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# Dust Ion-acoustic K-dV and Modified K-dV Solitons in a Dusty Degenerate Dense Plasma

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## Authors' contributions

This work was carried out in collaboration between all authors. Author KNM designed the study, performed the statistical analysis, wrote the protocol, managed the literature searches and wrote the first draft of the manuscript. Authors MSZ, NR and AAM managed the analyses of the study. All authors read and approved the final manuscript.

**Research Article** 

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## ABSTRACT

A theoretical investigation has been made of the roles of the degeneracy and the dynamics of electrons and ions on the DIA (dust ion-acoustic) Korteweg-de Vries (K-dV) and modified Kortewegde Vries (mK-dV) solitons that are found to exit in a dusty degenerate dense plasma containing non-relativistic degenerate ions and both non-relativistic and ultra relativistic electrons fluids, and the negatively charged dust grains. This fluid model, which is valid for both the non-relativistic and ultra-relativistic limits has been employed with the reductive perturbation method. The K-dV and modified K-dV equations have been derived, and numerically examined. The basic features of K-dV and modified K-dV solitons have been analyzed. It has been observed that the dusty degenerate plasma system under consideration supports the propagation of solitons obtained from the solutions of K-dV and modified K-dV equations. The relevance of our results obtained from this investigation in compact astrophysical objects is briefly discussed.

Keywords: Degenerate ions and electrons fluids; reductive perturbation method; degenerate pressure; solitons; negatively charged dust grains.

2010 Mathematics Subject Classification: 35L67; 65Z05; 68U20; 83A05; 85A15.

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## **1. INTRODUCTION**

Recently, the physics of dusty plasma is receiving a great deal of attention. Dusty plasmas are characterized as a low temperature multispecies ionized gas comprising electrons, protons, and negatively (or positively) charged grains of micrometer or submicrometer size. The study of the collective effects in dusty plasmas is of significant interest. Charged dust grains are found to modify or even dominate wave propagation [1-5], wave scattering [6-7] wave instability [8], ion trapping [9]. However, most of the studies on wave motions [1-4] in dusty plasma assume constant charge on the dust grains. Now-a-days a number of authors have become interested to study the properties of matter under extreme conditions [10-12], which occur due to the combine effect of Pauli's exclusion principle and Heisenberg's uncertainty principle, depends only on the number density of constituent particles, but independent on its own temperature [13,14]. This degenerate pressure has an important role to study the electrostatic perturbation in matters which exist in extreme conditions [10-12]. Electron degenerate pressure will halt the gravitational collapse of a star if its mass is below the Chandrasekhar limit (i.e. 1.44 solar masses) [15]. This is the pressure that prevents a white dwarf star from collapsing. Astrophysical aspects of high density like in many cosmic environments, compact astrophysical objects [16-19]. Examples of the latter are white and brown dwarf stars [20], as well as massive Jupiter which serves as the super-Earth terrestrial planets around other stars, and the benchmark for giant planets. In case of such a compact object the degenerate electron number density is so high (in white dwarfs it can be of the order of  $10^{30}$  cm<sup>3</sup>, even more [13,14] that the electron Fermi energy is comparable to the electron mass energy and as a result the electron speed becomes comparable to the speed of light in vacuum. For such interstellar compact objects the equation of state for degenerate ions and electrons are mathematically explained by Chandrasekhar [12] for two limits, named as nonrelativistic and ultra-relativistic limits. Chandrasekhar [21,22] presented a general expression for the relativistic ion and electron pressures in his classical papers. The pressure for ion fluid can be given by the following equation

$$P_i = K_i n_i^{\alpha}, \tag{1.1}$$

where

$$\alpha = \frac{5}{3}; K_i = \frac{3}{5} \left(\frac{\pi}{3}\right)^{\frac{1}{3}} \frac{\pi \hbar^2}{m} \simeq \frac{3}{5} \Lambda_c \hbar c, \qquad (1.2)$$

for the non-relativistic limit (where  $\Lambda_c = \frac{\pi\hbar}{mc} = 1.2 \times 10^{-10} \ cm$ , and  $\hbar$  is the Planck constant divided by (2  $\pi$ ). While for the electron fluid,

$$P_e = K_e n_e^{\gamma},\tag{1.3}$$

for non-relativistic limit

$$\gamma = \alpha; \ K_e = K_i, \tag{1.4}$$

and

$$Y = \frac{4}{3}; K_e = \frac{3}{4} \left(\frac{\pi^2}{9}\right)^{\frac{1}{3}} \hbar c \simeq \frac{3}{4} \hbar c,$$
(1.5)

in the ultra-relativistic limit [10-12].

