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Stability Analysis of Chain, Mild and Passive Smoking Model

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Abstract

In this paper, the global stability of free smoking equilibrium point was evaluated and presented graphically. The linear stability of a developed mathematical model illustrates the effect on the population of chain, mild and passive smokers. MATLAB programming was used to simulate the solutions, the reproduction number R_0 and the nature of the equilibria.

Keywords

Smoking Model, Stability Analysis, Global Stability and Lyapunov Method, Qualitative Behavior, Passive Smokers

1. Introduction

Castillo *et al.* [1] proposed a simple mathematical model for giving up smoking. They assumed that the total population consists of three populations: potential smokers, *i.e.* people who do not smoke yet and may become smokers in the future (P), smokers (S) and people who have already quit smoking permanently (Q). Since then, many articles were published further developing this model. The modifications were produced either by adding a factor or a new population (see e.g. [2] [3] [4] [5]). However, in 2008, Sharomi and Gumel [6] modified the model by including a class of smokers who temporarily quit smoking. They studied the global stability of smoker-free equilibrium (SFE). In 2011, Zaman [7] derived the giving-up smoking model taking into account the occasional smoking population and presented its qualitative behavior. In 2014, Z. Alkhudhari *et al.* [8] modified the model by assuming that the smoking classes are divided into occasional and chain smokers and they studied the existence and stability of equilibrium points. In 2017, Matintu [9] proposed a new modification by considering the moderate, chain smokers, potential smokers, and temporarily quit

smokers. In 2019, Ana *et al.* [10] constructed a new smoking model where the total population was partitioned into three classes: passive smoker E_1 (those at risk from others' smoking), infected I (people who are addicted to tobacco and can pass on the habit) and E_2 (people who have stopped smoking but are at risk of relapse) described as:

$$\dot{E_1} = \Delta - \frac{\beta}{N} E_1 I - (\mu + v_1) E_1, \tag{1a}$$

$$\dot{I} = \frac{\beta}{N} E_1 I - (\mu + \epsilon + \theta) I, \tag{1b}$$

$$\dot{E_2} = \theta I - \frac{\beta}{N} E_2 I - (\mu + v_2 + \omega) I, \tag{1c}$$

where $E_1 = E_1(t)$, $I = I(t)$ and $E_2 = E_2(t)$. The parameters $(\mu, v_1, v_2, \omega, \beta, \epsilon)$ and θ represent respectively the rate of natural death, exit rate from E_1 to healthy population, exit rate form E_2 to healthy population, death rate of ex-smoker, infection rate of smoking, death rate of smokers and exit rate from I to E_2. Δ is the average number of healthy people who are at risk of becoming active smokers.

Our paper is divided into: section (2), evaluating the global stability of the model in [10]. The modification of the model was stated in section (3) to include two classes of smokers: chain smokers S_1 and mild smokers S_2, with the full study of stability analysis and effect of each group on the population behavior (qualitative behavior).

2. The Global Stability of Equilibria

The equilibria of smoking-free equilibrium point in system (1a)-(1b), are given in [10] as

$$P_1 = (E_{1,0}, I_0, E_{2,0}) = \left(\frac{\Delta}{\mu + v_2}, 0, 0\right), \tag{2}$$

and the basic reproduction number $\mathcal{R}_0$ is defined as $\mathcal{R}_0 = \dfrac{\beta}{\mu + \epsilon + \theta}$.

Theorem 1.
If $\beta \leq \delta_4$ and $\mathcal{R}_0 \leq 1$, then P_1 is globally asymptotically stable.
Proof.
The Lyapunov function is defined as

$$V_1 = E_{1,0}\left(\frac{E_1}{E_{1,0}} - 1 - \ln\frac{E_1}{E_{1,0}}\right) + E_2 + I,$$

hence,

$$\dot{V_1} = \left(1 - \frac{E_{1,0}}{E_1}\right) + \dot{I} + \dot{E_2}, \tag{3}$$

By substituting (1a)-(1b) in (3),

$$\dot{V}_1 = \left(1 - \frac{E_{1,0}}{E_1}\right)\left(\Delta - \frac{\beta}{N}E_1 I - (\mu + v_1)E_1\right) + \frac{\beta}{N}I(E_1 + E_2)$$

$$-(\mu + \epsilon + \theta)I + \theta I - \frac{\beta}{N}E_2 I - (\mu + v_2 + \omega)E_2,$$

$$\dot{V}_1 = \left(1 - \frac{E_{1,0}}{E_1}\right)\left(\Delta - (\mu + v_1)E_1\right) - \frac{\beta}{N}E_1 I + \frac{\beta}{N}E_{0,1}I + \frac{\beta}{N}I(E_1 + E_2)$$

$$-(\mu + \epsilon + \theta)I + \theta I - \frac{\beta}{N}E_2 I - (\mu + v_2 + \omega)E_2,$$

