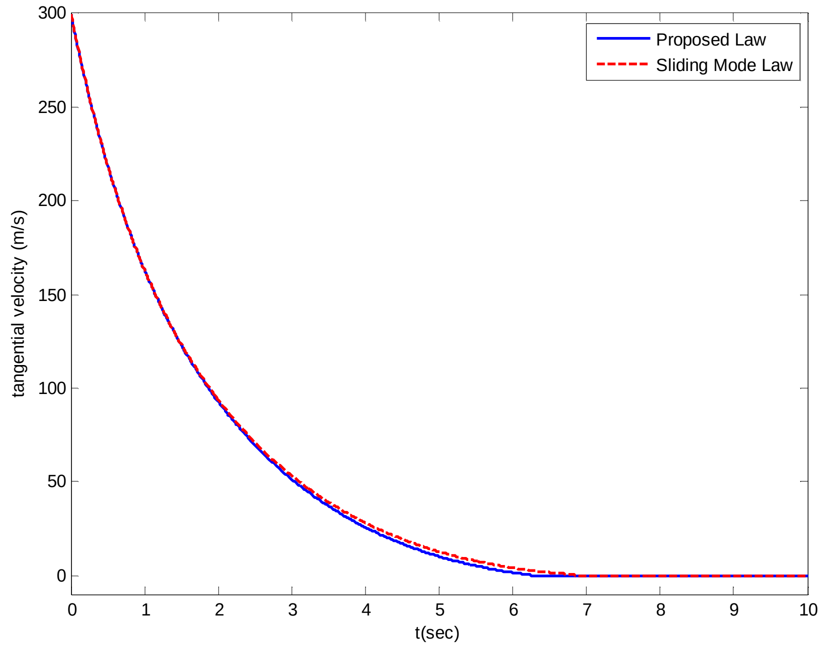

Fig. 17. Tangential relative velocities for H_∞ law and sliding mode law with initial conditions of case 1

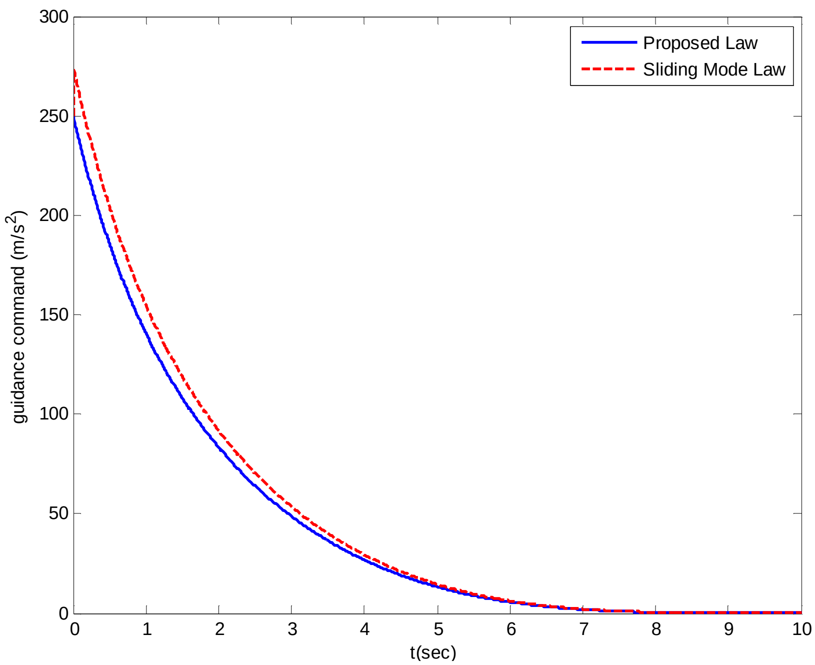

Fig. 18. Guidance commands for H_∞ law and sliding mode law with initial conditions of case 1c