Recently, a large number of authors [23-25], etc. have used the pressure laws 1.3 to 1.5 investigate the linear and nonlinear properties of electrostatic and electromagnetic waves, by using the non-relativistic quantum hydrodynamic (QHD) and quantum magnetohydrodynamic(Q-MHD) [23] models and by assuming either immobile ions or non-

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degenerate uncorrelated mobile ions. It turns out that the presence of the latter and degenerate ultra relativistic electrons with the pressure law (1.3-1.5) admits one-dimensional localized ion models (IMs) supported by linear and non linear ion inertial forces and the pressure of degenerate electron fluids in a dense quantum plasma that is unmagnetized. Again in this present days, some authors [26,27]; has made a number of theoretical investigations on the nonlinear propagation of electrostatic waves in degenerate quantum plasma. Again there are some works on electron-positron degenerate plasma with magnetic field [28]. These investigations are mainly based on the electron equation of state, which are only valid for the non-relativistic limit. Some investigations have been also made of the nonlinear propagation of electrostatic waves in a degenerate dense plasma which are mainly based on the degenerate electron equation of state valid for ultra-relativistic limit [13,14]. Still now, there is no theoretical investigation has been made to study the extreme condition of matter for both non-relativistic and ultra-relativistic limits on the propagation of electrostatic solitary waves in a dusty degenerate dense plasma system. Therefore, in our paper we study the properties of the solitons considering a dusty degenerate dense plasma containing degenerate electron-ion fluid (both non-relativistic and ultra-relativistic limits) with the negatively charged dust grains to study the basic features of the electrostatic solitary structures with the solutions of modified K-dV equation. Our considered model is relevant to compact interstellar objects (i.e. white dwarf, neutron star, black hole, etc.).

## 2. GOVERNING EQUATIONS

We consider a one-dimensional, unmagnetized dusty degenerate electron-ion plasma system containing non-relativistic degenerate cold ion and both non-relativistic and ultra-relativistic degenerate electron fluids with arbitrary charged dust grains. We are interested in the propagation of electrostatic perturbation in such a dusty degenerate dense plasma. Thus, at equilibrium condition we have  $n_{i0} = n_{e0}$  where  $n_{i0}(n_{e0})$  is the ion (electron) number density at equilibrium. The nonlinear dynamics of the electrostatic waves propagating in such a degenerate plasma is governed by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x} (n_i u_i) = 0, \tag{2.1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial \phi}{\partial x} + \frac{K_1}{n_i} \frac{\partial n_i^{\alpha}}{\partial x} = 0,$$
(2.2)

$$n_e \frac{\partial \phi}{\partial x} - K_2 \frac{\partial n_e^{\gamma}}{\partial x} = 0,$$
(2.3)

$$\frac{\partial^2 \phi}{\partial x^2} = -\rho, \tag{2.4}$$

$$\rho = n_i - n_e (1 - \mu), \tag{2.5}$$

where  $n_i(n_e)$  is the ion (electron) number density normalized by its equilibrium value  $n_{i0}(n_{e0})$ ,  $u_i$  is the ion fluid speed normalized by  $C_i = (m_e c^2 = m_i)^{1/2}$  with  $m_e(m_i)$  being the electron (ion) rest mass, c being the speed of light in vacuum,  $\phi$  is the electrostatic wave potential normalized by  $m_e c^2 = e$  with e being the magnitude of the charge of an electron, the time variable (t) is normalized by  $\omega_{pi} = (\frac{4\pi n_0 e^2}{m_i})^{1/2}$ , and the space variable (x) is normalized by ,  $\lambda_s = (\frac{m_e c^2}{4\pi n_0 e^2})^{1/2}$  and  $\mu$  is the ratio of the number density to the ion