Since $E_{1,0} = N$, then we have

$$\dot{V}_1 = \left(1 - \frac{E_{1,0}}{E_1}\right)\left(\Delta - \delta_1 E_1\right) - \delta_2 E_2 + \left(\beta - (\mu + \epsilon)\right)I.$$

where $\delta_1 = \mu + v_1$ and $\delta_2 = \mu + v_2 + \omega$. Using (2) we get

$$\dot{V}_1 = -\delta_1 \frac{\left(E_{1,0} - E_1\right)^2}{E_1} - \delta_2 E_2 + \left(\beta - (\mu + \epsilon)\right)I.$$

By re-arranging the equation, we get

$$\dot{V}_1 = -\delta_1 \frac{\left(E_{1,0} - E_1\right)^2}{E_1} - \delta_2 \mathcal{R}_0 E_2 + \frac{\delta_2}{\delta_3}\left(\delta_3\left(\mathcal{R}_0 - 1\right)\right)E_2 + \left(\beta - \delta_4\right)I, \tag{4}$$

where $\delta_3 = \mu + \epsilon + \theta$ and $\delta_4 = \mu + \epsilon$.

It is clear that $\dot{V}_1 \leq 0$ when $\beta \leq \delta_4$ and $\mathcal{R}_0 \leq 1$ for all $E_1, I, E_2 > 0$. Hence, the solutions of systems (1) is limited to Ω, the largest invariant subset of $\dot{V}_1 = 0$, where Ω is the region of solutions given in [10]. From (4), we see that $\dot{V}_1 = 0$ if and only if $E_1 = E_{1,0}$ and $I = E_2 = 0$. By using LaSalle's invariance principle, the point P_1 is globally asymptotically stable.

□

3. The Modified Model of Smoking

In this section, system (1) was modified to include two classes of smokers, chain smokers S_1 and mild smoker S_2 (see **Figure 1**). Accordingly, our model of ordinary differential equations (ODEs) are
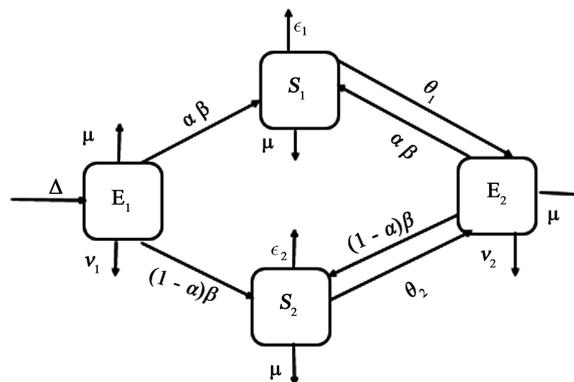

Figure 1. Graphic presentation of system (6).

$$\dot{E}_1(t) = \Delta - \frac{\beta}{N} E_1(t) \big(\eta S_1(t) + S_2(t)\big) - (\mu + v_1) E_1(t), \quad (5a)$$

$$\dot{S}_1(t) = a \frac{\beta}{N} \big(\eta S_1(t) + S_2(t)\big) \big(E_1(t) + E_2(t)\big) - (\mu + \epsilon_1 + \theta_1) S_1(t), \quad (5b)$$

$$\dot{S}_2(t) = (1-a)\frac{\beta}{N} \big(\eta S_1(t) + S_2(t)\big) \big(E_1(t) + E_2(t)\big) - (\mu + \epsilon_2 + \theta_2) S_2(t), \quad (5c)$$

$$\dot{E}_2(t) = \theta_1 S_1(t) + \theta_2 S_2(t) - \frac{\beta}{N} E_2(t) \big(\eta S_1(t) + S_2(t)\big) - (\mu + v_2 + \omega) E_2(t), \quad (5d)$$

where the parameters β, ω, v_1, v_2 and μ are defined as in the system (1a)-(1c), ϵ_1 and ϵ_2 are death rate of chain smokers S_1 and mild smokers S_2 respectively, while parameters θ_1 and θ_2 are exit rates from chain smokers and mild smokers to the healthy population (outside population N). We assume that the exposed people become either a chain or mild smoker at probabilities $(1 - a)$ and a with $0 < a < 1$). The chain smokers have a higher probability of generating more new smokers (by a factor $\eta \geq 1$ relative to the mild smokers). A simplified form of the model (5) can be written as