7. CONCLUSION

In this work, the nonlinear H_∞ guidance law is presented through application of H_∞ theory into the equations of 3-D relative motion. The guidance problem is considered as a nonlinear disturbance attenuation control problem by regarding target accelerations as unpredictable inputs. By using MPC, nonlinear relative motion equations of the target and missile only contain three state variables instead of six state variables used in polar coordinates leading to a less complex HJI equation. SPUA is used to solve the HJI equation where the disturbance and control policies are simultaneously updated. SPUA avoids solving the HJB equations because the HJI equation is directly successively approximated by a sequence of Lyapunov function equations. This results to one interactive loop instead of two. Hence, SPUA is much simpler and easier to implement than other existing methods. Galerkin's method is used to solve the LFEs and derive GSPUA. CESPUA is used to compute all the integrals at once. Furthermore, Monte Carlo integration is used together with LHS to evaluate all integrals, which further improved the efficiency of the CESPUA. The proposed H_∞ guidance law proved that, it not only satisfies the H_∞ robust stability but also obtains better performance and avoids control of relative velocity. An illustrative example is proposed to demonstrate the effectiveness of the developed guidance law. Compared with the sliding mode law, numerical simulations indicate that the H_∞ guidance law consumes less control energy, has reduced interception time, excellent tracking ability and offers better performance against uncertain target accelerations.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. George Siouris M. Missile guidance and control systems. Springer-Verlag New York. 2004;194-227.
2. Hough ME. Optimal guidance and nonlinear estimation for interception of decelerating targets. Journal of Guidance, Control and Dynamics. 1995;18(2):316-324.
3. Hough ME. Optimal guidance and nonlinear estimation for interception of accelerating targets. Journal of Guidance, Control and Dynamics. 1995;18(5):961-968.
4. Ben-Asher JZ, Yaesh I. Advances in missile guidance theory. Progress in Astronautics and Aeronautics, AIAA, New York; 1998.
5. Pastrik HL, Seltzer SM, Warren ME. Guidance laws for short-range tactical missiles. Journal of Guidance, Control, and Dynamics. 1981;4(2):98-108.
6. Zhou D, Mu C, Xu W. Adaptive sliding-mode guidance of a homing missile. Journal of Guidance, Control, and Dynamics. 1999;22(4)589–594.
7. Babu KR, Sarma IG, Swamy KN. Switched bias proportional navigation for homing guidance against highly maneuvering target. Journal of Guidance, Control, and Dynamics. 1994;17(6):1357–1363.
8. Moon J, Kim K, Kim Y. Design of missile guidance law via variable structure control. Journal of Guidance, Control, and Dynamics. 2001;24(4):659–664.
9. Brierly SD, Longchamp R. Application of sliding-mode control to air-air interception problem. IEEE Transactions of Aerospace and Electronic Systems. 1990;26(2):306-325.
10. Junhong S, Shenmin S. Three-dimensional guidance law based on adaptive integral sliding mode control. Chinese Journal of Aeronautics. 2016;29(1):202–214.
11. Liao F, Luo Q, Ji H, Gai W. Guidance laws with input saturation and nonlinear robust H∞ observers. ISA Transactions. 2016;63: 20–31.
12. Yang CD, Chen HY. Nonlinear H_∞ robust guidance law for homing missiles. Journal of Guidance, Control, and Dynamics. 1998;21(6):882-890.
13. Yang CD, Chen HY. Three-dimensional nonlinear H_∞ guidance law. International Journal of Robust and Nonlinear Control. 2001;11(2):109-129.
14. Chen BS, Chen YY, Lin CL. Nonlinear fuzzy H_∞ guidance law with saturation of actuators against manoeuvring targets. IEEE Transactions on Control Systems Technology. 2002;10(6):769–779.
15. Shieh CS. Design of three-dimensional missile guidance via tunable nonlinear H_∞ control with saturation constrain. IET Control Theory Application. 2007;1(3):756-763.
16. Li-Jun L, Yi S. Three-dimension H_∞ guidance law and capture region analysis. IEEE Transactions on Aerospace and Electronic Systems. 2012;48(1):419-429.
17. Tyan F, Shen JF. A simple adaptive GIPN missile guidance law. Proceedings of the American Control Conference, Minneapolis, MN. 2006;345-350.
18. Tyan F. Unified approach to missile guidance laws: A 3D extension. IEEE Transactions on Aerospace and Electronic Systems. 2005;41(4):1178-1199.

19. Del Shaft AJ. A L_2 -gain analysis of nonlinear systems and nonlinear state feedback H_∞control. IEEE Transactions on Automatic Control. 1992;37(6):770-784.
20. Biao Luo, Huai-Ning Wu. Computationally efficient simultaneous policy update algorithm for nonlinear H_∞ state feedback control with Galerkin's method. International Journal of Robust and Nonlinear Control. 2013;23(9):991-1012.
21. Biao Luo, Huai-Ning Wu, Tingwen Huang. Off-policy reinforcement learning for H∞ control design. IEEE Transactions on Cybernetics. 2015;45(1):65-76.
22. Beard RW, McLain TW. Successive Galerkin approximation algorithms for nonlinear optimal and robust control [J]. International Journal of Control. 1998;71(5): 717–743.
23. Finlayson BA. The method of weighted residuals and variational principles. Academic Press: New York; 1972.
24. McKay MD, Beckman R, Conover J. A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. Technometrics. 1979;21(2):239–245.

Peer-review history:
The peer review history for this paper can be accessed here:
http://sciencedomain.org/review-history/18859