number density ( $Z_{d}n_{d0}/n_{i0}$ ). The constant  $K_1 = \frac{n_0^{\alpha-1}K_i}{m_i^2}c_i^2$  and  $K_2 = \frac{n_0^{\gamma-1}K_e}{m_i^2}c_i^2 = \frac{m_0^{\gamma-1}K_e}{m_i^2}$ 

$$\left. n_0^{\gamma-1} K_p \right|_{m_i c_i^2}$$

## 3. DERIVATION OF K-DV EQUATION

Now we derive a dynamical K-dV equation for the nonlinear propagation of the DIA waves by using equations (2.1) - (2.5). To do so, we employ a reductive perturbation technique to examine electrostatic perturbations propagating in the relativistic dusty degenerate dense plasma due to the effect of dispersion, we first introduce the stretched coordinates [29]

$$\zeta = \varepsilon^{1/2} (x - V_p t), \tag{3.1}$$

$$\tau = \varepsilon^{3/2} t, \tag{3.2}$$

where  $V_p$  is the wave phase speed  $\omega/k$  with  $\omega$  being angular frequency and k being the wave number of the perturbation mode), and  $\epsilon$  is a smallness parameter measuring the weakness of the dispersion ( $0 < \epsilon < 1$ ). We then expand  $n_i, n_e, u_i, \rho, and \phi$ , in power series of  $\epsilon$ :

$$n_{i} = 1 + \epsilon n_{i}^{(1)} + \epsilon^{2} n_{i}^{(2)} + \cdots,$$
(3.3)

$$n_e = 1 + \epsilon n_e^{(1)} + \epsilon^2 n_e^{(2)} + \cdots,$$
(3.4)  
$$u_e = \epsilon u_e^{(1)} + \epsilon^2 u_e^{(2)} + \cdots$$
(3.5)

$$u_{i} = \epsilon u_{i}^{(1)} + \epsilon^{2} u_{i}^{(2)} + \cdots,$$
(3.5)

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots, \tag{3.6}$$

$$\rho = \epsilon \rho^{(1)} + \epsilon^2 \rho^{(2)} + \cdots, \tag{3.7}$$

and develop equations in various powers of  $\epsilon$ . To the lowest order in  $\epsilon$ , using equations (3.1)-(3.7) into equations (2.1) - (2.5) we get as,

$$u_{i}^{(1)} = V_{p}\phi^{(1)}/(V_{p}^{2} - K_{1}^{'}), \ n_{i}^{(1)} = \phi^{(1)}/(V_{p}^{2} - K_{1}^{'}), n_{e}^{(1)} = \phi^{(1)}/K_{2}^{'}, \text{ and}$$

$$V_{p} = \sqrt{K_{2}^{'}/(1 - \mu) + K_{1}^{'}}, \ \text{where} \ K_{1}^{'} = \alpha K_{1}/(\alpha - 1) \ \text{and} \ K_{2}^{'} = \gamma K_{2}/(\gamma - 1).$$
The relation
$$V_{p} = \sqrt{K_{2}^{'}/(1 - \mu) + K_{1}^{'}}, \ \text{represents} \ \text{the dispersion relation for the dust ion acoustic type}$$

electrostatic waves in the degenerate plasma under consideration. We are interested in studying the nonlinear propagation of these dissipative dust ion acoustic type electrostatic waves in a three components degenerate plasma. To the next higher order in  $\epsilon$ , we obtain a set of equations

$$\frac{\partial n_i^{(1)}}{\partial \tau} - V_p \frac{\partial n_i^{(2)}}{\partial \zeta} - \frac{\partial}{\partial \zeta} \left[ u_i^{(2)} + n_i^{(1)} u_i^{(1)} \right] = 0,$$
(3.8)

$$\frac{\partial u_{i}^{(1)}}{\partial \tau} - V_{p} \frac{\partial u_{i}^{(2)}}{\partial \zeta} - u_{i}^{(1)} \frac{\partial u_{i}^{(1)}}{\partial \zeta} - \frac{\partial \phi^{(2)}}{\partial \zeta} - K_{1}' \frac{\partial}{\partial \zeta} \left[ n_{i}^{(2)} + \frac{(\alpha - 2)}{2} (n_{i}^{(1)})^{2} \right] = 0,$$
  