$$\dot{E}_1(t) = \Delta - \frac{\beta}{N} E_1(t) \big(\eta S_1(t) + S_2(t)\big) - d_1 E_1(t), \quad (6a)$$

$$\dot{S}_1(t) = a \frac{\beta}{N} \big(\eta S_1(t) + S_2(t)\big) \big(E_1(t) + E_2(t)\big) - d_2 S_1(t), \quad (6b)$$

$$\dot{S}_2(t) = (1-a)\frac{\beta}{N} \big(\eta S_1(t) + S_2(t)\big) \big(E_1(t) + E_2(t)\big) - d_3 S_2(t), \quad (6c)$$

$$\dot{E}_2(t) = \theta_1 S_1(t) + \theta_2 S_2(t) - \frac{\beta}{N} E_2(t) \big(\eta S_1(t) + S_2(t)\big) - d_4 E_2(t), \quad (6d)$$

where $d_1 = \mu + v_1$, $d_2 = (\mu + \epsilon_1 + \theta_1)$, $d_3 = (\mu + \epsilon_2 + \theta_2)$ and $d_4 = (\mu + v_2 + \omega)$. The initial conditions of system (6), are given by

$$E_1(\alpha) = \varphi_1(\alpha), \quad S_1(\alpha) = \varphi_2(\alpha), \quad S_2(\alpha) = \varphi_3(\alpha), \quad E_2(\alpha) = \varphi_4(\alpha)$$

where $\alpha \in R$, $\big(\varphi_1(\alpha), \varphi_2(\alpha), \varphi_3(\alpha), \varphi_4(\alpha)\big) \in CR_+^4$ and C is the Banach space of continuous functions mapping the interval $(-\infty, 0]$ into R_+^4. By the fundamental theory of functional differential equations [11], system (6) has a unique solution satisfying the initial conditions above.

3.1. Non-Negativity and Boundedness of Solutions

Theorem 2. For system (6), there exist a positive number K such that the compact set Λ,

$$\Lambda = \big\{(E_1, S_1, S_2, E_2) \in R_+^4 : 0 \leq E_1, S_1, S_2, E_2 \leq K\big\}.$$

is positively invariant.

Proof. We have

$$\dot{E}_1(t)\big|_{E_1=0} = \Delta > 0,$$

$$\dot{S}_1(t)\big|_{S_1=0} = a\frac{\beta}{N} S_2(E_1 + E_2) > 0 \text{ for } S_2, E_1 \text{ and } E_2 > 0,$$

$$\dot{S}_2(t)\big|_{S_2=0} = (1-a)\frac{\beta}{N} \eta S_1(E_1 + E_2) > 0 \text{ for } S_1, E_1 \text{ and } E_2 > 0,$$

$$\dot{E}_2\left(t\right)\big|_{E_2=0} = \theta_1 S_1 + \theta_2 S_2 > 0 \quad \text{for} \quad S_1 \quad \text{and} \quad S_2 > 0,$$

so the solutions are non-negative. Now, to prove the boundedness of the solutions, we let $N(t) = E_1(t) + S_1(t) + S_2(t) + E_2(t)$, hence

$$\dot{N} = \Delta - \frac{\beta}{N}E_1\left(\eta S_1 + S_2\right) - d_1 E_1 + a\frac{\beta}{N}\left(\eta S_1 + S_2\right)\left(E_1 + E_2\right) - d_2 S_1 - d_3 S_2$$

$$+ (1-a)\frac{\beta}{N}\left(\eta S_1 + S_2\right)\left(E_1 + E_2\right) + \theta_1 S_1 + \theta_2 S_2 - \frac{\beta}{N}E_2\left(\eta S_1 S_2\right) - d_4 E_2$$

$$\leq \Delta - d_1 E_1 - d_5 S_1 - d_6 S_2 - d_4 E_2 \leq \Delta - \bar{d}N(t),$$

where $d_5 = \mu + \epsilon_1$, $d_6 = \mu + \epsilon_2$ and $\bar{d} = \min\{d_i\}$, $i = 1,4,5,6$. That is, if $N(0) \leq 0$ then $N(t) < K$ where $K = \Delta/\bar{d}$. It follows that if $E_1(0) + S_1(0) + S_2(0) + E_2(0) \leq K$ then $E_1(t), S_1(t), S_2(t), E_2(t) \leq K$.

$\square$

3.2. The Equilibria and Basic Reproduction Number $\mathcal{R}_0$

Theorem 3. 1) If $\mathcal{R}_0 \leq 1$, then there exists only one equilibrium point called smoking-free equilibrium P_0.

2) If $\mathcal{R}_0 > 1$, there exist two equilibria that are smoking-free equilibrium P_0 and smoking present equilibrium P_1.

Proof.

The system (6) has two equilibrium points which are: Smoking-free equilibrium point

$$P_0 = \left(E_{1,0}, S_{1,0}, S_{2,0}, E_{2,0}\right) = \left(\frac{\Delta}{d_1}, 0, 0, 0\right), \tag{7}$$

and smoking present equilibrium point

$$P_1 = \left(E_1^*, S_1^*, S_2^*, E_2^*\right)$$

where

$$E_1^* = \frac{\mathcal{R}_0 \Delta}{\left(\mathcal{R}_0 - 1\right)\beta + \mathcal{R}_0\left(\mu + v_1\right)},$$

$$S_1^* = a\frac{\beta\left(\mathcal{R}_0 - 1\right)}{\mathcal{R}_0 d_2}\left(x_1 + E_2^*\right),$$

$$S_2^* = (1-a)\frac{\beta\left(\mathcal{R}_0 - 1\right)}{\mathcal{R}_0 d_3}\left(x_1 + E_2^*\right),$$

$$E_2^* = \frac{\Delta\beta\left(\mathcal{R}_0 - 1\right)\left[(1-a)\theta_2 d_2 + a\theta_1 d_3\right]}{d_2 d_3 x_2\left[\beta\left(\mathcal{R}_0 - 1\right) + \mathcal{R}_0 d_1\right]},$$

and

$$x_1 = \left(\frac{\Delta\mathcal{R}_0}{\mathcal{R}_0 d_1 + \beta\left(\mathcal{R}_0 - 1\right)}\right),$$

$$x_2 = d_4 + \left(\frac{\beta\left(\mathcal{R}_0 - 1\right)}{\mathcal{R}_0 d_2 d_3}\right)\left(d_2 d_6\left(1-a\right) + a d_3 d_5\right).$$

Using the next generation method [12], the matrices F and V are

$$F = \begin{pmatrix} 0 & -\eta\beta & -\beta & 0 \\ 0 & a\eta\beta & a\beta & 0 \\ 0 & (1-a)\eta\beta & (1-a)\beta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, V = \begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{pmatrix},$$

and hence evaluate the reproduction number $\mathcal{R}_0$ which is the largest eigenvalue of $F(V)^{-1}$,

$$\mathcal{R}_0 = \beta \frac{(1-a)d_2 + a\eta d_3}{d_2 d_3}. \tag{8}$$

$\square$

3.3. Local Stability of Smoker Free Equilibrium

Theorem 4.

If $\mathcal{R}_0 < 1$, then P_0 is locally asymptotically stable.

Proof. The Jacobian matrix of system (6) at P_0 is

$$J(P_0) = \begin{pmatrix} -d_1 & -\eta\beta & \beta & 0 \\ 0 & \eta a\beta - d_2 & a\beta & 0 \\ 0 & (1-a)\beta\eta & (1-a)\beta - d_3 & 0 \\ 0 & \theta_1 & \theta_2 & -d_4 \end{pmatrix},$$

with the eigenvalues $\lambda_1 = -d_1$, $\lambda_2 = -d_4$, and $\lambda_{3,4} = c \pm b$, where

$$c = \left(\frac{1}{2} \left[\beta(1-a) - d_3 \right] + \left[\beta a\eta - d_2 \right] \right),$$

and

$$b = \frac{1}{2} \sqrt{2(\mathcal{R}_0 - 1) + (d_2 - a\beta\eta)^2 + (d_3 - (1-a)\beta)^2 + 2a\eta\beta(\beta(1-a))}.$$

It is easy to show that $c < 0$ when $\mathcal{R}_0 < 1$. To study the nature of b, we start with the fact that,

$$\beta((1-a)d_2 + a\eta d_3) < d_2 d_3,$$

hence,

$$\beta(1-a)d_2 < d_2 d_3, \text{ and } \beta a\eta d_3 < d_2 d_3,$$

$$\beta(1-a) < d_3, \text{ and } \beta a\eta < d_2.$$

Next, we use the numerical approach by assuming two small positive real numbers ξ_1 and ξ_2 such that $\beta\eta a + \xi_1 = d_2$ and $\beta(1-a) + \xi_2 = d_3$ hence