$$\frac{\partial \phi^{(2)}}{\partial \zeta} - K_{2}' \frac{\partial}{\partial \zeta} \left[ n_{e}^{(2)} - \frac{(\gamma - 2)}{2} (n_{e}^{(1)})^{2} \right] = 0,$$
 (3.9)

$$\frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} = -\rho^{(1)} , \qquad (3.10)$$

$$\rho^{(1)} = n_i^{(2)} - (1 - \mu) n_e^{(2)}. \tag{3.11}$$

Now, combining equations (3.8-3.11) we deduce a Korteweg-de Vries equation as

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} + B \frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} = 0, \qquad (3.12)$$

where the value of A and B are given by

$$A = \frac{(V_p^2 - K_1')^2}{2V_p} \left[ \frac{3V_p^2 + K_1'(\alpha - 2)}{(V_p^2 - K_1')^3} + \frac{(1 - \mu)(\gamma - 2)}{K_2'^2} \right], B = \frac{(V_p^2 - K_1')^2}{2V_p}.$$
(3.13)

The stationary solitary wave solution of equation (3.12) is

$$\phi^{(1)} = \phi_m sech^2 \frac{\xi}{\Delta} \quad , \tag{3.14}$$

where the special coordinate,  $\xi = \zeta - u_0 \tau$ , the amplitude ,  $\phi_m = 3u_0/A$ , and the width,  $\Delta = (4B/u_0)^{1/2}$ .

## 4. DERIVATION OF MK-DV EQUATION

To examine electrostatic perturbations propagating in the relativistic degenerate dense plasma due to the effect of dispersion by analyzing the outgoing solutions of equations (2.1-2.5), we now introduce the new set of stretched coordinates for the modified K-dV equation is:

$$\zeta = e(x - V_p t), \tag{4.1}$$

$$\tau = \epsilon^3 t. \tag{4.2}$$

To the lowest order in  $\epsilon$ , using equations (4.1,4.2, and 3.3-3.7), into the equations (2.1-2.5), we find the same results as we have had for the solitons for K-dV equation.

To the next higher order in  $\epsilon$ , we obtain a set of equations, which, after using the values of  $u_i^{(1)}$ ,  $n_i^{(1)}$ , and  $n_e^{(1)}$  can be simplified as

$$u_{i}^{(2)} = \frac{V_{p}\phi^{(2)}}{V_{p}^{2}-K_{1}^{'}} + \left[V_{p}K_{1}^{'} + \frac{V_{p}^{3}}{2} + \frac{K_{1}V_{p}(\alpha-2)}{2}\right] \frac{(\phi^{(1)})^{2}}{(V_{p}^{2}-K_{1}^{'})^{3}},$$
(4.3)

$$n_i^{(2)} = \frac{\phi^{(2)}}{v_p^2 - \kappa_1'} + [3V_p^2 + K_1'(\alpha - 2)] \frac{(\phi^{(1)})^2}{2(v_p^2 - \kappa_1')^3} , \qquad (4.4)$$

$$n_e^{(2)} = \frac{\phi^{(2)}}{\kappa_2} - \frac{(\gamma - 2)(\phi^{(1)})^2}{2(\kappa_2)^2} \quad , \tag{4.5}$$

$$\rho^{(2)} = \frac{1}{2} A(\phi^{(1)})^2 \tag{4.6}$$

where

$$A = \frac{(\gamma - 2)(1 - \mu)}{(K_2')^2} + \frac{3V_p^2 + K_2'(\alpha - 2)}{(V_p^2 - K_1')^3} , \qquad (4.7)$$