$$b = \frac{1}{2} \sqrt{2(\mathcal{R}_0 - 1) + \xi_1^2 + \xi_2^2 + 2(d_2 - \xi_1)(d_3 - \xi_2)}$$

$$= \frac{\sqrt{2}}{2} \sqrt{(\mathcal{R}_0 - 1) + \frac{1}{2}\xi_1^2 + \frac{1}{2}\xi_2^2 + d_2 d_3 - \xi_2 d_2 - \xi_1 d_3 + \xi_1\xi_2}$$

$$= \frac{\sqrt{2}}{2} \sqrt{(\mathcal{R}_0 - 1) + \frac{1}{2}(\xi_1 + \xi_2)^2 + d_2 d_3 - \xi_2 d_2 - \xi_1 d_3}.$$

Since $\frac{1}{2}(\xi_1 + \xi_2)^2 \cong 0$ and $d_2 d_3 < \xi_1 d_2 + \xi_2 d_3$ then $d_2 d_3 < d_2 + d_3$ and

$$(\mathcal{R}_0 - 1) + \frac{1}{2}(\xi_1 + \xi_2)^2 + d_2 d_3 - \xi_2 d_2 - \xi_1 d_3 < 0.$$

Thus $\lambda_{3,4}$ are complex numbers. Finally, we see that all the real parts of $\lambda_i, i = 1, 2, 3, 4$ are negative so the point P_0 is locally asymptotically stable.

□

3.4. The Global Stability of Smoker Free Equilibrium

Theorem 5. If $\beta\eta \le d_5, \beta \le d_6$ and $\mathcal{R}_0 \le 1$, then P_0 is globally asymptotically stable.

Proof. As done in Theorem (1), we defined the Lyapunov function as

$$V_2 = E_{1,0}\left(\frac{E_1}{E_{1,0}} - 1 - \ln\frac{E_1}{E_{1,0}}\right) + S_1 + S_2 + E_2,$$

hence

$$\dot{V}_2 = \left(1 - \frac{E_{1,0}}{E_1}\right)\dot{E}_1 + \dot{S}_1 + \dot{S}_2 + \dot{E}_2. \tag{9}$$

By substituting (6) in (9),

$$\dot{V}_2 = \left(1 - \frac{E_{1,0}}{E_1}\right)\left(\Delta - \frac{\beta}{N}E_1(\eta S_1 + S_2) - d_1 E_1\right) + \frac{a\beta}{N}(\eta S_1 + S_2)(E_1 + E_2) - d_2 S_1$$

$$+ \frac{(1-a)\beta}{N}(\eta S_1 + S_2)(E_1 + E_2) - d_3 S_2 + \theta_1 S_1 + \theta_2 S_2 - \frac{\beta}{N}E_2(\eta S_1 + S_2) - d_4 E_2$$

$$= \left(1 - \frac{E_{1,0}}{E_1}\right)(\Delta - d_1 E_1) + \frac{\beta}{N}E_{1,0}(\eta S_1 + S_2) - d_2 S_1 - d_3 S_2 + \theta_1 S_1 + \theta_2 S_2 - d_4 E_2$$

By using the equilibrium conditions $\Delta = d_1 E_{1,0}$ and $E_{1,0} = N$,

$$\dot{V}_2 = -d_1\left(\frac{(E_1 - E_{1,0})^2}{E_1}\right) + (\beta\eta - d_5)S_1 + (\beta - d_6)S_2 - d_4 E_2$$

$$= -d_1\left(\frac{(E_1 - E_{1,0})^2}{E_1}\right) + (\beta\eta - d_5)S_1 + (\beta - d_6)S_2 \tag{10}$$

$$+ \frac{d_4}{c_1}(c_1[\mathcal{R}_0 - 1])E_2 - \mathcal{R}_0 d_4 E_2,$$

where $c_1 = d_2 d_3$ and $c_2 = \beta(1-a)d_2 + a\eta d_3$. Clearly $\dot{V}_2 \le 0$, if $\beta\eta \le d_5$, $\beta \le d_6$ and $\mathcal{R}_0 \le 1$, for all $E_1, S_1, S_2, E_2 > 0$. Hence, the solutions of system (6) are limited to ω, the largest invariant subset of $\dot{V}_2 = 0$. From (10) we see that $\dot{V}_2 = 0$ if and only if $E_1 = E_{1,0}$ and $S_1 = S_2 = E_2 = 0$. Using LaSalle's invariance principle, we show that P_0 is globally asymptotically stable.

□

3.5. Numerical Simulations and Discussion

Using MATLAB programming, the solution of system (6) is evaluated by as-

suming values for the parameters as $\Delta = 50$, $\epsilon_1 = 0.03$, $\epsilon_2 = 0.02$, $\theta_1 = 0.4$, $\theta_2 = 0.5$, $\mu = 0.05$, $v_1 = 0.2$, $v_2 = 0.3$, $N = 200$, and $\omega = 0.001$ with two sets of values for β, η and a.

Set (1): $\mathcal{R}_0 \leq 1$, $\beta = 0.05$, $\eta = 1.5$, and $a = 0.4$.

Set (2): $\mathcal{R}_0 > 1$, $\beta = 0.5$, $\eta = 2.5$, and $a = 0.6$.

A three sets of values are applied and tested for the initial conditions:

IC1: $E_1(0) = 890, S_1(0) = 120, S_2(0) = 90, E_2(0) = 40$.

IC2: $E_1(0) = 480, S_1(0) = 60, S_2(0) = 40, E_2(0) = 30$.

IC3: $E_1(0) = 290, S_1(0) = 30, S_2(0) = 20, E_2(0) = 10$.

In **Figures 2-5**, the behavior of each smoking population is plotted as time increases. **Figure 2** shows the fast decrease of passive smokers for both $\mathcal{R}_0 \leq 1$ and $\mathcal{R}_0 > 1$ within a short period of time and reaches or approaches the steady
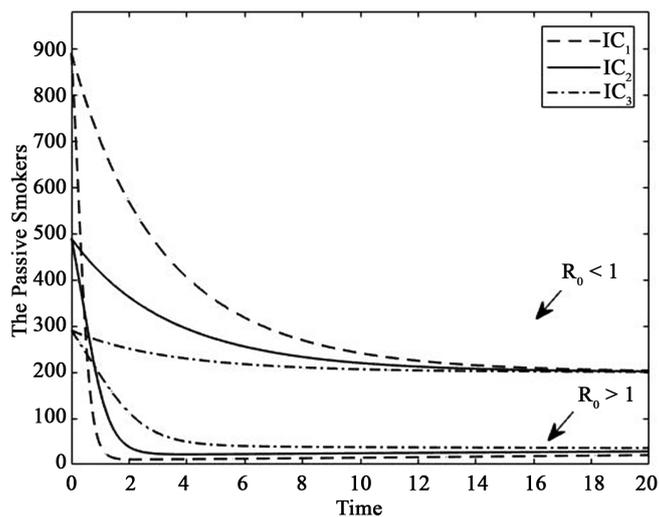

Figure 2. The plot of the passive smokers (E_1), for set (1), $\mathcal{R}_0 < 1$ and set (2), $\mathcal{R}_0 > 1$ together with IC1 (dash), IC2 (solid) and IC3 (dash dot).
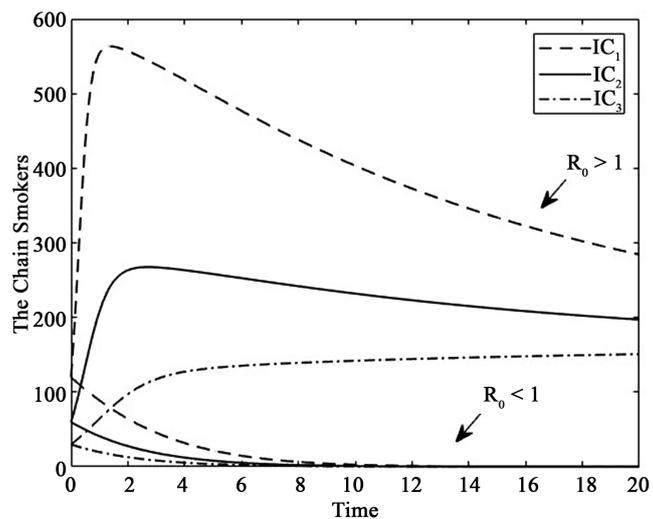