To further higher order of  $\epsilon$ , we obtain a set of equations

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$$\frac{\partial n_i^{(1)}}{\partial \tau} - V_p \frac{\partial n_i^{(3)}}{\partial \zeta} + \frac{\partial u_i^{(3)}}{\partial \zeta} + \frac{\partial}{\partial \zeta} \left[ u_i^{(2)} n_i^{(1)} + n_i^{(2)} u_i^{(1)} \right] = 0,$$
(4.8)

$$\frac{\partial u_i^{(1)}}{\partial \tau} - V_p \frac{\partial u_i^{(3)}}{\partial \zeta} + \frac{\partial}{\partial \zeta} \left[ u_i^{(1)} u_i^{(2)} \right] + \frac{\partial \phi^{(3)}}{\partial \zeta} + \frac{\partial \phi^{(3)}}{\partial \zeta}$$

$$K_{1}^{\prime} \frac{\partial}{\partial \zeta} \left[ n_{i}^{(3)} + (\alpha - 2) \left( n_{i}^{(1)} n_{i}^{(2)} \right) + \frac{(\alpha - 2)(\alpha - 3)}{6} \left( n_{i}^{(1)} \right)^{3} \right] = 0,$$

$$\frac{\partial \phi^{(3)}}{\partial \zeta} = \left( \frac{\partial}{\partial \zeta} - \frac{(\beta)}{2} \right) \left( \frac{(1)}{2} - \frac{(\gamma - 2)(\gamma - 3)}{6} \right) \left( \frac{(1)}{2} - \frac{(\gamma - 2)(\gamma - 3)}{6} \right) \left( \frac{(1)}{2} - \frac{(\gamma - 2)(\gamma - 3)}{6} \right) \left( \frac{(1)}{2} - \frac{(\gamma - 2)(\gamma - 3)}{6} \right) \left( \frac{(1)}{2} - \frac{(\gamma - 2)(\gamma - 3)}{6} \right) \left( \frac{(1)}{2} - \frac{(\gamma - 2)(\gamma - 3)}{6} \right) \left( \frac{(1)}{2} - \frac{(1)(\gamma - 3)}{6} \right) \left( \frac{(1)}{2} - \frac{(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)}{2} - \frac{(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \left( \frac{(1)(\gamma - 3)(\gamma - 3)}{6} - \frac{(1)(\gamma - 3)(\gamma - 3)}{6} \right) \right)$$

$$\frac{\partial \phi^{(3)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta} [n_e^{(3)} + (\gamma - 2) (n_e^{(1)} n_e^{(2)}) + \frac{(\gamma - 2)(\gamma - 3)}{6} (n_e^{(1)})^3] = 0,$$
(4.10)
$$\frac{\partial^2 \phi^{(1)}}{\partial \zeta} - \rho^{(3)}$$
(4.11)

$$\frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} = -\rho^{(3)}, \tag{4.11}$$

$$o^{(3)} = n_i^{(3)} - (1 - \mu)n_e^{(3)}.$$
(4.12)

Now combining equations (4.8) - (4.12) and using the values of  $n_i^{(1)}$ ,  $n_i^{(2)}$ ,  $u_i^{(1)}$ ,  $n_i^{(2)}$ , and  $\rho^{(2)}$ , we obtain an equation of the form

$$\frac{\partial \phi^{(1)}}{\partial \tau} + \beta \{\phi^{(1)}\}^2 \frac{\partial \phi^{(1)}}{\partial \zeta} + B \frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} = 0,$$
(4.13)

where the value of *B* is as before and  $\beta$  is given by

$$\beta = B\alpha \tag{4.14}$$

where  $\alpha$  is given by

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$$\alpha = \frac{15V_p^4 + 12K_1^{'}V_p^2 + 18K_1^{'}V_p^{2}(\alpha-2) + 3(K_1^{'}(\alpha-2))^2}{2(V_p^2 - K_1^{'})^3} + \frac{K_1^{'}(\alpha-2)(\alpha-3)}{2(V_p^2 - K_1^{'})^4} - \frac{(1-\mu)(6-7\gamma+2\gamma^2)}{2(K_2^{'})^3}$$
(4.15)

We call equation (4.13) as modified K-dV equation for planner geometry. The stationary solitary solution of equation (4.13) is given by

$$\phi^{(1)} = \phi_m \operatorname{sech}\left(\frac{\xi}{\Delta}\right),\tag{4.16}$$

where the special coordinate,  $\xi = \zeta - u_0 \tau$ , the amplitude ,  $\phi_m = \sqrt{\frac{6u_0}{\beta}}$ , the width,  $\Delta = \sqrt{\frac{1}{\gamma \phi_m}}$ ,  $\gamma = \frac{\alpha}{6}$  and  $u_0$  is the plasma species speed at equilibrium.