Figure 3. The plot of the chain smokers (S_1), for set (1), $\mathcal{R}_0 < 1$ and set (2), $\mathcal{R}_0 > 1$ together with IC1 (dash), IC2 (solid) and IC3 (dash dot).
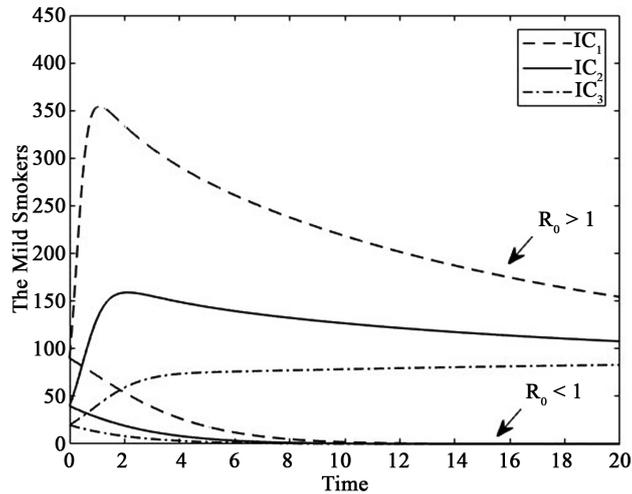
Figure 4. The plot of the mild smokers (S_2), for set (1), $\mathcal{R}_0 < 1$ and set (2), $\mathcal{R}_0 > 1$ together with IC1 (dash), IC2 (solid) and IC3 (dash dot).
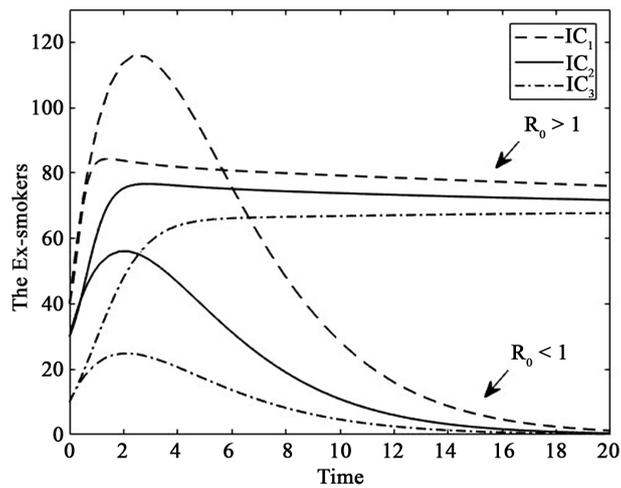

Figure 5. The plot of the ex-smokers (E_2), for set (1), $\mathcal{R}_0 < 1$ and set (2), $\mathcal{R}_0 > 1$ together with IC1 (dash), IC2 (solid) and IC3 (dash dot).