Fig. 1. Showing the effect of  $\mu$  on soliton (potential structure) obtained from eq.(3.14) for both electron-ion being non-relativistic degenerate when  $u_0$  is 0.1

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Fig. 2. Showing the effect of  $\mu$  on soliton (potential structure) obtained from eq.(3.14) for ion being non-relativistic degenerate and electron being ultrarelativistic degenerate when  $u_0$  is 0.1

## 5. NUMERICAL ANALYSIS

By the careful observation on the Fig. 1-4 it has become clear that the term  $\mu$  have an great effect on the potential,  $\phi^{(1)}$  of the K-dV and modified K-dV solitons. Because the potential,  $\phi^{(1)}$  increases more rapidly for ion being non-relativistic degenerate and electron being ultra-relativistic degenerate



Fig. 3. Showing the effect of  $\mu$  on soliton (potential structure) obtained from eq.(4.16) for both electron-ion being non-relativistic degenerate when  $u_0$  is 0.1

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## Fig. 4. Showing the effect of $\mu$ on soliton (potential structure) obtained from eq.(4.16) for ion being non-relativistic degenerate and electron being ultrarelativistic degenerate when $u_0$ is 0.1

than for both electron-ion being non-relativistic degenerate. Again in the same case (either ion being non-relativistic degenerate and electron being ultra-relativistic degenerate or electron-ion both being non-relativistic degenerate) the width,  $\Delta$  of the solitons obtained from the solutions of K-dV and modified K-dV equations (3.14) and (4.16) decreases sharply in all conditions whatever  $\mu$  increases with the term  $\xi$ . The most interesting point to note that the polarity of potential structure are different i.e. opposite for solitary waves and solitons. It is become more clear from observing the figures. The potential of solitary waves increases with the value of  $\mu$  with position  $\xi$  (figures 1 and 2). But the potential of solitons decreases with the value of  $\mu$  with position  $\xi$  (figures 3 and 4). It is the most important significance of this theoretical investigation.

## 6. DISCUSSION

To summarize, we have carried out solitons by deriving the K-dV and modified K-dV equations for a planar geometry in an unmagnetized plasma system containing degenerate electrons (non-relativistic or ultra relativistic limits) and degenerate ions being non-relativistic limit and the arbitrary charged dust grains. We have shown the existence of compressive (hump shape) DIA modified K-dV solitons. It can be noted here that rarefactive (dip shape) also may be occurs. We have identified the basic features of potential DIA solitons, which are found to exist beyond the K-dV limit. Generally the DIA modified K-dV solitons are completely different from the K-dV solitary waves. The plasma system under consideration supports finite potential modified K-dV solitons, whose basic features depend much on the degenerate pressure of ion and electron and the presence of arbitrary charged dust grains. It may be stressed here that the results of this investigation should be useful for understanding the nonlinear features of electrostatic disturbances in laboratory plasma conditions. Our investigation would also be useful to study the effects of degenerate pressure in interstellar and space plasmas [30], particularly in stellar polytropes [31], hadronic matter and quarkgluon plasma [32], protoneutron stars [33], darkmatter halos [34] etc. Further it can be said that the analysis of shock structures, vortices, double-layers etc. in a nonplanar geometry where the degenerate pressure can play the significant role, are also the problems of great importance but beyond the scope of the present work. To conclude, we propose to perform a laboratory experiment which can study such special new features of the DIA solitons propagating in dusty plasma in presence of degenerate electrons and ions.

#### 7. CONCLUSIONS

**a)** In the section (1), a brief discussion has been made about the validity of our this theoretical investigation.

**b)** In the section (2), we have represented the governing equation of our considered model which we have assumed theoretically.

**d)** In the section (3), we have derived the K-dV equation with the help of strong mathematical tools; reductive perturbation method.

e) In the section (4), we have derived the modified K-dV (mK-dV) equation with the help of strong mathematical tools; reductive perturbation method.

**f)** In the section (5), we have made a general analysis that what the results we have found from this investigation.

**g)** In the section (6), we have made some strong points in our favor to prove that our assumption for this model and this corresponding theoretical investigation are totally valid on the basic of the results.

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## **COMPETING INTERESTS**

Authors have declared that no competing interests exist.

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