state. Similar behavior was observed on the chain and mild populations for $\mathcal{R}_0 \leq 1$, **Figure 3** and **Figure 4** which reached zero as time increased. For $\mathcal{R}_0 > 1$, a peak appears at the beginning of the time and decreases to the steady state as time increases. This behavior was seen on the ex-smokers' population for both $\mathcal{R}_0 \leq 1$ and $\mathcal{R}_0 > 1$. The choice of initial values of the three sets has no effect on the behaviour of the different population types. It tends to zero for the chain, mild and ex-smokers for $\mathcal{R}_0 < 1$ as time increases. E_2 behaves similarly for any initial conditions, where we observe that E_2 tends to zero ($\mathcal{R}_0 \leq 1$) or steady state value ($\mathcal{R}_0 > 1$). Similarly for other types of population which means that P_0 and P_1 are globally asymptotically stable. **Figure 6** shows the phase plane space plots that explain the relationships between the different populations. These relationships are the same for different initial conditions. Next, we consider the relationships of ex-smokers' population versus the chain/mild smokers
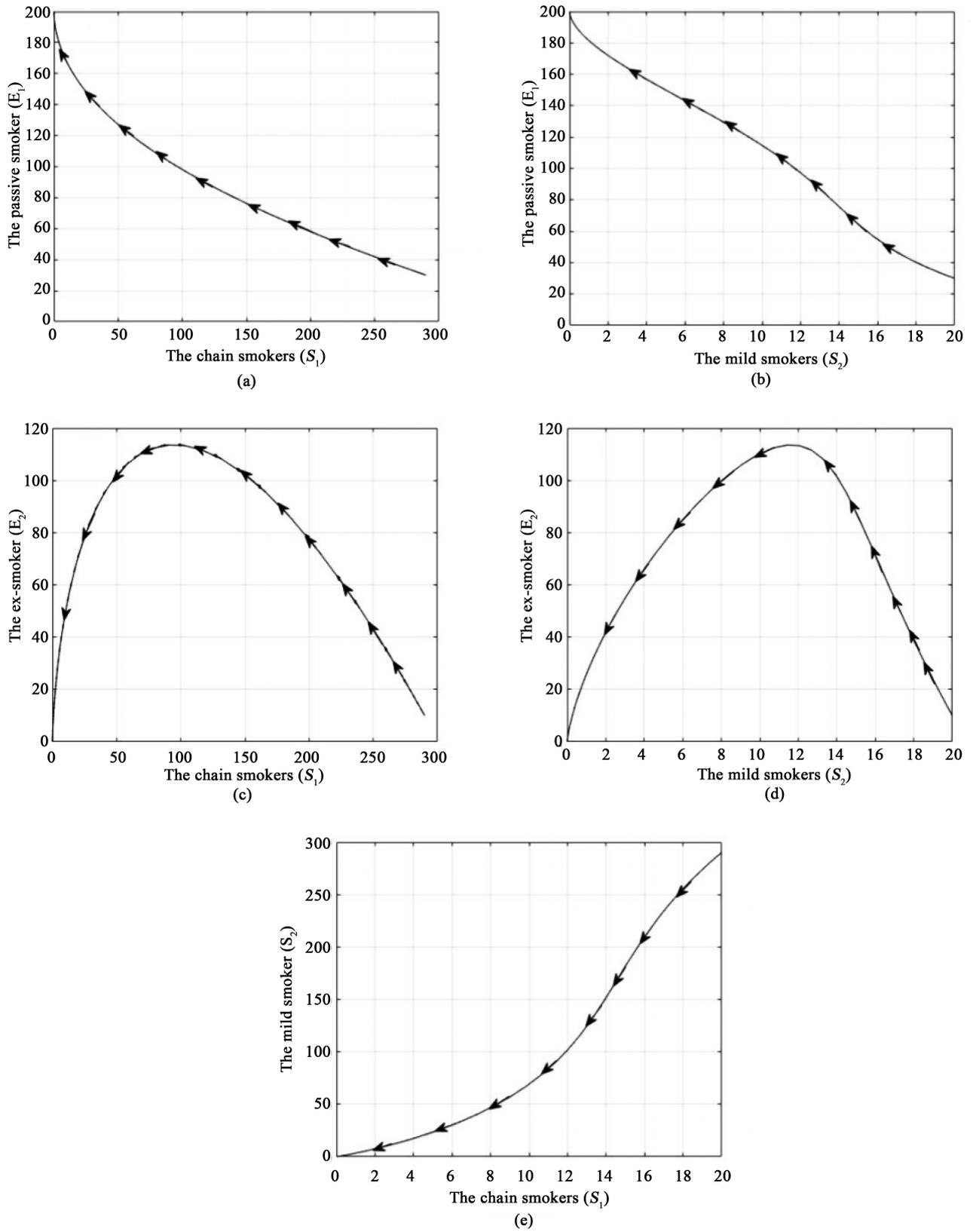
Figure 6. The phase plane plot of the populations of system (6) using the values of the parameters of set (1) with the initial conditions $E_1(0) = 30$, $S_1(0) = 290$, $S_2(0) = 20$ and $E_2(0) = 10$.

S_1 and S_2. As the chain smokers' number decreases, we notice an increase on the population of ex-smokers (E_2) up to a maximum value at around one third of chain smokers' initial values and around two-thirds of the mild smokers' initial value, Figure 5(c) and Figure 5(d). Finally, Figure 5(e) illustrates the impact of decreasing the number of chain smokers which leads to a decrease in the number of mild smokers.

As we observe from the Figures, we stress the importance of decreasing the number of chain smokers (either by using the media or educational direct seminars) on increasing the number of passive smokers (as the ex-smokers and mild are both joined) to create a healthy action.

Finally, we observe that when the contact between the two classes of smokers with the other populations is weak (small β) then the effect of the smoker's classes (chain/mild) was not being enough to generate the new individuals to the smokers' classes. So, the number of smoker's groups decreased and approached to zero (because the death or stop smoking). On the other hand, if β increase, we found that the number of smoker's groups increased also, since the rate of interaction between them and other groups (or the effect by smokers) was growing. However, we can see that, one of the solutions which is introduced to reduce the spread of smoking is prevent it in the larger places (such as educations) in order to minimize the rate of contact to be very small or zero.

4. Conclusion

The modified system/model shows interesting behavior between the different types of smoker's populations. The main advantage was the more decrease in the size of heavy/mild smokers in comparison with earlier studies. This should encourage others (ex- or passive) smokers to interact better with chain/mild smokers to increase the number of quitters. By comparing our results with the results in [10], we see that they are approximately similar to giving the behavior of each group, and the difference is that, our model explains and shows the effect of both of the chain smokers and mild smokers separately on the exposed population. Hence, we get that the chain smokers affect more than the mild smokers on the exposed groups. So, our model is more real.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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